Poisson-Laguerre and E-Kautz Models

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Abstract: - New models, called Poisson-Laguerre and E-Kautz models, are obtained by a re-parameterization of the so called Laguerre and Kautz series. The truncation error is conserved. The quantification of this error is possible by using the orthogonality property of Laguerre and Kautz functions. The state space representations are also given and compared. The particular structure of these new models is more suitable in computation, it allows particularly an analytic computation of the corresponding zero order hold discrete-time state space representation; that of Laguerre and Kautz models are also deduced.

Key-words: Poisson pulse functions; Laguerre functions; Kautz functions; Identification; continuous-time systems.

1 Introduction

Several papers have used Laguerre and Kautz series to approximate complex systems, namely infinite dimensional, time-delay and resonant systems [2], [5] and [8]-[11]. They have been found useful in the model order reduction. The property of being orthogonal allows the quantification of the truncation error.

Laguerre models give an excellent low-order approximation of a well-damped system, [9] and [8]. The Kautz functions are useful to cope with resonant systems [8] and [10]-[12].

Essentially the discrete-time version of these models has been used in system identification. One of our motivations in this work is to be able to identify the parameters of a continuous-time model in online way.

The use of Laguerre models for this objective has been proposed in [3]. It is based on the discretization of the state space representation, obtained from the expansion in Laguerre series of the system transfer function. The trapezoidal approximation has been used to compute the matrices of the zero order hold discrete-time representation. In this paper, an analytic value of these matrices is computed for the new models, called Poisson-Laguerre and E-Kautz models, thanks to their particular form. The corresponding matrices of Laguerre and Kautz models are deduced.

Markov models have also been used in system identification of continuous-time models in [7]. The main problems encountered with this method are the high approximation order of the model which induces a large number of parameters to identify and the numerical sensitivity problems of the identification algorithms due to the fact that the data vector contains the input signals and their integrals. To deal with this problems, the authors have modified, in an empirical way, the Markov operators $\{1/s^i\}$ as $\{[\beta/(s+\lambda)]^i\}$, where s is the Laplace operator, β(real) > 0 and λ(real) > 0. In this paper, we show that the model based on this latter operators can be obtained by carrying out a transformation on the Laguerre model. This new model have the advantage of preserving the same truncation error as Laguerre series.

The same remarks remain valid in the case of Kautz and E-Kautz models.

The parameters of Poisson-Laguerre and E-Kautz continuous-time models can be identified in an online way via their zero order hold discrete-time state space representation. This makes their use in adaptive control of continuous-time systems attractive

The paper is organised as follows: in Section 2, the Poisson-Laguerre models and their continuous-time and zero order hold discrete-time state space representations are presented. The Section 3 deals with the E-Kautz models. The concluding remarks are given in Section 4.

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2 Poisson-Laguerre Models

2.1 Laguerre functions

The orthonormal Laguerre functions, with respect to the standard time domain L_2 inner product, are given by the following equation

$$l_k(t) = \sqrt{2\lambda}e^{-\lambda t}\frac{e^{2\lambda t}}{k!}\frac{d^k}{dt^k}[(2\lambda t)^k e^{-2\lambda t}],\ k = 0,1,2,... \quad (1)$$

where λ (real) > 0.

The Laplace transform of $\{l_k(t)\}$ are orthonormal rational functions $\{L_k(s)\}$ given by

$$L_k(s) = \frac{\sqrt{2\lambda}}{(s+\lambda)}\left[\frac{s-\lambda}{s+\lambda}\right]^{k-1},\ k = 1,2,\ldots \quad (2)$$

2.2 Poisson pulse functions [6]

The classical Poisson pulse functions are defined as

$$p_k(t) = \frac{t^k e^{-\lambda t}}{k!},\ k = 0,1,2,... \quad (3)$$

where λ(real) > 0.

The Laplace transform of $p_k(t)$ is given by

$$\pi_k(s) = \frac{1}{(s+\lambda)^k} \quad (4)$$

2.3 Poisson-Laguerre series

A stable transfer function G(s) can be developed as a Laguerre series [9]

$$G(s) = \sum_{k=1}^{+\infty}\alpha_k L_k(s) \quad (5)$$

where $\{\alpha_k\}$ are the Laguerre coefficients.

The k^{th} order Laguerre function $L_k(s)$ can be decomposed in terms of $\{\pi_i(s),\ i = 1, ...,k\}$ functions as

$$L_k(s) = \sqrt{2\lambda}\sum_{i=1}^{k}(-2\lambda)^{i-1}\binom{k}{i-1}\pi_i(s) \quad (6)$$

with $\binom{q}{p} = \frac{q!}{p!(q-p)!}$

The substitution of (6) in (5) gives

$$G(s) = \sum_{k=1}^{\infty} g_k \pi_k(s) \quad (7)$$

Where $$g_k = \sqrt{2\lambda}(-2\lambda)^{k-1}\sum_{i=k}^{+\infty}\binom{i}{k-1}\alpha_i \quad (8)$$

Therefore, truncate the series (5) at a certain order n amounts to truncate (7) and (8) at the same order n. This means also that these two series, (5) and (7), have the same truncation error. The relationship between the Laguerre coefficients $\{\alpha_k\}$ and those of the series (7), $\{g_k\}$, is given by

$$g = \Gamma(\lambda)\alpha \quad (9)$$

where

$$g = [g_1 \quad g_2 \quad \cdots \quad g_n]^T,\ \alpha = [\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_n]^T \quad (10)$$

$$\Gamma(\lambda) = \sqrt{2\lambda}\begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ 0 & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma_{nn} \end{bmatrix} \quad (11)$$

and $$\gamma_{ij} = (-2\lambda)^{i-1}\binom{j-1}{i-1} \quad (12)$$

The non-orthogonal series (7) is called Poisson-Laguerre series. To approximate a stable transfer function by one of this series, they should be truncated at a certain approximation order n. Low values of n are obtained when λ is chosen close to the dominating pole of the system [8]. Some methods to compute an optimum λ can be found in [1] and [4].

When λ is chosen close to zero, the Poisson-Laguerre series becomes close to Markov series [7].

2.4 State space representations

Let $$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = C^T x(t) \end{cases} \quad (13)$$

be the single input single output state space representation of the system described by truncating the series (5) and (7), where $x^T(t) = [x_1(t) \quad \cdots \quad x_n(t)]$, y(t) and u(t) are respectively the state vector, the output and the input signals.

The parameters of (13) with respect to each expansion are given in table1.

	Laguerre	Poisson-Laguerre
A	$\begin{bmatrix} -\lambda & 0 & \cdots & & 0 \\ -2\lambda & -\lambda & \ddots & & \vdots \\ \vdots & \ddots & \ddots & & \\ & & & & 0 \\ -2\lambda & \cdots & & -2\lambda & -\lambda \end{bmatrix}$	$\begin{bmatrix} -\lambda & 0 & \cdots & & 0 \\ 1 & -\lambda & \ddots & & \vdots \\ 0 & \ddots & \ddots & & \\ \vdots & \ddots & & & 0 \\ 0 & \cdots & 0 & 1 & -\lambda \end{bmatrix}$
B^T	$[\sqrt{2\lambda} \quad 0 \quad \cdots \quad 0]$	$[1 \quad 0 \quad \cdots \quad 0]$
C	α	g
x(t)	$x_i^L(t) = \frac{\sqrt{2\lambda}}{(s+\lambda)}\left[\frac{s-\lambda}{s+\lambda}\right]^{i-1} u(t),\ i = 1,2,...,n$	$x_i^{PL}(t) = \frac{u(t)}{(s+\lambda)^i},\ i = 1,2,...,n$

Table 1: Comparison of Laguerre and Poisson-Laguerre state space representations.

One can check that the Poisson-Laguerre and Laguerre state space representations are linked by the relationship

$$x^{PL}(t) = \Gamma(\lambda)^T x^L(t) \qquad (14)$$

Note that the matrix $\Gamma(\lambda)$ is upper triangular with non-zero diagonal entries and thus invertible.
Since the matrices A and B are known, an online identification of the unknown parameters vector C of the continuous-time models can be performed by using the zero order hold discrete-time version of (13), say

$$\begin{cases} x_{k+1} = A_d x_k + B_d u_k \\ y_k = C^T x_k \end{cases} \qquad (15)$$

In the case of Poisson-Laguerre models, it can easily be shown that

$$A_d = e^{-\lambda T_s}\begin{bmatrix} 1 & 0 & \cdots & 0 \\ \frac{T_s^1}{1!} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \frac{T_s^{n-1}}{(n-1)!} & \cdots & \frac{T_s^1}{1!} & 1 \end{bmatrix} \qquad (16)$$

$$B_d = \begin{bmatrix} b_0 \\ \vdots \\ \vdots \\ \vdots \\ b_{n-1} \end{bmatrix} \quad \text{and} \quad C = g \qquad (17)$$

where

$$b_i = \frac{1}{\lambda^{i+1}} - e^{-\lambda T_s}\sum_{j=0}^{i}\frac{T_s^j}{j!\,\lambda^{i-j+1}} \quad i = 0,1,...,n\text{-}1. \quad (18)$$

and T_s is the sampling time interval.
The analytic value of the zero order hold discrete-time state space representation matrices of Laguerre models is deduced as follows

$$\Gamma(\lambda)^T A_d [\Gamma(\lambda)^T]^{-1} \quad \text{and} \quad \Gamma(\lambda)^T B_d \qquad (19)$$

Thus, the trapezoidal approximation used in [3] is avoided.

3 E-Kautz Models

The so-called (orthogonal) Kautz functions are given by

$$\begin{cases} \psi_{2j-1}(s) = \frac{s\sqrt{2q}}{s^2+ps+q}\left[\frac{s^2-ps+q}{s^2+ps+q}\right]^{j-1} \\ \psi_{2j}(s) = \frac{\sqrt{2pq}}{s^2+ps+q}\left[\frac{s^2-ps+q}{s^2+ps+q}\right]^{j-1} \end{cases} \qquad (20)$$

j=1,2,3, ...

where p(real) > 0 and q(real) > 0, [9]-[12]
The transfer function of a resonant system can be approximated by a Kautz series as

$$\hat{G}(s)=\sum_{k=1}^{n}\overline{\alpha}_k\psi_k(s) \tag{21}$$

where n is the approximation order and $\{\overline{\alpha}_k\}$ are the Kautz parameters. A low approximation order can be obtained by choosing p and q so that the roots of $s^2+ps+q=0$ are close to the complex poles of the system to be approximated.

Let [12] $$\zeta=s+\frac{q}{s} \tag{22}$$

Without loss of generality, we consider n = 2m (even). Then, taking into account of (22) in (20) and substituting in (21) lead to

$$\hat{G}(\zeta)=\sum_{k=1}^{m}\beta_k(s)\frac{\sqrt{2p}}{\zeta+p}\left[\frac{\zeta-p}{\zeta+p}\right]^{k-1} \tag{23}$$

where $$\beta_k(s)=\sqrt{\frac{q}{p}}\overline{\alpha}_{2k-1}+\frac{\sqrt{q}}{s}\overline{\alpha}_{2k} \tag{24}$$

One can remark that Equation (23) looks like Laguerre expansion described by Equations (5) and (2) with respect to the variable ζ.

In the same way as in §2.3, Equation (23) can be transformed as

$$\hat{G}(s)=\sum_{k=1}^{m}\xi_k(s)\frac{1}{(\zeta+p)^k} \tag{25}$$

where $$\xi(s)=[\xi_1 \quad \cdots \quad \xi_m]^T=\overline{g}_{odd}+\frac{1}{s}\overline{g}_{even} \tag{26}$$

and $$\begin{aligned}\overline{g}_{odd}&=\sqrt{2p}\,\Gamma(p)[\overline{\alpha}_1\ \overline{\alpha}_3 \ldots \overline{\alpha}_{2m-1}]^T\\ \overline{g}_{even}&=\sqrt{2pq}\,\Gamma(p)[\overline{\alpha}_2\ \overline{\alpha}_4 \ldots \overline{\alpha}_{2m}]^T\end{aligned} \tag{27}$$

and the matrix $\Gamma(p)$ is obtained by replacing λ by p in $\Gamma(\lambda)$, given in Equation (11).

The substitution of (27), (26) and (22) in (25) leads to

$$\hat{G}(s)=\sum_{k=1}^{n}\overline{g}_k\varphi_k(s) \tag{28}$$

where
$$\begin{cases}\varphi_{2j-1}(s)=\dfrac{s^j}{(s^2+ps+q)^j}\\[2mm] \varphi_{2j}(s)=\dfrac{s^{j-1}}{(s^2+ps+q)^j}\end{cases} \quad j=1,2,\ldots\ m. \tag{29}$$

The relationship between $\{\overline{g}_k, k=1,2,...,n\}$ and $\{\overline{\alpha}_k, k=1,2,...,n\}$ is given by

$$[\overline{g}_1\cdots\overline{g}_n]^T=\Lambda(p,q)[\overline{\alpha}_1\cdots\overline{\alpha}_n]^T \tag{30}$$

with $$\Lambda(p,q)=\sqrt{2p}\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \cdots & \Lambda_{1m}\\ 0 & \Lambda_{22} & & \vdots\\ \vdots & \ddots & \ddots & \\ 0 & \cdots & 0 & \Lambda_{mm}\end{bmatrix} \tag{31}$$

and $$\Lambda_{ij}=\gamma_{ij}(p)\begin{bmatrix}1 & 0\\ 0 & \sqrt{q}\end{bmatrix},\quad \gamma_{ij}(p)=(-2p)^{i-1}\binom{j-1}{i-1} \tag{32}$$

We can then notice that the series (28)-(29) have the same truncation error as Kautz series. They can be considered as equivalent and the model described by (28)-(29) is called E-Kautz model, *E-* means *Equivalent to.*

The state space representation based on Kautz and E-Kautz models can be written as

$$\begin{cases}\begin{bmatrix}\dot{Z}(t)\\ \dot{X}(t)\end{bmatrix}=\begin{bmatrix}A_{11} & -qI_{2m}\\ I_{2m} & \mathbf{0}\end{bmatrix}\begin{bmatrix}Z(t)\\ X(t)\end{bmatrix}+\begin{bmatrix}B_1\\ B_2\end{bmatrix}u(t)\\ y(t)=C^TX(t)\end{cases} \tag{33}$$

I_{2m} and **0** are 2m×2m identity and zero matrices. The parameters of (33) are given in table2.

	Kautz	E-Kautz
A_{11}	$\begin{bmatrix} -p & 0 & & \cdots & & & 0 \\ 0 & -p & & & & & 0 \\ -2p & 0 & & \ddots & & & \\ 0 & -2p & & \ddots & & & \vdots \\ -2p & 0 & & \ddots & & & \\ 0 & \ddots & \ddots & \ddots & -p & 0 & \\ \vdots & \ddots & & & 0 & -p & 0 \\ -2p & \cdots & 0 & -2p\ \ 0 & -2p & 0 & -p \end{bmatrix}$	$\begin{bmatrix} -p & 0 & & \cdots & & 0 \\ 0 & -p & & & & \\ 1 & & \ddots & \ddots & & \vdots \\ 0 & \ddots & \ddots & & & \\ \vdots & \ddots & & & & 0 \\ 0 & \cdots & 0 & 1 & 0 & -p \end{bmatrix}$
B_1^T	$[-p\sqrt{2q} \quad \sqrt{2pq} \quad -2p\sqrt{2q} \quad \sqrt{2pq} \quad -3p\sqrt{2q} \quad \cdots \quad -(2m-1)p\sqrt{2q} \quad \sqrt{2pq}]$	$[-p \quad 1 \quad 1 \quad 0 \quad \cdots \quad 0]$
B_2^T	$[\sqrt{2q} \quad 0 \quad \sqrt{2q} \quad 0 \quad \sqrt{2q} \quad \cdots \quad \sqrt{2q}]$	$[1 \quad 0 \quad \cdots \quad 0]$
C	$\overline{\alpha}$	$\overline{g}$
X(t)	$x_{2i-1}(t) = \frac{s\sqrt{2q}}{s^2+ps+q}\left[\frac{s^2-ps+q}{s^2+ps+q}\right]^{i-1} u(t)$ $x_{2i}(t) = \frac{\sqrt{2pq}}{s^2+ps+q}\left[\frac{s^2-ps+q}{s^2+ps+q}\right]^{i-1} u(t)$ $i = 1,2,\dots,m.$	$x_{2i-1}(t) = \frac{s^i}{(s^2+ps+q)^i} u(t)$ $x_{2i}(t) = \frac{s^{i-1}}{(s^2+ps+q)^i} u(t)$ $i = 1,2,\dots,m.$

Table 2: Comparison of Kautz and E-Kautz state space representations.

One can check that E-Kautz and Kautz state space representations are linked by the relationship

$$\begin{bmatrix} Z(t) \\ X(t) \end{bmatrix}_{Kautz} = \begin{bmatrix} \Lambda^T & 0 \\ 0 & \Lambda^T \end{bmatrix} \begin{bmatrix} Z(t) \\ X(t) \end{bmatrix}_{E-Kautz} \qquad (34)$$

Note that the matrix $\Lambda(p,q)$ is upper triangular with non zero diagonal entries and thus invertible.

Let $$\begin{cases} \begin{bmatrix} Z_{k+1} \\ X_{k+1} \end{bmatrix} = \begin{bmatrix} K_1^* & -qK_2^* \\ K_2^* & K_3^* \end{bmatrix} \cdot \begin{bmatrix} Z_k \\ X_k \end{bmatrix} + \begin{bmatrix} B_1^s \\ B_2^s \end{bmatrix} u_k \\ y_k = C^T X_k \end{cases} \qquad (35)$$

be the zero order hold discrete-time version of (33). In the case of E-Kautz models and using the fact that $e^{At} = \mathcal{L}^{-1}[(sI-A)^{-1}]$, where $\mathcal{L}^{-1}(*)$ indicates the inverse Laplace Transform, one can show that

$$K_1^* = \begin{bmatrix} \varphi_1^* & 0 & & \cdots & & 0 \\ 0 & \varphi_1^* & & & & \\ \varphi_3^* & & & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & & \\ \vdots & \ddots & & & & 0 \\ \varphi_{2m-1}^* & \cdots & 0 & \varphi_3^* & 0 & \varphi_1^* \end{bmatrix} \quad K_2^* = \begin{bmatrix} \varphi_2^* & 0 & & \cdots & & 0 \\ 0 & \varphi_2^* & & & & \\ \varphi_4^* & & & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & & \\ \vdots & \ddots & & & & 0 \\ \varphi_{2m}^* & \cdots & 0 & \varphi_4^* & 0 & \varphi_2^* \end{bmatrix} \qquad (36)$$

$$K_3^* = \begin{bmatrix} \varphi_1^*+p\varphi_2^* & 0 & & & \cdots & & & 0 \\ 0 & \varphi_1^*+p\varphi_2^* & & & & & & \\ -q\varphi_2^{*2} & 0 & & & & & & \\ 0 & & & & \ddots & & & \vdots \\ -q\varphi_2^*\varphi_4^* & \ddots & \ddots & & \ddots & & & \\ 0 & \ddots & & & & & & \\ \vdots & \ddots & & 0 & & & & 0 \\ -q\varphi_2^*\varphi_{2m}^* & \cdots & 0 & -q\varphi_2^*\varphi_4^* & 0 & q\varphi_2^{*2} & 0 & \varphi_1^*+p\varphi_2^* \end{bmatrix} \qquad (37)$$

$$B_1^s = [(\varphi_1^* - 1) \quad \varphi_2^* \quad 0 \quad \varphi_4^* \quad 0 \quad \cdots \quad \varphi_{2m}^*]^T \tag{38}$$

and
$$B_2^s = [\varphi_2^* \quad -\frac{1}{q}(\varphi_2^* - 1) \quad \varphi_4^* \quad -\frac{1}{q}\varphi_4^* \quad \varphi_6^* \quad \cdots \quad -\frac{1}{q}\varphi_{2m-2}^* \quad \varphi_{2m}^*]^T \tag{39}$$

where φ_i^* denotes $\varphi_i^*(T_s)$ and $\varphi_i^*(t) = \mathcal{L}^{-1}(\varphi_i(s))$; $\varphi_i(s)$ is defined in (29).

The analytic value of the zero order hold discrete-time state space representation matrices of Kautz models is deduced as follows

$$\begin{bmatrix} \Lambda^T & 0 \\ 0 & \Lambda^T \end{bmatrix} \begin{bmatrix} K_1^* & -qK_2^* \\ K_2^* & K_3^* \end{bmatrix} \begin{bmatrix} \Lambda^T & 0 \\ 0 & \Lambda^T \end{bmatrix}^{-1} \tag{40}$$

$$\begin{bmatrix} \Lambda^T & 0 \\ 0 & \Lambda^T \end{bmatrix} \begin{bmatrix} B_1^s \\ B_2^s \end{bmatrix} \quad \text{and} \quad C = \bar{g} \tag{41}$$

To obtain these matrices directly from Kautz matrices, given in the first column of table 2, seems to be less obvious.

4 Conclusions

New models, called Poisson-Laguerre and E-Kautz models, have been deduced respectively from Laguerre and Kautz series. The truncation error is conserved. It could be quantified by using the orthogonality property of Laguerre and Kautz series, via Equations (9) and (25).

As can be observed in table 1 and 2, the particular form of the state space representation obtained with Poisson-Laguerre and E-Kautz models is more suitable in computation. It allows particularly an analytic computation of their zero order hold discrete-time state space representation. The corresponding analytic expression of Laguerre and Kautz models is easily deduced from that of Poisson-Laguerre and E-Kautz models; this seems to be less obvious directly from Laguerre and Kautz continuous-time state space representation.

An online identification of the unknown parameter vectors g and $\bar{g}$ of the continuous-time models could be achieved by using the zero order hold discrete-time state space representations, for which these vectors remain the same

Poisson-Laguerre and E-Kautz models have been tested, in simulation, by identifying in online way a variety of continuous-time systems, namely time-delay, infinite-dimensional and resonant systems. Reasonably low approximation orders are used. A fast convergence rate of the recursive last squares algorithm is observed. These models appears thus to be particularly suitable for adaptive control of continuous-time systems.

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