Modelling plankton dynamics in brackish waters

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Abstract: - In this study, a four-compartment non-linear mathematical model is proposed to understand plankton dynamics in brackish water bodies. Local stability criteria are derived which are also used to estimate the range of values of model parameters. Numerical experiments and sensitivity tests with different parameters show that of all the seventeen parameters in the model the critical parameters affecting the stability of the solution are the growth rate of the phytoplankton which depends on light and nutrients.

As a case study, the model is then applied to Chilika Lagoon (19⁰28'N-19⁰54'N and 85⁰06'E-85⁰36'E), the largest brackish water lagoon with estuarine character on the east coast of India. A distinct salinity gradient exists along the lagoon due to the mixing of freshwater and saline water from the adjoining sea. This salinity gradient has a pronounced effect on the seasonal variability of the freshwater and marine phytoplankton species in the lagoon. The model is tested for different sectors of the lagoon which have distinct characteristics in terms of depth, light penetration, nutrients, salinity and hence planktonic growth. A fine tuning of the model parameters was done in order to validate the model results with the available data of the lagoon. Even in its simplest form, the model is able to reproduce most of the significant characteristics of the planktonic distribution in the lagoon-the bimodal oscillations, major plankton peaks etc.

Key-words: Plankton, Local Stability, Seasonal variability, Chilika Lagoon

1 Introduction

Several difficulties arise while developing an ecological model for an aquatic/marine ecosystem. In addition to the practical difficulties of getting an accurate data for validation modelers are often faced with the formidable task of selecting the right functional representation of the complex ecosystem. The complexity of the model should depend upon the purpose of the model and the availability of data [1]. A simple ecological model can be formulated on the basis of only three primary dependent systems: phytoplankton, herbivorous zooplankton and the nutrient system. Any complexity due to the addition of more state variables in such a simple model will influence the dynamics of the model. A lot of research has been done in order to examine the changes in the dynamics of the ecosystem models due to the incorporation of more variables in such models [1]; [2]; [3]; [4].

An ecosystem model is largely dependent upon the values of the parameters included in the model equations. The model can exhibit different dynamics depending upon the changes in the parameter values. Depending on the response of the model to the changes in the parameter values, the parameters can be classified as critical or non-critical. For analyzing the changes in dynamics in accordance with the changes in the parameter values a local stability analysis of the model equations is essential. A local stability analysis also helps in restricting the degrees of freedom of the parameter values [5]; [6]; [7]; [8].

In the present paper the effect of salinity changes on the ecosystems is analysed which is an important aspect of the brackish water ecosystems. In brackish waters there are no firm salinity boundaries between fresh and brackish waters. In principle, therefore, a complete continuum of concentration must exist between freshwater and sea water. It is for this reason that brackish waters may develop much greater ecological diversity than fully marine water bodies and have communities which comprise representatives from both fresh and marine waters. Changes in salinity in brackish waters serve as indirect switch mechanisms which may cause the lagoon community to change from submerged plant dominance to phytoplankton dominance [9]; [10].

Despite longstanding recognition of the role of salinity as a primary influence on plankton in various types of aquatic ecosystems, there have been very few experimental studies of the effects of salinity on plankton communities [11]; [12]; [13]. Ecological models developed for modelling phytoplankton distributions are either for freshwaters or for marine ecosystems and hence do not take into account the effect of salinity variations on the plankton community.

In this paper a model is developed to study the seasonal variability of plankton in shallow brackish water lagoons using a four-compartment ecological model. An important aspect of the study is estimating the range for parameter values using a local stability analysis for the system of equations. The model is essentially a four-compartment (NP_1P_2Z) model which includes nutrients(N), freshwater plankton (P_1) , marine plankton (P_2) and zooplankton (Z). Section 2 gives the formulation of the mathematical model and the details of the local stability analysis. This is followed by section 3 which deals with the case study of Chilika lagoon in India. Section 4 gives numerical experiments, model validation and a discussion about the range of parameter values used in the model. Finally, the important conclusions are presented in section 5.

2 **Problem Formulation**

The basic general equations that give the time evolution of a chemical or biological quantity are given by

$$\frac{dB_i}{dt} = S_i + D_i$$
 i=1, 2, (1)

where, Bi represents the concentration of the ith chemical or biological species, Si is the source term and Di the decay term which is defined for each variable B_i by a function depending upon the concentration of some other variables B_j as well as B_i itself.

We consider two interacting phytoplankton species P_1 (freshwater) and P_2 (marine). The model simulates the effect of salinity on P_1 and P_2 as well the interaction and coexistence of the two species. The model equations are:

$$\frac{dN}{dt} = \left[\frac{\overline{\alpha}_{1}N}{K_{N,1} + N} - r_{1} \times f_{1}(S)\right] P_{1} - \left[\frac{\overline{\alpha}_{2}N}{K_{N,2} + N} - r_{2} \times f_{2}(S)\right] P_{2} + \frac{m_{1}}{D}N_{0}(t) \quad (2)$$

$$\frac{dP_{1}}{dt} = \left[\frac{\overline{\alpha}_{1} N}{K_{N,1} + N} - r_{1} \times f_{1}(S)\right] P_{1} - \frac{p_{1} \times P_{1}}{A} \frac{c (A - P_{0}) Z}{(K_{Z} + A - P_{0})}$$
(3)

$$\frac{\mathrm{dP}_2}{\mathrm{dt}} = \left[\frac{\overline{\alpha}_2 N}{K_{N2} + N} - r_2 \times f_2(S)\right] P_2 - \frac{p_2 \times P_2}{A} \frac{c(A - P_0)Z}{(K_z + A - P_0)}$$
(4)

$$\frac{dZ}{dt} = \frac{e \times c \times (A - P_0) \times Z}{(K_z + A - P_0)} - g \times Z$$
(5)

where, the concentrations of nutrients, phytoplankton and zooplankton (N, P₁, P₂, Z) are measured in mg/l, t is time, $\overline{\alpha}_i$ (i=1,2) (d⁻¹) is the light limited growth rate of phytoplankton, r_i (i=1,2)

 (d^{-1}) is the metabolic, respiratory loss rate of phytoplankton (P₁ and P₂). K_{N,i} (i=1,2)(mg/l) and K_Z (mg/l) are the half saturation coefficients for nutrient uptake and zooplankton grazing respectively, c (d^{-1}) is the grazing rate and P₀ (mg/l) the grazing threshold. N₀(t) is the source of nutrients, m₁ (m d^{-1}) is the vertical diffusion rate and D the depth in meters. e is the grazing efficiency and g (d^{-1}) is the loss of zooplankton to carnivores. p_i (i=1,2) is the zooplankton. 'A' is the phytoplankton perceived by the herbivorous zooplankton and is defined

as $\sum_{i=1}^{2} p_i \times P_i$. The terms $f_i(S)$ (i=1,2) representing the

respiration response to salinity of P_1 and P_2 repectively [14], are a function of salinity S(ppt) and vary depending on the type of phytoplankton species. The present model reduces to the model defined by [15] with $f_i(S)=1$ for a single planktonic species. The effect of $f_1(S)$ is to increase the respiration rate as salinity increases above the optimal level of salinity for the freshwater phytoplankton and hence decrease the plankton. Similarly $f_2(S)$ also leads to an increase in the respiration rate as the salinity levels decrease from the optimal salinity levels for the marine phytoplankton and cause a decrease in the plankton. The first term within the brackets in equation (3) is the growth term for phytoplankton. It is represented as the product of the light dependent phytoplankton growth rate or photosynthesis ($\overline{\alpha}_i$) and the nutrient limited phytoplankton growth rate $\left(\frac{N}{K_{N,i} + N}\right)$ (Holling

type II functional response). The functional form for the light limited growth rate is taken from [16] which is adopted from [17]. The second term within the brackets in equation (3) gives the respiratory loss from the phytoplankton population. The third term $\left(-\frac{p_i \times P_i}{A} \frac{c(A - P_0)Z}{(K_z + A - P_0)}\right)$ represents the loss of the ith

phytoplankton due to grazing by zooplankton. This, again, is introduced following Holling type II functional response. A grazing threshold parameter P_0 is introduced in this term, it ensures the survival of at least P_0 amount of phytoplankton in spite of the grazing by zooplankton. The nutrient equation (equation (2)), the first term is the loss of nutrients due to the growth of phytoplankton. The second term $\left(\frac{m_1}{D}N_0(t)\right)$ represents the addition of nutrients

into the system through vertical diffusion from the sediments. $N_0(t)$ is the source from the sediments for the nutrients. The first term in equation (5) is the

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increase in zooplankton due to take up of phytoplankton. However, not all of the consumed phytoplankton is converted to useful energy by zooplankton, the amount of phytoplankton used by zooplankton is given by their assimilation efficiency 'e'. The second term is the loss of zooplankton due to predation by higher carnivores.

2.2 Local stability analysis: Routh-Hurwitz conditions

A local stability analysis of the system of equations is very essential for parameter estimation. The parameter values which are not taken from the literature need to be fixed. For making a rough estimate of the range of parameters involved, a local stability analysis of the system of equations is undertaken.

To determine the equilibrium points of the system of equations (2)-(5) we put

$$\frac{dN}{dt} = 0, \frac{dP_1}{dt} = 0, \frac{dP_2}{dt} = 0, \frac{dZ}{dt} = 0$$
(6)

Two equilibrium points exist for the above set of equations (2)-(5). These points are given by:

$$N^{1*} = \frac{K_{N,1}[R_1 \times S_1 + r_1 f_1(S)]}{[\alpha_1 - (R_1 \times S_1 + r_1 f_1(S))]}$$
(7)

where S_1 is defined according to (1) and

$$R_{1} = \frac{P_{1}}{\left(\frac{g}{ec-g} + P_{0}\right)}$$
(8)

Similarly we can get the second equilibrium value for N i.e., N²* by substituting the values for P₂. The equilibrium values of P₁ⁱ*(i=1,2) are given by

$$P_{1}^{i*} = \frac{1}{p_{1}} \left(\frac{g}{ec - g} + P_{0} - p_{2} \left(\frac{S_{1} - \frac{k_{1}}{p_{1}} \times \left[\frac{gK_{Z}}{(ec - g)} + P_{0} \right]}{\left[k_{2} - \frac{p_{2}}{p_{1}} k_{1} \right]} \right) \right) (i = 1, 2)$$
(9)

where,

$$\mathbf{k}_{1} = \left[\frac{\overline{\alpha}_{1} \mathbf{N}^{i} *}{\mathbf{K}_{N,1} + \mathbf{N}^{i} *} - \mathbf{r}_{1} \times f_{1}(\mathbf{S})\right] \text{and}$$
(10(a))

$$\mathbf{k}_{2} = \left[\frac{\overline{\alpha}_{2} \, \mathbf{N}^{i} *}{\mathbf{K}_{N,2} + \mathbf{N}^{i} *} - \mathbf{r}_{2} \times f_{2}(\mathbf{S}) \right]$$
(10(b))

The equilibrium values of $P_2^{1*}(i=1,2)$ are given by

$$P_{2}^{i*} = \frac{S_{1} - \frac{k_{1}}{p_{1}} \times \left[\frac{gK_{Z}}{(ec - g)} + P_{0} \right]}{\left[k_{2} - \frac{p_{2}}{p_{1}} k_{1} \right]}$$
(11)

The equilibrium values for Z are given by

$$Z^{i*} = \frac{\mathrm{em}_1 \mathrm{N}_0}{\mathrm{gD}} \tag{12}$$

In order to find about the stability of the

equilibrium points we need to linearize the system of equations (2)-(5) and find the Jacobian matrix. For this purpose we define the following: Let

$$\mu^{i}(t) = N(t) - N^{i} *$$

$$\eta^{i}_{1}(t) = P_{1}(t) - P_{1}^{i} *$$

$$\eta^{i}_{2}(t) = P_{2}(t) - P_{2}^{i} *$$

$$v^{i}(t) = Z(t) - Z^{i} *$$
(13)

where for the sake of simplicity we have dropped the superscript i (i=1,2) from N^{i*} , P_1^{i*} , P_2^{i*} and Z^{i*} . Substituting in equations (2)-(5) and neglecting the product terms, we get

$$= \begin{bmatrix} -(k_{3}+k_{4}) & -k_{1} & -k_{2} & 0\\ k_{3} & k_{1}-k_{5}P_{2}*+p_{1}^{2}k_{6}P_{1}* & k_{5}P_{1}*+p_{1}p_{2}k_{6}P_{1}* & -\frac{A}{P_{2}}k_{5}P_{1}*\\ k_{4} & k_{5}P_{2}*-p_{1}p_{2}k_{6}P_{2}* & k_{2}-k_{5}P_{2}*-p_{2}^{2}k_{6}P_{2} & -\frac{A}{P_{1}}k_{5}P_{2}*\\ 0 & p_{1}k_{7} & p_{2}k_{7} & k_{8} \end{bmatrix}$$
(15)

is the Jacobian matrix and

$$k_{3} = \frac{\overline{\alpha}_{1} N * P_{1} *}{(K_{N,1} + N *)^{2}}, \quad k_{4} = \frac{\overline{\alpha}_{2} N * P_{2} *}{(K_{N,3} + N *)^{2}}$$

$$k_{5} = \frac{p_{1}p_{2}cZ * (A - P_{0})}{A^{2}(K_{Z} + A - P_{0})}, \quad k_{6} = \frac{cZ * K_{Z}}{A(K_{Z} + A - P_{0})^{2}}$$

$$k_{7} = \frac{ecZ * K_{Z}}{(K_{Z} + A - P_{0})^{2}}, \quad k_{8} = \frac{ec(A - P_{0})}{(K_{Z} + A - P_{0})} - g$$
(16)

The eigenvalues λ of the matrix A now satisfy $det(J - \lambda I) = 0$, where I is a 4x4 identity matrix. Therefore λ must satisfy a characteristic equation of the form

$$\lambda^4 + a_1^i \lambda^3 + a_2^i \lambda^2 + a_3^i \lambda + a_4^i = 0, \quad (i = 1, 2) \quad (17)$$

where (suppressing the superscript).

$$a_{1} = -(k_{8} + k_{2} - k_{5}P_{1} * -p_{2}^{2}k_{6}P_{2} * -k_{1})$$
(18(a))

$$a_{2} = a_{1}([(k_{3} + k_{4}) - [p_{1}k_{6}(p_{1}P_{1} - p_{2}P_{2})] - k_{8} - k_{1}) - k_{4} + k_{2}(k_{5}P_{2} * -p_{1}p_{2}k_{6}P_{2} * -k_{4})$$
(18(b))

$$a_{3} = -(p_{1}k_{6})(p_{1}P_{1}*-p_{2}P_{2}*)(a_{1}(k_{3}+k_{4})+ -a_{1}k_{8} + \frac{k_{7}p_{2}Ak_{5}P_{2}}{p_{1}} - k_{2}k_{4}] + a_{1}k_{8}(1+k_{4}) + k_{4}k_{8}(1+k_{2}) - k_{1}\frac{k_{7}p_{2}Ak_{5}P_{2}}{p_{1}} + k_{2}P_{2}*[k_{8}k_{5} - p_{1}p_{2}k_{8}k_{6}P_{2}*+k_{7}Ak_{5}P_{2}* + k_{4}p_{2}k_{6}(p_{1}P_{1}*-p_{2}P_{2}*)$$
(18(c))

$$\begin{aligned} a_{4} &= -(p_{1}k_{6})(p_{1}P_{1}*-p_{2}P_{2}*)[(k_{3}+k_{4})\left(a_{1}-\frac{k_{7}p_{2}Ak_{5}P_{2}*}{p_{1}}\right) \\ &-k_{1}k_{4}k_{8}\frac{p_{2}}{p_{1}}]+A^{2}k_{4}k_{5}k_{7}\left(\frac{1}{p_{2}}-\frac{k_{1}}{p_{1}}\right) \end{aligned} \tag{18(d)}$$

For the stability of the solutions of the system of equations (2)-(5) all the roots of the characteristic equation (15) or all the eigenvalues of the Jacobian matrix (15) should have negative real parts. This can be done without actually solving for all the roots of the characteristic equation by applying the Routh-Hurwitz stability criterion [18]; [19]. The Routh-Hurwitz conditions which give the necessary and sufficient condition for all the roots of the characteristic equations to have negative real roots thus implying asymptotic stability are as follows:

$$a_1^i > 0, a_3^i > 0 \text{ and } a_1^i a_2^i a_3^i > a_3^{i^2} + a_1^{i^2} a_4^i \text{ (i=1,2)}$$
 (19)

The inequalities given in (19) for the two equilibrium points, pose a restriction on the permissible degrees of freedom for the parameters which can be modeled in the equations (2)-(5). This is very essential for the parameter estimation. The solutions for equations (2)-(5), keeping the restrictions of the inequalities posed by the Routh-Hurwitz conditions will all be asymptotically stable.

3 CHILIKA LAGOON-a case study

3.1 Description of Study Area

Chilika lagoon $(19^{0}28^{\circ}N-19^{0}54^{\circ}N)$ and $85^{0}06^{\circ}E-85^{0}36^{\circ}E)$ on the east coast of India (Figure 1), is one of the unique ecospheres in the world. It is the largest brackish water lagoon with estuarine character. On account of its rich biodiversity and socio-economic importance, it was designated as a 'Ramsar site'- a wetland of international importance in 1981 [20].

The water-spread area of the Chilika Lagoon varies between 1165 to 906 sq km during the monsoon and summer respectively. A significant part of the fresh water and silt input to the lagoon comes from river Mahanadi and its distributaries. Based on the physical and dynamical characteristics of the lagoon, the lagoon is divided into four sectors (Fig 1). The northern sector receives discharge of the floodwaters from the rivers. It is the shallowest region of the lagoon and has the highest nutrient concentrations, which are brought in by the river waters draining in the lagoon basin. The southern sector is relatively smaller and does not show much seasonal variation in any of the hydrographic parameters. Southern sector is also observed to be poor in nutrients. The central sector has features intermediate of the other two sectors. The lagoon is shallow (average depth 2.5 m), the northern sector is the shallowest region of the lagoon and hence light penetration is higher. The southern sector is the deepest out of the three sectors and light penetration in this region of the lagoon is the lowest. The lagoon is separated from the Bay of Bengal by a sand bar 60 km in length. A distinct salinity gradient exists along the lagoon due to the influx of fresh water from the rivers and the inflow of seawater through the outer channel. The width of the original and natural inlet is about 1.5 km [21].

3.2 Plankton Characteristics of Chilika Lagoon

Chilika Lagoon constitutes a typical environment for the study of marine algae because of its variable hydrological conditions from place to place and from season to season. A number of algal forms belonging to Cyanophyta, Chlorophyta, Bacillariophyta, Dinophyta and Rhodophyta are found in the different sectors of the lagoon [22]. Biological production in Chilika was studied in the form of total Chlorophyll pigment found in the lagoon waters. Chlorophyll-a showed much higher concentrations in the northern sector as compared to the southern sector [23]; [24]; [25]. Chlorophyll-b and c concentrations are seen to be much lower than the Chlorophyll-a concentrations in all the sectors so their role in the productivity is not very significant.

Distribution of phytoplankton in the different sectors shows wide range of variations in physicochemical features and habitat preferences of these organisms. The total biomass of phytoplankton is observed to be the highest in the northern sector followed by the central sector and lowest in the southern sector. The annual cycle of phytoplankton densitv shows bimodal oscillations. Also. phytoplankton are seen to be maximum in summer followed by winter and the rainy seasons. The primary peak is found to occur during March-May followed by a secondary peak in November [26]. The time series data of plankton analysed by [27] shows that there are two distinct peaks of phytoplankton, one between March to June and the other in October depending on the sector considered. The minimum is observed during monsoon months (July-September).

Many field studies have also been conducted to analyze the chemical nature of Chilika Lagoon. These studies show that nitrate, phosphate and silicate are the predominant type of nutrients occurring within the lagoon. Nutrient dynamics in Chilika is controlled by two factors namely: addition by the river runoff and depletion caused due to the uptake by phytoplankton. Maximum concentrations of these nutrients are observed during the rainy season and the lowest during summer [26]. April and May are seen to be the months of a rapid depletion of nutrients, caused by the rapid growth of phytoplankton. Among the three nutrients, silicate showed more pronounced spatial variations [28]. Sectorwise, more nutrients concentration was observed in the northern sector where the effect of freshwater was more [28]. Apart from the data of [27], which is for an entire year taken at specific sites in different sectors, rest of the data is patchy. Also, the other data sets follow a different unit of measurement i.e., mgChl-a/l, mgC/litre etc, as opposed to number of phytoplankton per litre used by [27]. Moreover, all the data is not for the same year. Since the data of [27] is a continuous data, it is their data which have been mainly used in this study to validate the model.

Several field studies have been conducted to study the zooplankton distribution of Chilika lagoon [23]. Copepods are seen to be the most abundant among all the zooplankton species in the lagoon. Zooplankton were found to be abundant in the central sector and very low in numbers in the southern sector and near the river opening in the northern sector. Seasonally, zooplankton population is higher during the pre-monsoon period as compared to post-monsoon [23].

The above mentioned studies about the plankton population of Chilika Lagoon are based on the observations taken from various sites from the lagoon. Though such studies are very helpful in understanding the distribution of the different phytoplankton species within the lagoon, they need to be consolidated and augmented using time series data. Moreover, the studies are not uniform in terms of methodology and units of measurements. For example, [27] give a time series data for plankton concentration measured as average number of plankton found per litre. Other available data sets are patchy and follow different units of measurement of plankton densities like mgChl-a/litre, mgC/litre etc. However, to be able to predict the future distributions of the phytoplankton species and also to study the effect of different restorative measures taken by the Chilika Development Authority (CDA) on the biological species it is essential to develop a model for biological production in the lagoon.

4 Numerical Experiments and Prediction of Range of Values for the Parameters

The model discussed in section 2 is used to simulate the effect of salinity on the annual distribution of plankton and nutrients in the different sectors of Chilika Lagoon. Comparisons are made between the distributions of the freshwater and marine phytoplankton species in each of the sectors. Since data is not available for the annual distribution of freshwater (P₁) and marine (P₂) phytoplankton separately, for the purpose of validation, the total phytoplankton population (P₁+P₂) is considered.

There are, in total, seventeen parameters involved in the model equations. Out of these, the parameters for which the value is taken from literature and observations are depth (D), low light photosynthetic slope (σ_i , i=1,2), maximum photosynthetic rate (Q_i i=1,2) and light attenuation by phytoplankton (k_i , i=1,2), optimal salinity (S_{OPT}), β , the palatability of different phytoplankton species to herbivores (p_i, i=1,2). These values are considered to be fixed and have not been tuned during the model simulations. The value for the nutrient source term N_0 is taken from the available data given by [29]. Fine tuning for the rest of the parameters was required. The model is seen to be very sensitive to the value of parameters $r_{i,}$, $K_{N,i}$, m_1 , c and e hence they are considered to be critical for Chilika. The model results along with the reasoning for the choice of the range of the parameters are discussed as follows:

The model results are given in figs 2(a-c) and 3(a-c). Figs 2(a-c) depict the total phytoplankton in the northern, central and southern sectors of the lagoon. Figs 3(a-c) give a comparison between the freshwater and marine phytoplankton in each of the sectors. The northern sector is full of freshwater throughout the year and hence the salinity level in the northern sector is lower than the optimal level required for freshwater and marine phytoplankton. Hence, the value for $f_1(S)=1$ for the freshwater phytoplankton. Marine phytoplankton are almost negligible in this region [22].

Central sector has freshwater coming in from the northern sector and saline water coming from the outer channel and hence has brackish water characteristics. The salinity levels of the central sector are higher than the northern sector but they are still lower than the optimal salinity levels for the freshwater and marine phytoplankton species. Therefore, for the central sector $f_1(S)=1$ whereas $f_2(S)>1$. The results for the central sector are given

in figs 2(b) and 3(b).

The southern sector is found to be a low salinity area and the salinity range in the southern sector is also seen to be lower than the optimal salinity levels required for both the freshwater and marine phytoplankton species. So, for the southern sector $f_1(S)=1$ whereas $f_2(S)>1$. The results of the numerical simulations are given in figs 2(c) and 3(c). Figs 2(a-c) depict the total annual phytoplankton distribution in the northern, central and southern sectors respectively along with the observed values from [27]. For the validation of the simulated results, fine tuning and sensitivity analysis of the parameters were required.

In the northern sector, since the marine phytoplankton are almost negligible. The sensitivity of the model to the parameters for the freshwater phytoplankton are only discussed here. Through the sensitivity analysis it was found that the growth rate of P₁ was the most effective parameter in controlling the phytoplankton distribution. Since the growth rate is expressed as a product of different terms (first term within the bracket of equation (3)), an individual discussion about the relative importance of each of the terms is required. The parameter involved in the nutrient limited growth rate is the half saturation coefficient for nutrient uptake $K_{N,1}$. As expected, increase in the half saturation of nutrient uptake will lead to a decrease in the phytoplankton population. The admissible range for $K_{N,1}$ in the Northern sector is (10.0-12.0). Decrease in the value of $K_{N,1}$ below the specified range leads to an increase in the phytoplankton and instability of the solution. An increase in $K_{N,1}$ above the specified range results in an increase in the peak value of the phytoplankton population. Another controlling factor for phytoplankton is the respiration rate ' r_1 '. Since respiration rate is a sink for phytoplankton, any increase in r_1 leads to a decrease in phytoplankton population. The range for r1 in the Northern sector is found to be (0.22-0.3). The model is seen to be very sensitive to both of the above parameters. The value of the vertical diffusivity 'm₁' in the northern sector is found to be 0.05. Phytoplankton distribution is found to be very sensitive to the values 'm1' since it controls the input of nutrients into the system. Any decrease in m_1 i.e., $m_1 < 0.05$ results in lower number of nutrients entering into the system and hence a lower productivity of phytoplankton. Even a slight increase in m1 results in the instability of the solution.

Increase in the grazing rate leads to a decrease in the phytoplankton and a corresponding increase in the zooplankton populations. The range for 'c' in the northern sector is (0.058-0.06). Increase in the value of c leads to an increase in the zooplankton population and the solution does not reach steady state. Changes in the assimilation efficiency 'e' also have similar effects as the grazing rate i.e., an increase in e results in an increase in zooplankton population (without reaching steady state) and a decrease in e leads to the death of zooplankton population. The range of e in the northern sector is found to be (0.8-0.92).

Changes in the value of the grazing threshold P_0 do not show significant changes in the phytoplankton population. The range for P0 in the northern sector is found to be (5.0-9.0). Decrease in the value of P_0 results in the increase of zooplankton population and the solution does not reach a steady state. Increase in P_0 i.e., P_0 >8.0 leads to the extinction of zooplankton.

The half saturation coefficient for grazing, K_Z , also shows some effect on the phytoplankton population but its major effect is seen only on the zooplankton population. The range for K_Z in the northern sector is obtained as (30.0-65.0). A decrease in the value of K_Z results in the increase in zooplankton population without reaching a steady state solution.

Another factor affecting the zooplankton population is the loss rate of zooplankton to carnivores 'g'. The range of 'g' in the northern sector is obtained as (0.03-0.045). As expected, increase in g beyond this range leads to a decrease in the zooplankton and a decrease in g results in the increase of zooplankton population. Although the value of g has some effect on the zooplankton population, its effect on phytoplankton is not considerable. The sensitivity of the model to parameter values in the central and southern sector is the same as that in the northern sector. The range of parameter values for all sectors are listed in Table 1.

5 Conclusions

Our model simulation results show that out of the different terms in the growth rate term, changes in light limited growth rate and nutrient limited growth rate are the controlling factors for phytoplankton population. However the zooplankton population is seen to be affected by the grazing rate and the grazing threshold. The model does not show very significant changes due to the changes in zooplankton grazing half saturation, grazing threshold and loss of zooplankton to carnivores.

The model proposed has the potential to simulate the effect of salinity on the phytoplankton in different sectors. The model results have been validated against the available observations from [27]. With the importance and encouragement given to the preservation of Chilika Lagoon and the efforts and plans of Chilika Development Authority (C.D.A.), there are reasons to believe that more researchers will get interested. There will be more funding in future to support research in this area and more species specific data will be available. The present model, which makes a distinction between freshwater and marine plankton can be of great use for predicting the effect of changes in salinity of the lagoon once the model is calibrated with the data.

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Fig 1: Map of Chilika showing different



Fig 2(a)









Fig 3(a): Freshwater and Marine Phytoplankton in Northern Sector



Fig 3(b): Freshwater and Marine Phytoplankton in Central Sector



Fig 3(c): Freshwater and Marine Phytoplankton in Southern Sector

Parameters	Definition	Northern Sector	Central Sector	Southern Sector
$K_{\rm M}$ (mg/l)	Nutrient uptake half saturation	(10.0-12.0)	(10.0 - 12.0)	(11.5-17.0)
$\mathbf{K}_{N,1}$ (ing/1)		10.0	10.0	15.0
$K_{N,2}$ (mg/l)		(10.0-15.0)	(17.0-22.0)	(17.0-22.0)
<u>N,2</u> (<u>8</u> ,-)		10.0	20.0	20.0
$N_0(mg/l)$	Nutrient source ¹¹	150.89	107.95	91.25
$r_1 (d^{-1})$	Respiration rate of phytoplankton	(0.22-0.3)	(0.1-0.12)	(0.2-0.3)
		0.25		0.3
$r_2(d^{-1})$		(0.22-0.3)	(0.12-0.16)	(0.1-0.12)
$m (d^{-1})$	Vertical diffusion rate	0.23	0.15	0.1
m ₁ (u)	vertical diffusion rate	(0.058-0.06)	(0.07 - 0.076)	(0.07 - 0.076)
$c(d^{-1})$	Grazing rate	0.06	0.07	0.07
p1	Palatability of freshwater plankton to zooplankton **	0.4	0.4	0.4
p ₂	Palatability of marine plankton to zooplankton **	0.4	0.4	0.4
K _Z (mg/l)	Grazing half saturation	(30.0-65.0)	(55.0-82.0)	(50.0-75.0)
		30.0	70.0	50.0
$P_{o}(m\sigma/l)$	Grazing threshold	(5.0-9.0)	(5.0-8.0)	(1.0-3.0)
1 0(119/1)	Giuzing unconoiu	8.0	5.0	1.0
е	Assimilation efficiency	(0.8-0.92)	(0.8-0.85)	(0.3-0.6)
		0.9	0.8	0.4
g (d ⁻¹)	Loss rate of zooplankton	(0.03-0.045)	(0.01-0.018)	(0.01-0.013)
D (m)		0.03	0.015	0.01
D(m)	Low light photosynthetic slope § Maximum photosynthetic rate §	1.5	2.0	2.5
$\sigma_1(ly)$		0.04	0.04	0.04
$O_1(ly)$		0.02	0.02	0.02
$Q_1(d)$ $Q_2(d^{-1})$		2.3	2.3	2.3
$\frac{\sqrt{2}}{k_1} \left(d^{-1} \right)$	Light attenuation by phytoplankton	0.0	0.0	0.0
$k_1 (d^{-1})$		0.1	0.1	0.1
S _{OPT} (ppt)	Optimal salinity (freshwater plankton) ^{\perp}	14.0	14.0	14.0
S _{OPT} (ppt)	Optimal salinity (marine plankton) \downarrow	20.0	20.0	20.0
β	Value of $f(S)$ when $S = 2S_{OPT}^{\perp}$	2.0	2.0	2.0
β	Value of $f(S)$ when $S = 0^{\perp}$	2.5	2.5	2.5

 Table 1: Values of parameters used in the model

Source: *[30], [§][15], [⊥][14], **[3], ^{⊥⊥}[29]