Methodology for Load Matching and Optimization of Directly Coupled PV pumping systems

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Abstract: The aim of this work is to develop a methodology for the assessment of load matching and further estimation of the optimum photovoltaic (PV) arrays arrangement for water pumping systems over a prolonged period of time. The method calls for the calculation of the appropriate effectiveness factor defined as the ratio of the load energy over the maximum energy that can be produced by the PV array for a specific time period. The effectiveness factor depends on the PV array characteristics, the load characteristics, and the solar irradiance conditions. To produce realistic predictions for the effectiveness factor and the PV arrays arrangement with validity over long periods of time, the present model takes into account the stochastic variation of solar irradiation over a long period of time and not just a fixed diurnal variation as was traditionally done in the past. In order to generalize the analysis, simulation results must be presented in a reduced form based on the values of the voltage and current corresponding to the maximum power of the PV array. The results can be expressed in multiple-curve comprehensive plots, which allow determining the optimum photovoltaic array panel's arrangement without engaging sophisticated mathematical calculations.

Keywords: PV, Water pumping systems, Load matching, Optimization.

1. Introduction

In many stand-alone PV systems, the solar modules array is designed to power specific single loads, such as lights (resistive loads), electromechanical loads coupled to dc motors, electrolysis loads, etc. [1]. Two different load configurations are currently in use for PV systems. One is the direct-coupled systems which are simple and reliable, but do not operate at the maximum power operating point of the array due to the continuous variation of solar radiation. The other uses a maximum power point tracker (MPPT) to maintain the PV array at a voltage for which it produces maximum power. The latter is the most efficient configuration of the two but it is less reliable in many occasions. The quality of load matching in a PV system determines the system performance and its degree of utilization. An optimum PV panels' arrangement results in more accurate sizing, reduction of the rating of the subsystems and maximum utilization of the costly solar array generator.

Several studies investigated the direct-coupled PV-load configuration and obtained useful information regarding the adaptability of a PV system to various loads [2-6]. The design methodology in these studies was either based on the diurnal variation of solar radiation so their results could not be safely applied over a period of time or produced results for specific input radiation time series and so their results were not of general validity.

Electric power production by a PV system depends greatly on insolation, which varies continuously with time. Therefore, the design of such a system involves a stochastic parameter and differs from the design of a conventional power production system. This work aims to provide a generalized methodology for analyzing the direct coupling of a PV system to water pumping loads. The optimum design of the system is based on the maximization of the effectiveness factor that is defined as the ratio of the load input energy to the PV array maximum energy over a time period. The advantage of this approach lies in the fact that it takes into account the variation of solar radiation over a long time period and not in just one day as previous studies has done [2,3].

Recently, the problem of load matching for Thevenin's equivalent loads has been dealt with using a similar methodology [7]. The great advantage of that work was that the optimum photovoltaic array panel's arrangement could be found by using generalized plots which allows the design engineer to avoid sophisticated simulations. In this work, an effort is made to approximate the more complicated water pumping loads to Thevenin's equivalent loads with constraints. If this attempt is successful then similar generalized plots can be also used for PV matching to water pumping systems.

2. Model development

A single PV unit is a nonlinear electric power source whose characteristic equation depends chiefly on the intensity of solar radiation and to a lesser extent on temperature. Putting together in series and in parallel several such units results in a PV system whose characteristic equation depends, along with the above, also on the arrangement of the connected units. Thus, the characteristic equation of a PV system has the general form:

$$I = f_{PV}(U, G, T, p, s) \tag{1}$$

where I is the current of the system, U is the voltage of the system, G is the solar radiation, T is the temperature of the PVs, s is the number of the PV units connected in series (s-chains) and p is the number of the s-chains connected in parallel.

The characteristic equation of an electric load is of the form:

$$I = f_L(U) \tag{2}$$

When the load is coupled directly to the PV system then the power delivered to the load is the product of the voltage times the current of the system:

$$P_L = I(G, T, p, s) \cdot U(G, T, p, s) \tag{3}$$

The magnitudes of U and I are obtained from the solution of the system of equations (1) and (2).

Considering that the variation of the PV temperature is chiefly a function of radiation, the average power for this period is:

$$\overline{P}_L = \int n_{em}(G, p, s) \cdot P_L(G, p, s) \cdot f_G(G) dG \tag{4}$$

where $f_G(G)$ is the probability density function of solar radiation, n_{em} is the overall efficiency of the electromechanical system. In the most general case n_{em} can be also considered as a function of *G, p, s*.

In a similar manner, the average maximum power produced by the PVs can be estimated from equation (1):

$$P_{PV}^{\max} = (I \cdot U)_{\max} \tag{5.1}$$

$$\overline{P}_{PV}^{\max} = \int P_{PV}^{\max}(G) \cdot f_G(G) dG \tag{5.2}$$

The ratio of the two quantities is defined as an *effectiveness factor*:

$$n_{ef} = \frac{\overline{P}_L}{\overline{P}_{PV}^{\max}} \tag{6}$$

The optimum matching of the PV system to the coupled load corresponds to the particular combination of in-series (s) and in-parallel (p) connected units that yields the maximum n_{ef}.

2.1 Characteristic equation of a PV system

The electrical behavior of a PV unit is represented in the equivalent electric circuit of Figure 1.

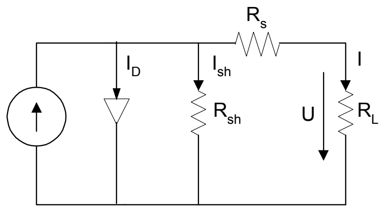

Fig. 1 Equivalent electrical circuit of a solar unit.

The relationship between current and voltage is [7]:

$$I = I_L - I_D - I_{sh} = I_L - I_0 \left\{ \exp\left[\frac{U + IR_S}{U_T} \right] - 1 \right\} - \frac{U + IR_S}{R_{sh}} \tag{7}$$

where I_L is the short-circuit current in (A), I_D is the diode current of the equivalent circuit, I_0 is the inverse polarization current in (A), I is the load current in (A), U is the load voltage in (V), R_s is the series resistance in (Ω), R_{sh} is the shunt resistance in (Ω) and U_T is the thermal voltage in (V)

In practice and particularly for the case of single crystalline silicon cells, the resistance R_{sh} is much higher than R_s and therefore equation (7) can be reduced as follows:

$$I = I_L - I_D = I_L - I_0\left\{\exp\left[\frac{U+IR_s}{U_T}\right]-1\right\} \quad (8)$$

Parameters I_L, I_0, R_s and U_T depend on solar radiation and the temperature of the PV unit. A method to determine these four parameters is presented by Duffie and Beckman [8]. This method is based on information for I and U given by the manufacturer of a PV unit for irradiance, G_{ref}, at a reference temperature, T_{ref}, and is described briefly by the following relations:

$$I_{L,ref} = I_{sc,ref} \quad (9)$$

$$U_{T,ref} = \frac{\mu_{U,oc}T_{c,ref} - U_{oc,ref} + E_q N_s}{\frac{\mu_{I,sc}T_{c,ref}}{I_{L,ref}} - 3} \quad (10)$$

$$I_{0,ref} = \frac{I_{L,ref}}{\exp\left(\frac{U_{oc,ref}}{U_{T,ref}}\right)-1} \quad (11)$$

$$R_{s,ref} = \frac{U_{T,ref}\ln\left(1-\frac{I_{mp,ref}}{I_{L,ref}}\right) - U_{mp,ref} + U_{oc,ref}}{I_{mp,ref}} \quad (12)$$

The subscripts *oc, sc, mp* and *ref* refer to open circuit, short circuit, maximum power and reference conditions, respectively. In addition, E_q is the energy gap of silicon (eV), N_s is the number of cells connected in series in a single unit of the PV system and $\mu_{U,oc}$, $\mu_{I,sc}$, are the temperature coefficients of the open circuits voltage and closed circuits current, respectively.

For varying insolation and temperature conditions the above parameters change according to the following relations:

$$\frac{U_T}{U_{T,ref}} = \frac{T_C}{T_{C,ref}} \quad (13)$$

$$I_L = \frac{G}{G_{ref}}\left[I_{L,ref} + \mu_{I,sc}\left(T_C - T_{C,ref}\right)\right] \quad (14)$$

$$I_0 = I_{0,ref}\left(\frac{T_C}{T_{C,ref}}\right)^3 \exp\left[\frac{E_q N_s}{U_T}\left(1-\frac{T_{C,ref}}{T_C}\right)\right] \quad (15)$$

$$R_s = R_{s,ref} \quad (16)$$

The unit temperatures, T_C and $T_{C,ref}$ are computed from the relation:

$$T_C = T_\alpha + G\cdot\frac{T_{C,NOCT} - T_\alpha}{G_{NOCT}}\cdot\left(1-\frac{\eta_C}{\tau\alpha}\right) \quad (17)$$

using G and T_a or G_{ref} and $T_{a,ref}$, respectively. In equation (17) T_a is the ambient temperature, the subscript NOCT refers to the unit temperature for nominal operation, n_c is the efficiency of the unit at NOCT conditions and $\tau\alpha$ is the product of the unit coefficients of transmittance and absorption.

In brief, the evaluation of the characteristic equation of a PV system is as follows: First, equations (9) to (12) are used to estimate the values of the four aforementioned parameters at reference conditions. Next, these values are adjusted to the actual operating conditions with equations (13) to (17). Finally, the system current, I, is calculated from equation (18) which is derived from equation (8) accounting for the PV units that are connected in parallel *(p)* and in series *(s)*:

$$I = p\cdot I_L - p\cdot I_0\left\{\exp\left[\frac{U + I\cdot\frac{s}{p}\cdot R_s}{s\cdot U_T}\right]-1\right\} \quad (18)$$

2.2 Voltage-current characteristics of motor-pump loads

In such systems the voltage-current characteristic depends on the kind of motor excitation and the type of load.

2.2.1 Electrical motor

For the four basic types of dc motors, schematically presented in Figure 2,

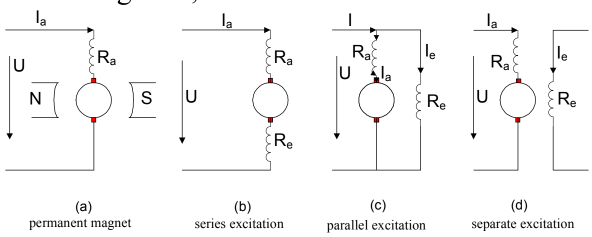

Fig. 2 Equivalent electrical circuits of dc electrical machines for four different kinds of excitation

the following relations hold:

$$\left.\begin{aligned} U &= E + I_a\cdot R_a \\ E &= c_m\cdot\Phi\cdot\omega_m \\ T &= c_m\cdot\Phi\cdot I_a \end{aligned}\right\} \quad \text{electrical motor} \quad (19)$$

$$T = f(\omega_m) \quad \text{mechanical load} (20)$$

where U is the voltage, E is the emf of the motor, I_α is the armature current, I_e is the field-winding current, Φ is the magnetic flux, c_m is the motor

constant, R_a is the armature resistance, R_e is the field-winding resistance, ω_m is the motor angular speed and T is the motor torque.
Specifically, for the motors of Figure 2 it is:
(a) for a permanent magnet motor, $I=I_a$, $\Phi=constant$,
(b) for a series excitation motor, $I=I_a$, $\Phi = k_1 \cdot I$, $T = k_2 \cdot I^2$ and $R = R_a + R_e$,
(c) for a parallel excitation motor $I_a = I - \frac{U}{R_e}$ and the magnetic flux is a function of excitation: $\Phi = \Phi\left(\frac{U}{R_e}\right)$
(d) for a separate excitation motor $I=I_a$, $\Phi= constant$,
The voltage-current characteristic of each load is derived if the appropriate relation from the above is replaced in equations (19) και (20).

2.2.2 Pump

The behavior of any type of pump can be described by its constant speed characteristics $H_{ref}(Q)$ and $P_{m,ref}(Q)$ where Q is the volumetric flow rate of the pump whereas H_{ref} is the total head and $P_{m,ref}$ the mechanical power of the pump for a reference motor angular speed. Using the above constant speed characteristics, one can approximately describe the variation of the total head and the power of the pump for different angular speeds as follows:

$$H(Q,\omega_m) = H_{ref}(Q)\left(\frac{\omega_m}{\omega_r}\right)^2 \quad \text{(a)}$$

$$P_m(Q,\omega_m) = P_{m,ref}(Q)\left(\frac{\omega_m}{\omega_r}\right)^3 \quad \text{(b)} \qquad (21)$$

where ω_r is a reference motor angular speed.
Based on hydraulic considerations, the total head that a pump must overcome is given approximately by a relation of the form [10]:

$$H(Q) = H_g + hQ^2 \qquad (22)$$

Where H_g is the static head and h a coefficient that accounts for flow resistances. Usually, H_g is a known design parameter while the value of h must be estimated from the actual pipe geometry [10].

Combining equation (21a) and (22) yields

$$Q = \begin{cases} Q(\omega_m) & \omega_m \geq \omega_{\min} \\ 0 & \omega_m < \omega_{\min} \end{cases} \qquad (23)$$

where ω_{min} is the minimum motor angular speed for which $Q = 0$
Knowing that the mechanical power of the motor P_m is the product of the torque T times the angular speed of the motor, the following holds for the torque:

$$T(Q,\omega_m) = T_{ref}(Q)\left(\frac{\omega_m}{\omega_r}\right)^2 \qquad (24)$$

where T_{ref} is the torque of the pump for a reference motor angular speed ($=P_{m,ref}/\omega_r$).
Taking into account equation (23), a relation between T και ω_m is obtained i.e., equation (20). This describes the mechanical load of the system.
It has been shown that the combination of equations (19) with (20) incorporating the (21), (22), (23) and (24) leads eventually to a relation between voltage and current of the form [11, 12]:

$$U = I \cdot R_{th}(H) + U_{th}(H) \qquad (25)$$

This is a linear relation with coefficients that are functions of the total head of the pump.
Thus, for a specific total head the relation (25) corresponds to a Thevenin equivalent load like the one examined in reference [7].

2.3 Variability of solar radiation

In order to describe the temporal variability of daily solar radiation the equations proposed by Bendt et al. [9] are employed. These equations provide the probability distribution of the daily clearness index, $K_T = G_d/G_o$, over a period of time when the average clearness index is $\overline{K}_T$:

$$F(K_T) = \frac{\exp(\gamma K_{T,\min}) - \exp(\gamma K_T)}{\exp(\gamma K_{T,\min}) - \exp(\gamma K_{T,\max})} \qquad (26)$$

where γ is computed from the following relations:

$$\overline{K}_T = \frac{(K_{T,\min} - \frac{1}{\gamma})\exp(\gamma \cdot K_{T,\min}) - (K_{T,\max} - \frac{1}{\gamma})\exp(\gamma \cdot K_{T,\max})}{\exp(\gamma \cdot K_{T,\min}) - \exp(\gamma \cdot K_{T,\max})} \qquad (27)$$

$$\gamma = -1.498 + \frac{1.184\xi - 27.182\exp(-1.5\xi)}{K_{T,\max} - K_{T,\min}},$$

$$\xi = \frac{K_{T,\max} - K_{T,\min}}{K_{T,\max} - \overline{K}_T} \qquad (28)$$

$$K_{T,\max} = 0.6313 + 0.267\overline{K}_T - 11.9(\overline{K}_T - 0.75)^8,$$
$$K_{T,\min} = 0.05 \qquad (29)$$

Where $\overline{K}_T$, $K_{T,\max}$ and $K_{T,\min}$ are the average, maximum and minimum daily clearness index for a specific location and time period whereas G_d and G_o are the daily and extraterrestrial solar radiation, respectively.

The corresponding diurnal variation of solar radiation obeys the relations [8]:

$$r_t = \frac{G}{G_d} \qquad (30)$$

$$r_t = \frac{\pi}{24}\left(z + b\cos\omega\right)\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \frac{\pi\omega_s}{180}\cos\omega_s} \qquad (31)$$

$$z = 0.409 + 0.5016\sin\left(\omega_s - 60\right) \qquad (32)$$

$$b = 0.6609 - 04767\sin\left(\omega_s - 60\right) \qquad (33)$$

where G is the hourly radiation. The quantities ω (hour angle) and ω_s (sunrise hour angle) are common solar engineering parameters that can be computed from analytical expressions, e.g., [8]. It must be noted that the sunrise hour angle depends greatly on the *latitude* of the measuring location.

Total irradiance on the PV array plane is calculated using an isotropic model for both the diffuse irradiation and the ground reflected irradiation. Calculations by this model take into account the location *latitude* and the *tilt* of the array plane. Details on the above can be found in classic solar engineering books, e.g. [8].

3. Simulation methodology

It was shown above that when a pump is the load to a PV system this load can be approximated by a Thevenin equivalent load. This means that there is a simple linear relation between voltage and current with coefficients that depend solely on the total head of the pump. The only difference from an ordinary Thevenin equivalent load is that in the case of a pump the overall efficiency n_{em} is zero for $\omega_m < \omega_{min}$, (or $I<I_{min}$ or $G<G_{min}$, respectively), so the equivalent characteristic holds only for $\omega_m \geq \omega_{min}$ (or $I \geq I_{min}$, or $G \geq G_{min.}$, respectively).

Therefore, this work suggests that a similar simulation algorithm may be applied as in [7] in order to examine the effectiveness factor of a PV system directly coupled to water pump loads and also optimize an appropriate (per unit) *design parameter* defined as the ratio of load resistance to equivalent resistance of the PV system:

$$R_{pu} = \frac{R_{th}}{R_{eq,PV}} \qquad (34)$$

where the equivalent resistance of the PV system is given as:

$$R_{eq,PV} = \frac{U_{mp,PV} - U_{th}}{I_{mp,PV}} = \frac{s \cdot U_{mp,u} - U_{th}}{p \cdot I_{mp,u}} \qquad (35)$$

$U_{mp,PV}$ ($U_{mp,u}$) and $I_{mp,PV}$ ($I_{mp,u}$) are the voltage and current for the maximum power of the PV system (unit) when the solar radiation is equal to the hourly radiation at solar noon for the specific location, season, clearness index and tilt of PV units.

According to the proposed methodology, plots will be constructed describing the design parameter with respect to the clearness index and latitude for different seasons of the year as well as for the entire year. Evidently, these plots must be produced for the optimum ratio of the maximum power of the PV array over the nominal power of the pumping system.

The values of s and p for the optimum arrangement of the PV system is estimated based on the values of a single PV unit according to the following relations,

$$s \cdot U_{mp,u} = U_{th} + R_{eq,PV} \cdot p \cdot I_{mp,u} \qquad (36)$$

$$s \cdot p = ct = No\ \ of\ \ units \qquad (37)$$

4. Conclusions

A methodology has been developed for the investigation of the effectiveness factor and optimum arrangement of a PV system directly coupled to a water pump load. The main advantage of the method is that it accounts for the variance of solar radiation not from just a single day but from a longer period of time as it should be in a real application. The methodology is based on the use of per unit reduced parameters which leads to a generalized designing procedure. The comprehensive plots that can be produced by appropriate simulations will allow the reduced design parameter, R_{pu}, (dictating the optimum PV system arrangement) to be easily deduced. Therefore, the present work represents a short-cut method for field engineers and PV experts.

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