Complex schemes of filtering in experimental data estimation

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Abstract: Scope of the work is the decision of a high accuracy experimental data reception problem in medical, ecological and chemical data processing software. Using complex schemes of filtering improves reception of experimental data with high reliability. Approach of data acquisition about random parameter with a required degree of reliability is investigated. The approach is used for control of experimental stochastic parameter on the basis of the found interval estimations regression dependences [1,2].

Key-Words: Complex scheme, Control system, Filtering, Kalman filter, Rate of convergence, Robustness, Sensor

1 Introduction

Alongside with systems, which properties can be specified by active experiment, there are systems accepting only possibility of passive overseeing by variables of their states. On the basis of the observational data about processes their current and prognosticated properties can be detected, that allows to accept adequate measures of counteraction to development of unsafe situations [2,3].

Main the requirement is the maximal reliability of situations distinguishing, and also lowering the probability of a unsafe situation. The problem of lowering the hazard includes both first and second requirements, which execution is reached by appropriate data processing of observations [2,3]. Thus one of the tasks of statistical estimation is solved.

For improvement the estimations accuracy of observable parameters it is possible to utilize two approaches:

• rise of accuracy by usage of more precise sensors;
• using the complex schemes in control systems with standard accuracy sensors.

The first approach has set of objective implementation difficulties. The second approach bases on usage of standard sensors and implementation (program or hardware) algorithm of processing. It requires only knowledge of structure and numerical characteristics of instrumental error of sensors, collection of the external factors influential on instrumentation indications. Thus the potential accuracy of estimations is reached using complex scheme in control systems [12, 15].

2 The Problem Statement

2.1 Main Assumptions

In composition of elementary complex scheme is two measuring systems – data sources about the parameter N. These sources produce the parameter *N* with errors Δ_1 and Δ_2 accordingly. The signal of measurements on a filter, is formed as a differential signal $\Delta_1 - \Delta_2$, which does not contain N. Kalman filter formed in view of statistical properties of errors Δ_1, Δ_2 and implemented in a computing system, using measurements, produces optimal estimations of a vector of a system condition, of which units the optimal estimations of separate errors are formed.

Consider main first meter, forming the parameter N with a resultant error equal error of an optimal estimation $\Delta_1 - \hat{\Delta}_1$ [5].

The problem is to *minimize* the function $\Delta_1 - \hat{\Delta}_1$.

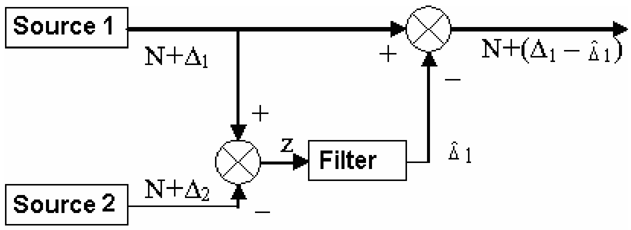

Fig.1: Complex filtering scheme with two experimental data sources

2.2 Proposed Method

Let's consider more operation of Kalman filter in complex scheme of control system and write its equation. Let measuring system intended for forming of the parameter N, will use in it 2 independent sources, which output parameters (Fig. 1):

$$Y_1(t) = N(t) + \Delta_1(t) \quad (1)$$
$$Y_2(t) = N(t) + \Delta_2(t)$$

Consider that structure of measurement errors like the following:

$$\Delta_1(t) = d_0 + d_1 \cdot t + e_1(t) \quad (2)$$
$$\Delta_2(t) = c + e_2(t),$$

where d_0 – component of an error of the first source using an initial parameter error; d_1 - linearly varying component of an error of the first source; d_0, d_1 – components of errors characterized by dispersions $(s_0)^2$, $(s_1)^2$ and zero average of distribution; $e_1(t)$ – stochastic process with zero average of distribution and correlation function:

$$K_{e_1(t)} = (s_{e_1})^2 \cdot \exp(-a_e |t|);$$

c – component of an error of the second source with a dispersion $(s_2)^2$ and zero average of distribution;

$e_2(t)$ – Stochastic process with zero expectation and correlation function:

$$K_{e_2(t)} = s^2 \cdot \exp(-a_e |t|) \cdot (\cos(bt) + (a/b) \cdot \sin(b|t|))$$

System state vector:

$$x(t) = |x_1, x_2, x_3, x_4, x_5, x_6| = |x_1, d_1, e_1, e_2, x_2, c|.$$

State vector components:

$$\dot{x} = Fx + Gw, \quad (3)$$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & -2\alpha & 1 & 0 \\ 0 & 0 & 0 & -\alpha^2-\beta^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{2\alpha\sigma^2} & 0 \\ 0 & 2\sqrt{\alpha\sigma^2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Covariance matrix for the system:

$$P_0 = \begin{bmatrix} 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma0^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2(\alpha^2+\beta^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma2^2 \end{bmatrix}.$$

3 Problem Solution

Let the dynamics of dispersions changes and correlation moments is described by the following matrix covariance equation (Riccati equation):

$$P_{k+1} = \Phi P \Phi^T + \Gamma Q_1 \Gamma^T \quad (4)$$

General errors model for complex scheme:

$$\dot{x} = F(t)x + G(t)w;$$
$$y = H(t)x, \quad (5)$$

where x – state vector (dimension (n × 1)); w –white noises vector (dimension (r × 1), having a matrix of intensity Q); y – system parameters errors vector (dimension (m × 1)); F, G, H – matrix of dimensions (n × n), (r × r), (m × m) (generally these matrixes depend on time).

This model generalizes different types of control systems with complex scheme, with two main. The first type systems concerned with the filters defined by transfer functions (or matrixes) with known parameters. In this using complex schemes with stationary filters, defined by operator form. Second type schemes have such model:

$$\dot{x} = Fx + Gw,$$
$$z = Hx + v,$$
$$y = H_0 x \quad (6)$$

satisfying to relations:

$$\dot{\hat{x}} = F\hat{x} + K(z - H\hat{x}), \quad (7)$$

in the same denotations with (5), where v – measurement noises vector (dimension (m × 1), with intensity R); z – vector of filter input measurements (dimension (m × 1)); K – amplification coefficient matrix (dimension (n × m)).

Consider feasibilities of suboptimal Kalman filters and suboptimization of the second type complex scheme. Let's consider varieties reduced orders suboptimal filters. The reduced filters (RF), simplified filters (SF) and their stationary modifications concern to reduced orders suboptimal filters. As any stationary filters of sort $\dot{\hat{x}} = F\hat{x} + K(z - H\hat{x})$, can be shown to appropriate transfer functions and matrixes on known relations

$$W(p) = H_0[pI_n - (F - KH)]^{-1}K, \quad (8)$$

the main attention will be given non-stationary Kalman filters of types of RF and SF. The filters of this sort have by common feature that they are intended for an estimation ($n_1 \times 1$) - subvector x_1 of a state vector x of model (6). So the structure of a matrix $K(t)$ of such filters has $n_2 = n - n_1$ of zero rows. Admitting, that the estimated part of a vector x is posed in its top, the appropriate extended vector of Kalman filter estimations $\hat{x}$ and matrix K^* look like:

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \dots \\ 0 \end{bmatrix}, \; K^* = \begin{bmatrix} K_1 \\ \dots \\ 0 \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ \dots \\ x_2 \end{bmatrix}, \tag{9}$$

where, x_1, $\hat{x}_1$ - vectors of dimension ($n_1 \times 1$); x_2 - vector of dimension ($n_2 \times 1$); K_1 - amplification coefficients matrix (with dimension ($n_1 \times m$)).

The analysis of accuracy and sensitivity of Kalman filter type (6) is grounded on obtaining covariance matrixes of estimations this error:

$$e = \begin{bmatrix} e_1 \\ \dots \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ \dots \\ x_2 \end{bmatrix} = x - \hat{x} \cdot \tag{10}$$

Covariance matrixes, appropriate to this vector:

$$P = M(\, e\, e^T\,) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}; \tag{11}$$

$$P_{11} = M(\, e_1\, e_1^T\,)\; ; \tag{12}$$

$$P_{12} = P_{21}^T = M(\, e_1\, x_2^T\,)\; ; \tag{13}$$

$$P_{22} = M(\, e_2\, e_2^T\,)\; . \tag{14}$$

has block structure.

At dimension of a matrix P ($n \times n$) it means solution of $n(n + 1)/2$ equations due to symmetry of this matrix.

After obtaining matrixes P_{11}, P_{12}, P_{22}, in each instant, the covariance matrix of output variables $y(t)$ is determined by standard method:

$$P_y(t) = H_0 P(t) H_0^T \cdot \tag{15}$$

One of suboptimal Kalman filter modifications is the approximated filter, in which model the amplification factors are approximated, forming a matrix of approximated amplification factors $K_a(t)$.

Let's substitute approximated amplification factors ($K(t) = K_a(t)$ in a continuous function case, $K_k = (K_a)_k$ in a discrete function case) in the filter equation:

$$\dot{P} = (F - KH)P + P(F - KH)^T + GQG^T + KRK^T,$$

$$P(0) = P_0 \text{ – continuous function.} \tag{16}$$

Similar to errors for discrete function case ($P_k = \operatorname{cov} e_k, S_k = \operatorname{cov} E_k$):

$$P_k = (E - K_k H) S_k (E - K_k H)^T + K_k R_k K_k^T, \tag{17}$$

$$S_{k+1} = F P_k F^T + G Q_k G^T \; ,$$

$$P(0) = P_0 \cdot$$

Considerable simplification of the equations P analysis, because of approximating of amplification factors matrixes. In this model Riccati equations are not written.

4 Simulation Results: Convergence and Robustness

The implementation of considered approach displays essential decrease of parameter estimation error.

Usage of the surveyed complex scheme in control systems using Kalman filters allows increasing accuracy of estimations. The error of estimation of the parameter decreases with time (Fig. 2).

Without application of a method there was an accumulation of instrumental error in closed-loop control systems.

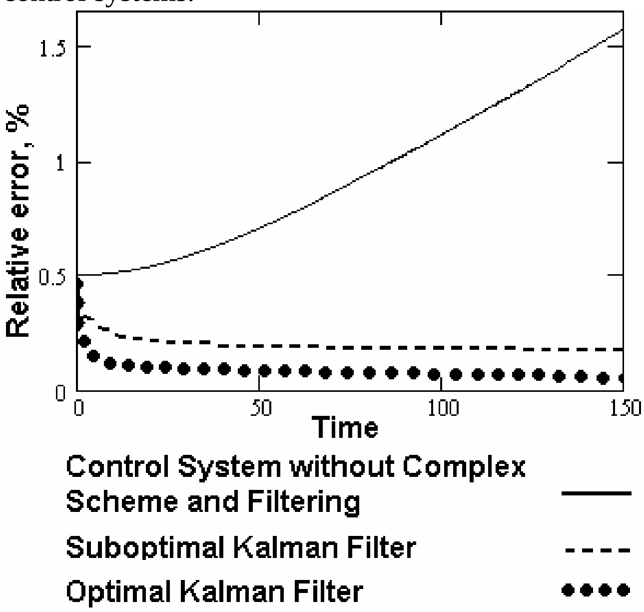

Fig.2: Convergence and robustness of complex Kalman filtering scheme in contrast with ordinary closed-loop control system (Matlab modeling: X-axis – time, Y-axis – relative error of parameter N estimation)

5 Conclusion

A very powerful method of using complex schemes in filtering control systems has following main advantages:

- comparative simplicity of information processing;
- sharp rise of estimations accuracy (on a short time interval);
- low cost of high accuracy and function stability.

Although performance of complex schemes in closed-loop control systems with Kalman filtering was best when we had knowledge of structure and numerical characteristics of instrumental error of sensors, this approach allows to provide increasing of estimation accuracy without replacement of sensors by more precise. Such schemes allow to apply scaling with major number of sensors.

I am looking forward to apply these methods in realizations in related areas like chemistry, medicine, and et cetera.

References:

[1] Thomas M. Hamill, Jeffrey S.Whitaker, and Chris Snyder. Distance-dependent filtering of background error covariance estimates in an ensemble kalman filter. *Monthly Weather Review*, Vol. 129. pp. 2776–2790, 2001.

[2] Data Assimilation via Error Subspace Statistical Estimation. P.F.Lermusiaux, A.R. Robinson, *Monthly Weather Review*, Vol. 127, July 1999.

[3]Y. Bar-Shalom, X.-R. Li, *Estimation and Tracking – Principles, Techniques ans Software*, Artech House, 1993.

[4] Carlson, H. A., Information-Sharing Approach to Federated Kalman Filtering, Proc. of *National Aerospace and Electronics Conference*, Dayton, OH, 1988.

[5] Brown, R. G., *Introduction to Random Signal Analysis and Kalman Filtering*, John Wiley & Sons, Inc., 1983.

[6] Gelb, A. (editor), *Applied Optimal Estimation*, The MIT Press, Cambridge, MA, 1974.

[7] Comparison and Analysis of Centralized, Decentralized, and Federated Filters. Y. Gao, E. J. Krakivsky, and M. A. Abousalem, *Navigation: Journal of The Institute of Navigation*, Vol. 40, No. 1, Spring 1993.

[8] E. A. Wan and R. van der Merwe, *Kalman Filtering and Neural Networks*, Eds. S. Haykin, 2001.

[9] Spall J. C. *Introduction to Stochastic Search and Optimization*. Wiley, New York, 2003.

[10] Kushner H.J., Yin G.G. *Stochastic Approximation and Recursive Algorithms and Applications*. New York: Springer - Verlag,, 2003.

[11] Granichin O.N. Linear regression and filtering under nonstandard assumptions (Arbitrary noise) // *IEEE Trans. on AC*, v.49, oct. 2004, pp.1830-1835.

[12] Granichin O.N. Estimating the parameters of linear regression in an arbitrary noise. *Automation and Remote Control*, 2002, v.63, No.1, pp.25-35.

[13] Kushner H.J., Yin G.G. *Stochastic Approximation Algorithms and Applications*. New York: Springer - Verlag, 1997, 415 p.

[14] S. Julier, J. Uhlmann, H. F. Durrant-Whyte, A new method for the nonlinear transformation of means and covariances in filters and estimators, *IEEE Trans. Automat. Contr.*, Vol. 45, No. 3, pp. 477-482, 2000.

[15] Simon J. Julier, Jeffrey K. Uhlmann, and Hugh F. Durrant-Whyte. A new approach for filtering nonlinear systems. In Proc. of the *American Control Conference*, pages 1628–1632, Seattle, WA, 1995.

[16] Geir Evensen. Sampling strategies and square root analysis schemes. *Ocean Dynamics*, 54:539–560, 2004.

[17] M. K. Tippett, J. L. Anderson, C. H. Bishop, T. M. Hamill, and J. S. Whitaker. Ensemble square-root filters. *Monthly Weather Review*, 131:1485–1490, 2002.

[18] A. Doucet, N. de Freitas, and N. Gordan, editors. *Sequential Monte Carlo Methods in Practice*, Springer, New York, NY, 2001.

[19] John Lewis, S. Lakshmivarahan, and Sudarshan Dhall. *Dynamic Data Assimilation: A Least Squares Approach*. Cambridge University Press, Cambridge, UK, 2006.

[20] Zorin A.V. Regression Method of Estimation in Ecological Observations Processing Software // In: Proc. of the *8-th St. Petersburg Conference on Soft Computing and Measurements*, St. Petersburg, 2005.