The role of viscous friction damping in adaptive output feedback tracking control of manipulators

JAVIER MORENO
Centro de Investigación y Desarrollo de Tecnología Digital del IPN
CITEDI–IPN, Ave. del Parque 1310, Mesa de Otay, Tijuana, B.C., 22510
MEXICO

Abstract: The trajectory tracking control of robot manipulators under the practical situation in that actuators have limited power (torque–bounded control) and that only position measurements are carried out (output feedback) is addressed in this paper. Specifically, we show that viscous friction damping is enough for global bounded adaptive output feedback control of robot manipulators.

Key–Words: Tracking control; Output feedback; Viscous friction; Robot manipulator; Lyapunov function.

1 Introduction

Adaptive control of robot manipulators has been an active research topic in the last 15 years. The main motivation of this has been the interest in adding a reliable degree of robustness to the closed–loop system, specially in manufacturing system, where the manipulator frequently achieves pick and place tasks of materials and parts, whereby adaptation of the payload changes helps to guarantee the motion control. See, e.g., [4] for a reference on globally asymptotically stable adaptive algorithms, which guarantee the asymptotic tracking of the joint desired trajectory. However, in most of the adaptive controllers the joint velocity is assumed to be measurable for feedback. In practice, the joint velocity measurement might be contaminated by noise, hence the control system performance may be reduced. The presence of noise in the measurements and the discretization of the controller also limits the values of the controller gains. For these reasons, the problem of designing motion control algorithms that deal with the velocity reconstruction (output feedback) and the parameter adaptation is important. Even in the trajectory design for

The problem of adaptive output feedback tracking control of manipulators consists in designing a control algorithm by using only joint position measurements and a parameter estimation update law, so that the error between the time–varying desired position and the position of the system goes asymptotically to zero for a set of initial conditions. This paper addresses this control problem.

As pointed out in [3], despite the numerous algorithms of adaptive controllers and of tracking controller–observer schemes, there are relatively few algorithms which combine both adaptive schemes and velocity reconstruction. Our literature review does not cover all previous work on adaptive output feedback tracking control for robots. Only a brief description of the key results proposed latter than 1996 is provided. In [3] an adaptive scheme with boundedness of the estimated parameters and uniform ultimate boundednes for tracking and observation errors was proposed. Inspired in the approach introduced by Loria [8], Zhang *et al.* [15] proposed a controller that depends in the initial condition of a dynamic extension. A redesign of the approach introduced in [15] is proposed in the work of Zergeroglu *et al.* [14], but presenting the global convergence of the position and tracking errors. Let us notice that these approaches were extended by Dixon *et al.* [5] to the problem of adaptive trajectory tracking control of manipulators with flexible joints by using only position measurements of the robot actuators and links. In a similar way, in this work the global convergence of the link position and velocity errors was shown.

The main aim of this paper is to deal the problem of adaptive output feedback tracking control de manipulators by introducing a controller which achieves the global convergence result of the position and velocity tracking error with the nice property of producing saturated torque input. More specifically, a robot parameter estimation update law to be used along with a torque–bounded controller is proposed. A special tuning of the gains involved in the controller , velocity error estimator and the update rule, guarantees the global convergence result.

Notation: Throughout this paper the following notation will be adopted. $\|x\| = \sqrt{x^T x}$ stands for the norm of vector $x \in \mathbb{R}^n$. $\lambda_{min}\{A(x)\}$ and $\lambda_{Max}\{A(x)\}$ denote the minimum and maximum eigenvalues of a symmetric positive definite matrix $A(x) \in \mathbb{R}^{n\times n}$ for all $x \in \mathbb{R}^n$, respectively. $\|B(x)\| = \sqrt{\lambda_{Max}\{B(x)^T B(x)\}}$ stands for the induced norm of a matrix $B(x) \in \mathbb{R}^{m\times n}$ for all $x \in \mathbb{R}^n$. In this paper, the notation $\mathrm{hypfunc}(x)$ denotes a hyperbolic function of $x \in \mathbb{R}$, e.g., $\tanh(x)$ and $\ln(\cosh(x))$, while the notation $\mathrm{Hypfunc}(z) = diag\{\mathrm{hypfunc}(z_1),\ldots,\mathrm{hypfunc}(z_n)\}$, $z = [z_1,\ldots,z_n]^T \in \mathbb{R}^n$, denotes a diagonal matrix containing as elements the hyperbolic function of each element of the vector z. As example, $\mathrm{Sech}^2(z) = diag\{\mathrm{sech}^2(z_1),\ldots,\mathrm{sech}^2(z_n)\}$. See the Appendix A for the hyperbolic function properties used in this paper.

2 Robot dynamics and control goal

The dynamics in joint space of a serial–chain n-link robot manipulator considering the presence of friction at the robot joints can be written as [12],[13]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F_v\dot{q} = \tau \qquad (1)$$

where $M(q)$ is the $n\times n$ symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n\times n$ vector of centripetal and Coriolis torques, F_v is a $n\times n$ constant positive definite diagonal matrix, which contains the viscous friction coefficients, τ is the $n\times 1$ vector of applied torques inputs and $g(q)$ is the $n\times 1$ vector of gravitational torques.

Assuming that only robot joint displacements $q(t) \in \mathbb{R}^n$ are available for measurement and uncertainty on the robot parameters is present. Then, the adaptive output feedback tracking control problem is to design a control input τ together with a parameter estimation update law so that the joint displacements $q(t) \in \mathbb{R}^n$ converge asymptotically to the desired joint displacements $q_d(t) \in \mathbb{R}^n$, i.e.,

$$\lim_{t\to\infty} \tilde{q}(t) = 0, \qquad (2)$$

where

$$\tilde{q}(t) = q_d(t) - q(t)$$

denotes the tracking error.

Throughout this paper we consider that $q_d(t)$ is three times differentiable and

$$\begin{aligned} \|\dot{q}_d(t)\| &\leq \|\dot{q}_d\|_M \ \forall\, t \geq 0, & (3)\\ \|\ddot{q}_d(t)\| &\leq \|\ddot{q}_d\|_M \ \forall\, t \geq 0, & (4)\end{aligned}$$

where $\|\dot{q}_d\|_M > 0$ and $\|\ddot{q}_d\|_M > 0$ denote known constants.

The following properties are satisfied for the dynamic model (1) (see e.g. [9], [4], [13], [12]):

- **Property 1.** For all q, $\dot{q}$, $\ddot{q} \in \mathbb{R}^n$ we have

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F_v\dot{q} = Y(q,\dot{q},\ddot{q})\theta$$

where $Y(q,\dot{q},\ddot{q})$ is a $n\times r$ regressor matrix and $\theta \in \mathbb{R}^r$ is the parameter vector containing the robot and payload parameters.

- **Property 2.** For all $q,\dot{q},x,y,z \in \mathbb{R}^n$, the inertia and Coriolis matrix (using Christoffel symbols) satisfy

$$\lambda_{Max}\{M(q)\}\|x\|^2 \geq x^T M(q) x \geq \lambda_{min}\{M(q)\}\|x\|^2, \qquad (5)$$

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T, \qquad (6)$$

$$C(x,y)z = C(x,z)y,$$

$$C(x,y+z) = C(x,y) + C(x,z),$$

$$\|C(q,\dot{q})\| \leq k_{C1}\|\dot{q}\|, \qquad (7)$$

$$x^T\left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right]x = 0$$

- **Property 3.** The so–called residual dynamics [1], [2]

$$h(\tilde{q},\dot{\tilde{q}}) = [M(q_d) - M(q)]\ddot{q}_d$$

$$+[C(q_d,\dot{q}_d) - C(q,\dot{q})]\dot{q}_d + [g(q_d) - g(q)], \qquad (8)$$

satisfies the following inequality [10]

$$\|h(\tilde{q},\dot{\tilde{q}})\| \leq c_1\|\dot{\tilde{q}}\| + \frac{\delta\alpha}{\tanh(\alpha\sigma)}\|\tanh(\sigma\tilde{q})\| \qquad (9)$$

where σ is a strictly positive constant,

$$\begin{aligned} c_1 &= k_{C1}\|\dot{q}_d\|_M,\\ \delta &= k_g + k_M\|\dot{q}_d\|_M + k_{C2}\|\dot{q}_d\|_M^2,\\ \alpha &= 2\frac{k_1 + k_2\|\ddot{q}_d\|_M + k_{C1}\|\dot{q}_d\|_M^2}{\delta}.\end{aligned}$$

The constants involved in the robot model properties are defined as follows [6], [10],

$$k_M \geq n^2\left[\max_{i,j,k,q}\left|\frac{\partial M_{ij}(q)}{\partial \mathrm{q_k}}\right|\right] \qquad (10)$$

$$k_{C1} \geq n^2\left[\max_{i,j,k,q} \mid c_{ijk}(q)\mid\right] \qquad (11)$$

$$k_{C2} \geq n^3\left[\max_{i,j,k,l,q}\left|\frac{\partial c_{ijk}(q)}{\partial q_l}\right|\right] \qquad (12)$$

$$k_g \geq n\left[\max_{i,j,q}\left|\frac{\partial g_i(q)}{\partial q_j}\right|\right] \qquad (13)$$

$$k_1 \geq \sup_{\forall\, q \in \mathbb{R}^n} \|g(q)\|, \tag{14}$$

$$k_2 \geq \lambda_{max}\{M(q)\} \tag{15}$$

where $M_{ij}(q)$ is the ij–element of matrix $M(q)$, $c_{ijk}(q)$ is the ijk Christoffel symbol, and $g_i(q)$ is the i–element of vector $g(q)$.

It is noteworthy that all the constants involved in Property 3, see equations (10)–(15), can be computed through rough estimations of the real robot parameters. Then, very poor information of the robot model is requested to compute the inequality (9). Let us notice that Property 3 has been derived from the results established in [10]. There, it is stated that

$$\|h(\tilde{q}, \dot{\tilde{q}})\| \leq c_1 \|\dot{\tilde{q}}\| + \delta sat(\|\tilde{q}\|; \alpha),$$

where

$$sat(\|\tilde{q}\|; \alpha) = \begin{cases} \alpha \|\tilde{q}\| & \forall \quad \|\tilde{q}\| \leq \alpha, \\ \alpha & \forall \quad \|\tilde{q}\| > \alpha. \end{cases}$$

By noting that

$$\frac{\alpha}{\tanh(\alpha\sigma)} \|\tanh(\sigma\tilde{q})\| \geq sat(\|\tilde{q}\|; \alpha)$$

for any strictly positive constant σ and α, the inequality (9) is derived.

3 Proposed controller and analysis

3.1 Adaptive output feedback tracking controller

Consider the control law given by

$$\tau = Y(q_d, \dot{q}_d, \ddot{q}_d)\hat{\theta} + K_v \tanh(\tilde{\vartheta}) + K_p \tanh(\sigma\tilde{q}), \tag{16}$$

where $\tilde{q} = q_d - q$ denotes the tracking error, $K_v = diag\{k_{v1}, \ldots, k_{vn}\}$ and $K_p = diag\{k_{p1}, \ldots, k_{pn}\}$ are positive definite matrices, σ is a strictly positive constant, $\tilde{\vartheta}$ is obtained from the following nonlinear filter

$$\begin{aligned} \dot{x} &= -A \tanh(\tilde{\vartheta}), & (17) \\ \tilde{\vartheta} &= x + B\tilde{q}, & (18) \end{aligned}$$

where $A = diag\{a_1, \ldots, a_n\}$, $B = diag\{b_1, \ldots, b_n\}$ are positive definite matrices, and $\hat{\theta} \in \mathbb{R}^r$ is the estimated parameter vector obtained from a update law. The real–time implementation in block diagram form is shown in Figure 1. The notation τ_i^{Max} refers to the maximum capability of torque provided by the robot actuator of the joint $i = 1, \ldots, n$.

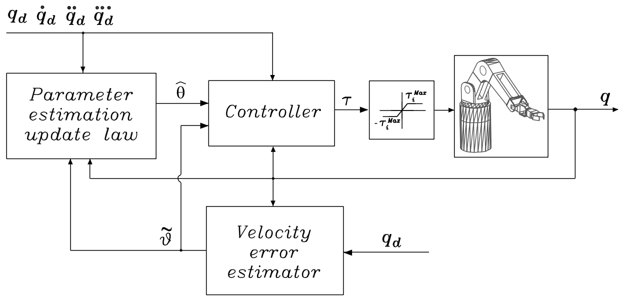

Figure 1: Block diagram of the adaptive output feedback tracking controller

3.2 Analysis

Our main results on the global adaptive output feedback tracking control of robot manipulators are stated in this Section. In particular, Proposition 1 provides sufficient conditions for the global convergence of the position and velocity tracking error by using the control law (16) together with a update law of the estimated robot parameters.

Consider the following estimated robot parameters update law:

$$\hat{\theta} = \Gamma \Big[Y^T(q_d, \dot{q}_d, \ddot{q}_d)\tilde{q} - \int_0^t \Big\{ \dot{Y}^T(q_d, \dot{q}_d, \ddot{q}_d)\tilde{q} - \xi Y^T(q_d, \dot{q}_d, \ddot{q}_d) \tanh(\sigma\tilde{q}) \Big\} dt \Big], \tag{19}$$

with Γ a positive definite matrix and the strictly positive constant $\xi \in (\xi_{min}, \xi_{Max})$. The explicit values of ξ_{min} and ξ_{Max} will be defined later.

The closed–loop system can be obtained by substituting the controller equation (16) into the robots dynamics (1), using Property 1 of the robot model, differentiating equation (18) with respect to time, and using the parameter estimation error definition

$$\tilde{\theta} = \theta - \hat{\theta} \ \in \mathbb{R}^r, \tag{20}$$

whose time derivative is $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. Then, we can write

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \\ \\ \tilde{\vartheta} \\ \tilde{\theta} \\ \\ \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}} \\ M(q)^{-1}[-C(q,\dot{q})\dot{\tilde{q}} - F_v\dot{\tilde{q}} - K_v \tanh(\tilde{\vartheta}) \\ -K_p \tanh(\sigma\tilde{q}) - h(\tilde{q}, \dot{\tilde{q}}) + Y(q_d, \dot{q}_d, \ddot{q}_d)\tilde{\theta}] \\ -A\tanh(\tilde{\vartheta}) + B\dot{\tilde{q}} \\ \Gamma[-Y^T(q_d, \dot{q}_d, \ddot{q}_d)\dot{\tilde{q}} \\ -\xi Y^T(q_d, \dot{q}_d, \ddot{q}_d)\tanh(\sigma\tilde{q})] \end{bmatrix} \tag{21}$$

The state space origin $[\tilde{q}^T \ \dot{\tilde{q}}^T \ \tilde{\vartheta}^T \ \tilde{\theta}^T]^T = 0 \in \mathbb{R}^{3n+r}$ is the unique equilibrium point of the closed–loop system (21).

Let us define the constants

$$\gamma_1 = \frac{\delta\alpha}{\tanh(\alpha\sigma)}, \quad (22)$$

$$\gamma_2 = 2c_1 + \lambda_{max}\{F_v\}, \quad (23)$$

$$\gamma_3 = k_{C1}\sqrt{n} + \sigma\lambda_{max}\{M(q)\}, \quad (24)$$

$$c_1 = k_{C1}\|\dot{q}_d\|_M, \quad (25)$$

which will be useful in the proof of the following Proposition.

Proposition 1. *Assume that the damping introduced by the viscous friction coefficients F_v satisfies*

$$\lambda_{min}\{F_v\} > c_1. \quad (26)$$

Then, there exist observer gain $\xi \in (\xi_{min}, \xi_{max})$ and large enough $\lambda_{min}\{K_p\}$ such that the closed–loop system (21) is globally stable in the Lyapunov sense [7]. In addition,

$$\lim_{t\to\infty} \begin{bmatrix} \tilde{q}(t) \\ \dot{\tilde{q}}(t) \\ \tilde{\vartheta}(t) \end{bmatrix} = 0,$$

while $\tilde{\theta}(t)$ remains bounded for all time $t \geq 0$.

Proof: We propose the following Lyapunov function candidate

$$V_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta}) = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \sum_{i=1}^{n} k_{v_i} b_i^{-1}\ln(\cosh(\tilde{\vartheta}_i))$$

$$+\sum_{i=1}^{n} k_{p_i}\sigma^{-1}\ln(\cosh(\sigma\tilde{q}_i))$$

$$+\xi\tanh(\sigma\tilde{q})^T M(q)\dot{\tilde{q}} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$$

where ξ is the positive constant involved in the update law (**??**). A lower bound on $V_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta})$ is given by

$$V_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta}) \geq \eta^T P\eta + \sum_{i=1}^{n} k_{vi} b_i^{-1}\ln(\cosh(\tilde{\vartheta}_i)) + \frac{1}{2}\tilde{\theta}^T\Gamma\tilde{\theta}, \quad (27)$$

where

$$\eta = \begin{bmatrix} \sqrt{\sum_{i=1}^{n}\ln(\cosh(\sigma\tilde{q}_i))} \\ \|\dot{\tilde{q}}\| \end{bmatrix}$$

$$P = \begin{bmatrix} \sigma^{-1}\lambda_{min}\{K_p\} & -\frac{\xi}{2}\sqrt{2}\lambda_{max}\{M(q)\} \\ -\frac{\xi}{2}\sqrt{2}\lambda_{max}\{M(q)\} & \frac{1}{2}\lambda_{min}\{M(q)\} \end{bmatrix},$$

Property 1 of the robot model and

$$\sqrt{\sum_{i=1}^{n}\ln(\cosh(z_i))} \geq \frac{1}{\sqrt{2}}\|\tanh(z)\| \;\forall\, z = [z_1 \cdots z_n]^T, \quad (28)$$

were used. If P is positive definite, then the function $V_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta})$ is globally positive definite and radially unbounded. The sufficient and necessary condition for P to be positive definite is

$$\xi < \frac{\sqrt{\sigma^{-1}\lambda_{min}\{K_p\}\lambda_{min}\{M(q)\}}}{\lambda_{Max}\{M(q)\}}.$$

The time derivative of $V_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta})$ along of the closed–loop systems trajectories (21) is given by

$$\dot{V}_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta}) = \xi\tanh(\sigma\tilde{q})^T\Big[-F_v\dot{\tilde{q}} - K_v\tanh(\tilde{\vartheta})$$

$$-K_p\tanh(\sigma\tilde{q}) + C(q,\dot{q})^T\dot{\tilde{q}} - h(\tilde{q},\dot{\tilde{q}})\Big]$$

$$+\xi\dot{\tilde{q}}^T M(q)\mathrm{Sech}^2(\sigma\tilde{q})\sigma\dot{\tilde{q}} - \dot{\tilde{q}}^T F_v\dot{\tilde{q}} - \dot{\tilde{q}}^T h(\tilde{q},\dot{\tilde{q}})$$

$$-\tanh(\tilde{\vartheta})^T K_v B^{-1} A\tanh(\tilde{\vartheta}).$$

To obtain further conclusions on the closed–loop stability we compute a upper bound on each term of the Lyapunov function time derivative:

$$-\dot{\tilde{q}}^T F_v\dot{\tilde{q}} \leq -\lambda_{min}\{F_v\}\|\dot{\tilde{q}}\|^2,$$

$$-\dot{\tilde{q}}^T h(\tilde{q},\dot{\tilde{q}}) \leq c_1\|\dot{\tilde{q}}\|^2 + \gamma_1\|\tanh(\sigma\tilde{q})\|\|\dot{\tilde{q}}\|,$$

$$-\tanh(\tilde{\vartheta})^T K_v B^{-1}A\tanh(\tilde{\vartheta}) \leq -\lambda_{min}\{K_v B^{-1}A\} \times\|\tanh(\tilde{\vartheta})\|^2,$$

$$\xi\tanh(\sigma\tilde{q})^T C(q,\dot{q})^T\dot{\tilde{q}} \leq \xi k_{C1}\|\dot{q}\|\|\tanh(\sigma\tilde{q})\|\|\dot{\tilde{q}}\|$$

$$\leq \xi k_{C1}\sqrt{n}\|\dot{\tilde{q}}\|^2 + \xi c_1\|\tanh(\sigma\tilde{q})\|\|\dot{\tilde{q}}\|,$$

$$\xi\dot{\tilde{q}}^T M(q)\mathrm{Sech}^2(\sigma\tilde{q})\sigma\dot{\tilde{q}} \leq \xi\sigma\lambda_{max}\{M(q)\}\|\dot{\tilde{q}}\|^2,$$

$$-\xi\tanh(\sigma\tilde{q})K_v\tanh(\tilde{\vartheta}) \leq \xi\lambda_{max}\{K_v\}\|\tanh(\sigma\tilde{q})\| \times\|\tanh(\tilde{\vartheta})\|,$$

$$-\xi\tanh(\sigma\tilde{q})^T K_p\tanh(\sigma\tilde{q}) \leq -\xi\lambda_{min}\{K_p\}\|\tanh(\sigma\tilde{q})\|^2,$$

$$-\xi\tanh(\sigma\tilde{q})^T F_v\dot{\tilde{q}} \leq \xi\lambda_{max}\{F_v\}\|\tanh(\sigma\tilde{q})\|\|\dot{\tilde{q}}\|,$$

$$-\xi\tanh(\sigma\tilde{q})^T h(\tilde{q},\dot{\tilde{q}}) \leq \xi c_1\|\tanh(\sigma\tilde{q})\|\|\dot{\tilde{q}}\|, +\xi\gamma_1\|\tanh(\sigma\tilde{q})\|^2. \quad (29)$$

The previous bounds have been obtained by using the inequalities of Property 2 of the robot model, Property 3 of the residual dynamics $h(\tilde{q},\dot{\tilde{q}})$, the fact $\|\dot{q}\| \leq \|\dot{q}_d\|_M + \|\dot{\tilde{q}}\|$, and properties of the hyperbolic functions, see the Appendix A.

On this way, we are able to write

$$\dot{V}_1(t,\tilde{q},\dot{\tilde{q}},\tilde{\vartheta},\tilde{\theta}) \leq -\begin{bmatrix}\|\tanh(\sigma\tilde{q})\| \\ \|\dot{\tilde{q}}\|\end{bmatrix}^T Q_1 \begin{bmatrix}\|\tanh(\sigma\tilde{q})\| \\ \|\dot{\tilde{q}}\|\end{bmatrix}$$

$$-\begin{bmatrix} \|\tanh(\sigma\tilde{q})\| \\ \|\tanh(\tilde{\vartheta})\| \end{bmatrix}^T Q_2 \begin{bmatrix} \|\tanh(\sigma\tilde{q})\| \\ \|\tanh(\tilde{\vartheta})\| \end{bmatrix} \tag{30}$$

where γ_1, γ_2, γ_3 and c_1 were defined in (22), (23), (24) and (25), respectively,

$$Q_1 = \begin{bmatrix} \frac{\xi}{4}[\lambda_{min}\{K_p\} - \gamma_1] & -\frac{1}{2}\gamma_1 \\ -\frac{1}{2}\gamma_1 & [1-\beta]\lambda_{min}\{F_v\} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\xi}{4}[\lambda_{min}\{K_p\} - \gamma_1] & -\frac{1}{2}\xi\gamma_2 \\ -\frac{1}{2}\xi\gamma_2 & \beta\lambda_{min}\{F_v\} - c_1 - \xi\gamma_3 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} \frac{\xi}{2}[\lambda_{min}\{K_p\} - \gamma_1] & -\frac{\xi}{2}\lambda_{max}\{K_v\} \\ -\frac{\xi}{2}\lambda_{max}\{K_v\} & \lambda_{min}\{K_vB^{-1}A\} \end{bmatrix}$$

where $\beta \in (0,1)$. Then, the sufficient condition for Q_1 to be positive definite are

$$\xi > \frac{\gamma_1^2}{[1-\beta][\lambda_{min}\{K_p\} - \gamma_1]\lambda_{min}\{F_v\}},$$

$$\xi < \frac{[\lambda_{min}\{K_p\} - \gamma_1][\beta\lambda_{min}\{F_v\} - c_1]}{[\lambda_{min}\{K_p\} - \gamma_1]\gamma_3 + \gamma_2^2},$$

which are subject to

$$\lambda_{min}\{K_p\} > \gamma_1.$$

Finally, the sufficient and necessary condition for Q_2 to be positive definite is

$$\xi < \frac{2[\lambda_{min}\{K_p\} - \gamma_1]\lambda_{min}\{K_vB^{-1}A\}}{\lambda_{max}\{K_v\}^2}$$

Then, for all

$$\frac{\gamma_1^2}{[1-\beta][\lambda_{min}\{K_p\} - \gamma_1]\lambda_{min}\{F_v\}} < \xi < \min\left\{ \frac{[\lambda_{min}\{K_p\} - \gamma_1][\beta\lambda_{min}\{F_v\} - c_1]}{[\lambda_{min}\{K_p\} - \gamma_1]\gamma_3 + \gamma_2^2}, \frac{2[\lambda_{min}\{K_p\} - \gamma_1]\lambda_{min}\{K_vB^{-1}A\}}{\lambda_{max}\{K_v\}^2}, \frac{\sqrt{\sigma^{-1}\lambda_{min}\{K_p\}\lambda_{min}\{M(q)\}}}{\lambda_{Max}\{M(q)\}} \right\}, \tag{31}$$

the matrices P, Q_1 and Q_2 are positive definite, which implies that the $V_1(t, \tilde{q}, \dot{\tilde{q}}, \tilde{\vartheta}, \tilde{\theta})$ is globally positive definite and radially unbounded function, and that its time derivative $\dot{V}_1(t, \tilde{q}, \dot{\tilde{q}}, \tilde{\vartheta}, \tilde{\theta})$ is globally negative definite. Therefore, the state space origin of the closed–loop system (21) is stable in the Lyapunov sense. Besides, we have that

$$[\tilde{q}(t)^T\ \dot{\tilde{q}}(t)^T\ \tilde{\vartheta}(t)^T\ \tilde{\theta}(t)^T]^T,$$

$$[\dot{\tilde{q}}(t)^T\ \ddot{\tilde{q}}(t)^T\ \dot{\tilde{\vartheta}}(t)^T\ \dot{\tilde{\theta}}(t)^T]^T \in L_\infty^{3n+r}.$$

From inequality (30), it can be show that $[\tilde{q}(t)^T\ \dot{\tilde{q}}(t)^T\ \tilde{\vartheta}(t)^T]^T \in L_2^{3n}$. Hence, by invoking Barbalat's lemma, we conclude that

$$\lim_{t\to\infty} \begin{bmatrix} \tilde{q}(t) \\ \dot{\tilde{q}}(t) \\ \tilde{\vartheta}(t) \end{bmatrix} = 0,$$

which completes the proof of Proposition 1.

△△△

It is possible to show that by using a large enough value of $\lambda_{min}\{K_p\}$ the inequality (31) is easily satisfied. In fact, the inequality (31) suggests a tuning procedure for obtaining a proper numerical value of $\lambda_{min}\{K_p\}$. This can be derived from relating the left and right hand–side of (31).

With relation to the result stated in Proposition 1, in the paper [11] the global stability of a non–adaptive controller was shown by using the assumption that high enough damping of the viscous friction coefficients is present.

4 Conclusion

This paper addressed the tracking control of robot manipulators by using only joint position measurements (output feedback tracking control). This paper proposed a new design considering the practical situation that the robot is subject to constrained torques. The stability analysis of the closed–loop system coming from a controller/observer scheme was shown. Besides, we also proved that the position and velocity error signals converge to zero for any initial condition of the closed–loop system.

Acknowledgements: The research was supported by the *Secretaría de Investigación y Postgrado del IPN* and *Consejo Nacional del Tecnología*, Mexico.

References:

[1] S. Arimoto, "Fundamental problems of robot control: part 1, innovations in the realm of robot servo-loops" , Robotica, Vol. 13, pp. 19–27, 1995.

[2] S. Arimoto, "Fundamental problems of robot control: part 2, a nonlinear circuit theory towards an understanding of dexterous motions", Robotica, Vol. 13, pp 111–122, 1995.

[3] M.A. Arteaga, "Robot control and parameter estimation with only joint position measurements", Automatica, Vol. 39, pp. 67–73, 2003.

[4] C. Canudas de Wit, B. Siciliano and G. Bastin (eds.), *Theory of robot control*, Springer-Verlag, London, 1996.

[5] W. Dixon, E. Zergeroglu, D.M. Dawson and M.W. Hannan, "Global adaptive partial state feedback tracking control of rigid–link flexible–joint robots", Robotica, Vol. 18, pp. 325-336, 2000.

[6] R. Kelly, R. Salgado, "PD control with computed feedforward of robot manipulators: A design procedure", IEEE Trans. Rob. Aut. Vol. 10 No. 4, pp. 566–571, 1994.

[7] H. Khalil, *Nonlinear Systems*, Prentice–Hall, Upper Saddle River, 1996.

[8] A. Loria, "Global tracking control of one degree of freedom Euler–Lagrange systems without velocity measurements", European Journal of Control, Vol. 2, No. 2, pp 144–151, 1996.

[9] R. Ortega, A. Loria, P.J. Nicklasson and H. Sira–Ramirez, *Pasivity–Based Control of Euler–Lagrange Systems*, Springer, London, 1998.

[10] V. Santibáñez and R. Kelly, "PD control with computed feedforward for robor manipulators: analysis and experimentation", Robotica, Vol. 19, pp. 11–19, 2001.

[11] V. Santibáñez and R. Kelly, "Global asymptotic stability of bounded output feedback tacking control for robot manipulators", 40th IEEE Conf. Decision and Control, Orlando FL, pp. 1378–1379, Dec. 2001.

[12] M.W. Spong and M. Vidyasagar, *Robot dynamics and control*, John Wiley and Sons, New York, 1989.

[13] L. Sciavicco and B. Siciliano, *Modeling and control of robot manipulators*, Springer, London, 2000.

[14] E. Zergeroglu, D. M. Dawson, M.S. de Queiroz, and M. Krstic, "On Global Output Feedback Tracking Control of Robot Manipulators", Proc. of the IEEE Conference on Decision and Control, Sydney, Australia, pp. 5073-5078, Dec. 2000.

[15] F. Zhang, D.M. Dawson, M.S. de Queiroz and W. Dixon, "Global adaptive output feedback tracking control of robot manipulators", IEEE Transactions on Automatic Control, Vol. 45, No.6 pp. 1203–1208, 2000.