COMPONENTWISE STABILITY OF 1D AND 2D LINEAR DISCRETE-TIME SINGULAR SYSTEMS

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Abstract: This paper deals with a special type of asymptotic (exponential) stability namely componentwise asymptotic (exponential) stability for 1-D and 2-D linear discrete-time singular systems. The main motivation for these results is the need, particularly felt in the evaluation in a more detailed manner of the dynamical behaviour of 1-D and 2-D linear discrete-time singular systems. Necessary and sufficient conditions for componentwise asymptotic (exponential) stability are given.

Key words: 1-D and 2-D discrete-time singular systems; componentwise asymptotic (exponential) stability; singular Fornasini-Marchesini model; Drazin inverse theory.

NOTATIONS

$x = (x_i) \in \Re^n$ (x a real vector);

$H = (h_{ij}) \in \Re^{nxn}$ (H a real matrix);

Int D interior of set D

δD boundary of set D

det(M) determinant of matrix $M \in C^{nxn}$;

H^+ matrix with component $h_{ij}^+ = \sup(h_{ij}, 0)$, i,j=1,2,..,n;

H^- matrix with components $h_{ij}^- = \sup(-h_{ij}, 0)$, i,j=1,2,..,n;

$|H|$ matrix with components $|h_{ij}|$, i,j=1,2,..,n,

x^+ vector with components $x_i^+ = \sup(x_i, 0)$, i=1,..,n;

x^- vector with components $x_i^- = \sup(-x_i, 0)$, i=1,..,n;

x, y vectors in $\Re^n$;

$x \le y$ if $x_i \le y_i$, i=1,2,..,n;

$x < y$ if $x_i < y_i$, i=1,..,n.

1. Introduction

Componentwise stability of linear continuous systems with or without time delay has been studied by Hmamed (1996), Voicu (1984, 1987) and Hmamed and Benzaouia (1997). The purpose of this note is to extend the concept of Componentwise asymptotic (exponential) stability of 1-D and 2-D linear discrete-time systems (see Hmamed, 1997) to the componentwise stability of of 1-D and 2-D discrete-time singular systems.

The paper is organized as follows. Section 2 deals with the main results, giving necessary and sufficient conditions for componentwise asymptotic (exponential) stability of 1D singular systems. This is then extended to the 2D singular Fornasini-Marchesini in Section 3.

2. 1D discrete singular systems

In this section, some results about componentwise asymptotic (exponential) stability of 1D singular systems are established. We focus on discrete-time singular systems, which are described by the implicit form

$$\begin{cases} Ex(k+1) = Ax(k) \\ x(0) = x_0 \end{cases}, \quad k > 0 \tag{1}$$

where $x \in \Re^n$ is the state vector, $E \in \Re^{nxn}$, $rank(E) = q \le n$, $A \in \Re^{nxn}$.

In certain applications, namely in electrical engineering and biology, dynamical systems have to satisfy some additional constraints of the form

$$x \in \Omega \subset \Re^n \tag{2}$$

where Ω is the set of admissible, states defined by

$$\Omega = \left\{ x \in \Re^n / -\rho_2(k) \le x(k) \le \rho_1(k); \rho_1(k), \rho_2(k) \in int \Re^n_+ \right\} \tag{3}$$

with

$$\lim_{k\to+\infty} \rho_1(k) = 0, \quad \lim_{k\to+\infty} \rho_2(k) = 0 \tag{4}$$

This is a variant nonsymmetrical polyhedral set, as is generally the case in practical situations.

In certain cases, $\rho_1(k)$ and $\rho_2(k)$ take the form

$$\rho_s(k) = \alpha_s \beta^k \tag{5}$$

with $0 < \beta < 1$ and $\alpha_s > 0$ for s=1,2.

The purpose of this section is to define a special type of asymptotic (exponential) stability of (1), namely the componentwise asymptotic (exponential) stability characterized by (3) and (4) ((3) and (5)). Necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (1) are given.

First, recall some important properties of implicit systems that are assumed intrinsic in the following analysis.

Definition 2.1: The system (1) is called componentwise asymptotically stable with respect to $\tilde{\rho}(k) = \left[\rho_1^T(k)\ \rho_2^T(k)\right]^T$ (CWAS $\tilde{\rho}$) if for every $-\rho_2(0) \le x_0 \le \rho_1(0)$, the response of (1) satisfies

$$-\rho_2(k) \le x(k) \le \rho_1(k), \quad \forall k \ge 0 \tag{6}$$

Definition 2.2 The system (1) is called componentwise exponential asymptotically (CWEAS) if there exist $\alpha_1 > 0$ and $\alpha_2 > 0$ such that, for every $-\alpha_2 \le x_0 \le \alpha_1$, the response of (1) satisfies

$$-\alpha_2 \beta^k \le x(k) \le \alpha_1 \beta^k, \quad \forall k \ge 0 \tag{7}$$

Definition 2.3 [Campbell et al., 1976]: The system (1) is said to be regular if $\det(sE - A) \ne 0$.

Definition 2.4 [Lewis, 1987]: The system (1) is said to be impulse-free if

$\deg \det(sE - A) = rank(E)$ or $(sE - A)^{-1}$ is proper.

Definition 2.5 [Chen, 1970] A rational function $G(s)$ is said to be proper if $G(\infty)$ is constant matrix, $G(s)$ is said to be strictly proper if $G(\infty) = 0$.

For the convenience of the later statements in this paper, we use the pair (E,A) to represent the system (1).

Theorem 2.1 [Campbell, 1991] Suppose that (E,A) is regular. Then $Ex(k+1) = Ax(k) + f(k)$, $\forall k \ge 0$ is solvable and the general solution is given by:

$$x(k) = \bar{A}^k \bar{P} q + \hat{E}^D \sum_{i=0}^{k-1} \bar{A}^{k-i-1} \hat{f}(i) - (I - \bar{P}) \sum_{i=0}^{\nu-1} \bar{E}^i \hat{A}^D \hat{f}(k+i) \tag{8}$$

where $\hat{f}(i) = (\lambda E - A)^{-1} f(i)$, $\hat{E} = (\lambda E - A)^{-1} E$, $\hat{A} = (\lambda E - A)^{-1} A$, $\bar{A} = \hat{E}^D \hat{A}$, $\bar{E} = \hat{E}\hat{A}^D$, $\bar{P} = \hat{E}\hat{E}^D$, $q \in \Re^n$, ν is the index of $\hat{E}$ and λ is a scalar such that $\lambda E - A$ is nonsingular. The projection $\bar{P}$ and $\bar{E}$, $\bar{A}$, ν are independent of λ.

Matrix $\hat{E}^D$ is the Drazin inverse of $\hat{E}$ and the index of a matrix is the size of the largest nilpotent block in its Jordan canonical form.

We now give necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (1).

Theorem 2.2: Suppose that (E,A) is regular, a necessary and sufficient condition for the system (1) to be CWAS $\tilde{\rho}$ is

$$\tilde{\rho}(k+1) \ge \tilde{H}\tilde{\rho}(k), \quad \forall k \ge 0 \tag{9}$$

with

$$\tilde{H} = \begin{pmatrix} H^+ & H^- \\ H^- & H^+ \end{pmatrix}, \quad \tilde{\rho}(k) = \left[\rho_1^T(k)\ \rho_2^T(k)\right]^T \tag{10}$$

and

$$H = \hat{E}^D \hat{A} \tag{11}$$

Proof: From Theorem 2.1 the solution of the system (1) is written in the form

$$x(k) = \left(\hat{E}^D \hat{A}\right)^k \hat{E}\hat{E}^D q = \left(\hat{E}^D \hat{A}\right)^k x_0,$$

$$q \in \Re^n \ \forall k \ge 0$$

then:

$$x(k+1) = \hat{E}^D \hat{A} x(k) \tag{12}$$

with the initial condition $x_0 = \hat{E}\hat{E}^D q$, $q \in \Re^n$.

At this step, we can use the proof given in Hmamed (1997) as the proof remains unchanged.

Remark 1: From Campbell et al. (1976) and Lewis (1986), we know that the consistent initial conditions $x_0 = x(0)$ of system (1) are defined by

$$x_0 = \hat{E}\hat{E}^D x_0$$

and then, there always exists a vector $q \in \Re^n$ such that $x_0 = \hat{E}\hat{E}^D q$ (Tarboureich et al. 1993).

Remark 2: In the case where matrix E is non-singular, then system (1) can be written as a classical autonomous linear system defined as

$$x(k+1) = E^{-1}Ax(k) \qquad (13)$$

Hence, if we apply the previous result to system (13), then the classical result of the componentwise stability of 1-D linear discrete-time systems is obtained, that is

$$\tilde{\rho}(k+1) \geq \tilde{H}\tilde{\rho}(k) \quad \forall k \geq 0$$

with $\tilde{H} = \begin{pmatrix} (E^{-1}A)^+ & (E^{-1}A)^- \\ (E^{-1}A)^- & (E^{-1}A)^+ \end{pmatrix}$.

For $E = I$, we obtain the result given in [Hmamed, 1997].

In the symmetrical case $\rho_1(k) = \rho_2(k) = \rho(k)$, we can deduce the following result.

Corollary 2.1: Suppose that (E,A) is regular , a necessary and sufficient condition for the system (1) to be CWAS ρ is

$$\rho(k+1) \geq |H|\rho(k) \quad \forall k \geq 0 \qquad (14)$$

matrix H is defined by (11).

Proof: By observing that $|H| = H^+ + H^-$, the proof follows from Theorem 2.6.

Using the techniques of Hmamed (1997), we can also extend the results of Theorem2.2 to the following Theorem which deals with the componentwise exponential asymptotic stability.

Theorem 2.3: Suppose that (E,A) is regular , the system (1) (system (12)) is CWEAS if and only if one of the following conditions holds:

i) $$\left(\beta I - \tilde{H}\right)\tilde{\alpha} \geq 0; \qquad (15)$$

ii)

$$1 > \beta \geq \max_i \max\left(h_{ii}^+ + h_{ii}^- \frac{\alpha_2^i}{\alpha_1^i} + \sum_{j\neq i} h_{ij}^+ \frac{\alpha_1^j}{\alpha_1^i} + h_{ij}^- \frac{\alpha_2^j}{\alpha_1^j},\ h_{ii}^+ + h_{ii}^- \frac{\alpha_1^i}{\alpha_2^i} + \sum_{j\neq i} h_{ij}^+ \frac{\alpha_2^j}{\alpha_2^i} + h_{ij}^- \frac{\alpha_1^j}{\alpha_2^j},\right) \qquad (16)$$

with $\tilde{\alpha} = \begin{bmatrix}\alpha_1^T & \alpha_2^T\end{bmatrix}^T$ and $H = \hat{E}^D\hat{A} = (h_{ij})$.

In the symmetrical case $\rho_1(k) = \rho_2(k) = \alpha\beta^k$, we can deduce the following result.

Corollary 2.3: The regular system (1) is CWEAS if and only if one of the following conditions holds:

(i) $\beta\alpha \geq |H|\alpha$;

(ii) $$1 > \beta \geq \max_i \left(\sum_{j=1}^n |h_{ij}| \frac{\alpha^j}{\alpha^i} \right)$$

with $H = \hat{E}^D\hat{A} = (h_{ij})$.

Remark 3: When $E = I$, we obtain the result given by Hmamed (1997).

2-D Fornasini- Marchesini model

In this section, we extend the notion of componentwise asymptotic (exponential) stability to implicit 2-D Fornasini- Marchesini model described by:

$$Ex(i+1, j+1) = Ax(i, j+1) + Bx(i+1, j) \qquad (17)$$

with the boundary conditions

$$x(i,0) \text{ and } x(0, j) \text{ for } i, j = 0,1,\ldots \qquad (18)$$

where $x \in \Re^n$ is the state vector.

Assume then that (E,A) is a regular pencil and impulse-free. A similar discussion applies if (E,B) is regular. Treat j as fixed, If the sequence $x(i, j)$ is considered known, then (17) is a difference equation

$$Ex(i+1, j+1) = Ax(i, j+1) + [Bx(i+1, j)],\quad i \geq 0 \qquad (19)$$

for $x(i, j+1)$ with the terms in square brackets known (see Campbell., 1991).

Since (E,A) is a regular pencil, we may apply Theorem 2.1, we have

$$x(i, j+1) = \left(\hat{E}^D\hat{A}\right)^i \hat{E}\hat{E}^D q + \hat{E}^D \sum_{k=0}^{i-1} \left(\hat{E}^D\hat{A}\right)^{i-k-1} \hat{B}x(k+1, j) - (I - \hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1} (\hat{E}\hat{A}^D)^r \hat{A}^D\hat{B}x(i+r, j) \qquad (20)$$

here ν is the index of $\hat{E}$

$$x(i+1, j+1) = \left(\hat{E}^D\hat{A}\right)^{i+1} \hat{E}\hat{E}^D q + \hat{E}^D \sum_{k=0}^{i} \left(\hat{E}^D\hat{A}\right)^{i-k} \hat{B}x(k+1, j) - (I - \hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1} (\hat{E}\hat{A}^D)^r \hat{A}^D\hat{B}x(i+1+r, j)$$

$$= \left(\hat{E}^D\hat{A}\right)\left(\hat{E}^D\hat{A}\right)^i \hat{E}\hat{E}^D q + \left(\hat{E}^D\hat{A}\right)\hat{E}^D \sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1}\hat{B}x(k+1,j) +$$

$$\left(\hat{E}^D\hat{B}\right)x(i+1,j) - (I-\hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r\hat{A}^D\hat{B}x(i+k+1+r,j)$$

$$= \left(\hat{E}^D\hat{A}\right)\left[x(i,j+1) + (I-\hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r\hat{A}^D\hat{B}x(i+k,j)\right]$$

$$+\left(\hat{E}^D\hat{B}\right)x(i+1,j) - (I-\hat{E}\hat{E}^D)\sum_{r=0}^{\nu-1}(\hat{E}\hat{A}^D)^r\hat{A}^D\hat{B}x(i+k+1+r,j)$$

$$= \left(\hat{E}^D\hat{A}\right)x(i,j+1) + \left(\hat{E}^D\hat{B}\right)x(i+1,j) +$$

$$(I-\hat{E}\hat{E}^D)\sum_{k=0}^{\nu-1}(\hat{E}\hat{A}^D)^k\hat{A}^D\hat{B}(\left(\hat{E}^D\hat{A}\right)x(i+k,j) - x(i+k+1,j))$$

$$= \left(\hat{E}^D\hat{A}\right)x(i,j+1) + \left(\hat{E}^D\hat{B}\right)x(i+1,j) -$$

$$(I-\hat{E}\hat{E}^D)\sum_{k=0}^{\nu-1}(\hat{E}\hat{A}^D)^k\hat{A}^D\hat{B}x(i+k+1,j)$$

Definition 3.1 : The regular system (17) is called componentwise asymptotically stable with respect to $\tilde{\rho}(i,j) = \left[\rho_1^T(i,j)\ \rho_2^T(i,j)\right]^T$ (CWAS $\tilde{\rho}$) if, for every

$$\begin{cases} -\rho_2(i,0) \le x(i,0) \le \rho_1(i,0) \\ -\rho_2(0,j) \le x(0,j) \le \rho_1(0,j) \end{cases} \text{ for i,j=0,1,2,.} \qquad (21)$$

The response of (17) satisfies

$$-\rho_2(i,j) \le x(i,j) \le \rho_1(i,j) \quad \forall (i,j) > (0,0) \qquad (22)$$

where

$$\rho_1(i,j) > 0,\ \rho_2(i,j) > 0\ \forall (i,j) > (0,0)$$

$$\lim_{i\to\infty \text{ and/or } j\to\infty} \rho_1(i,j) = 0\ , \quad \lim_{i\to\infty \text{ and/or } j\to\infty 2} \rho_1(i,j) = 0$$

Definition 3.2 : The regular system (17) is called componentwise exponential asymptotically stable (CWEAS) if there exist $\alpha_1 > 0$ and $\alpha_2 > 0$ such that, for every

$$\begin{cases} -\alpha_2\beta^i \le x(i,0) \le \alpha_1\beta^i \\ -\alpha_2\gamma^j \le x(0,j) \le \alpha_1\gamma^j \end{cases} \text{ for i,j=0,1,2,…} \qquad (23)$$

the response of (17) satisfies

$$-\alpha_2\beta^i\gamma^i \le x(i,j) \le \alpha_1\beta^i\gamma^i\ , \quad \forall (i,j) > (0,0)$$

where $0 < \beta < 1$ and $0 < \gamma < 1$.

We give now necessary and sufficient conditions for componentwise asymptotic (exponential) stability of the system (29) or (17).

Theorem 3.1: Suppose that (E,A) is regular and impulse-free, a necessary and sufficient condition for the system (17) to be CWAS $\tilde{\rho}$ is

$$\tilde{\rho}(i+1,j+1) \ge \tilde{H}_1\tilde{\rho}(i,j+1) + \tilde{H}_2\tilde{\rho}(i+1,j),\quad \forall (i,j) > (0,0) \qquad (24)$$

with

$$\tilde{H}_1 = \begin{pmatrix} (\hat{E}^D\hat{A})^+ & (\hat{E}^D\hat{A})^- \\ (\hat{E}^D\hat{A})^- & (\hat{E}^D\hat{A})^+ \end{pmatrix},\ \tilde{H}_2 = \begin{pmatrix} (\hat{E}^D\hat{B})^+ & (\hat{E}^D\hat{B})^- \\ (\hat{E}^D\hat{B})^- & (\hat{E}^D\hat{B})^+ \end{pmatrix},$$

$$\tilde{\rho}(i,j) = \left[\rho_1^T(i,j)\ \ \rho_2^T(i,j)\right]^T \qquad (25)$$

Proof: Since system (20) is impulse free, in this case, ν becomes 1 (Lewis, 1986).

Express $x(i,j+1)$ by using (20) with $\nu = 1$ as

$$x(i,j+1) = \left(\hat{E}^D\hat{A}\right)^i \hat{E}\hat{E}^D q + \hat{E}^D\sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1}\hat{B}x(k+1,j) \qquad (26)$$

then

$$x(i+1,j+1) = \left(\hat{E}^D\hat{A}\right)^{i+1}\hat{E}\hat{E}^D q + \hat{E}^D\hat{B}x(i+1,j) + \hat{E}^D(\hat{E}^D\hat{A})\sum_{k=0}^{i-1}\left(\hat{E}^D\hat{A}\right)^{i-k-1}\hat{B}x(k+1,j) \qquad (27)$$

From the Drazin inverse theory used in [Campbell et al., (1976)], we know that

$$\hat{E}\hat{A} = \hat{A}\hat{E}\ ,\ \hat{E}^D\hat{A} = \hat{A}\hat{E}^D \text{ and } \hat{E}^D\hat{A}^D = \hat{A}^D\hat{E}^D$$

then

$$x(i+1,j+1) = H_1\,x(i,j+1) + H_2\,x(i+1,j) \qquad (28)$$

with

$$H_1 = \hat{E}^D\hat{A},\ H_2 = \hat{E}^D\hat{B} \qquad (29)$$

and boundary conditions $x(i,0) = x_{i,0}$ and

$x(0,j) = \hat{E}^D\hat{E}q$, $q \in \Re^n$, $j \ge 1$.

At this step, this follows similar lines to the proof of Theorem 3.3 in Hmamed (1997).

Remark 4: The boundary values $x(i,0)$ may be taken arbitrary and the boundary values $\hat{E}^D\hat{E}x(0,j),\ j \ge 1$, are arbitrary (Campbell, 1991). Then, we can applied the results given in the last section, implying the existence of a vector $q \in \Re^n$ such that:

$$x(0,j) = \hat{E}^D\hat{E}q = x_{0,j},\ j \ge 1$$

By analogy with section 2, we give some definitions.

Remark 5: If E is nonsingular square matrix, then equation (24) is the classical condition given in Hmamed (1997).

In the symmetrical case $\rho_1(i,j) = \rho_2(i,j) = \rho(i,j)$, we can deduce the following result.

Corollary 3.1. Suppose that (E,A) is regular and impulse-free, a necessary and sufficient condition for the system (17) to be CWAS ρ is

$$\rho(i+1,j+1) \ge |H_1|\rho(i,j+1) + |H_2|\rho(i+1,j) \qquad (30)$$

matrices H_1 and H_2 are defined by (29).

Proof: On observing that $|H_1| = H_1^+ + H_2^-$ and $|H_2| = H_2^+ + H_2^-$, the proof follows from Theorem 3.1.

Theorem 3.2. Suppose that (E,A) is regular and impulse-free, the system (17) (system (28)) is CWEAS if and only if one of the following conditions holds:

i) $$\beta\gamma\tilde{\alpha} \geq (\tilde{H}_1\gamma - \tilde{H}_2\beta)\tilde{\alpha}\,; \qquad (31)$$

ii) $$1 > \beta\gamma \geq \max_i \max\left\{ [\sum_{j=1}^{n}(h_{ij}^{1+}\gamma + h_{ij}^{2+}\beta)\alpha_1^j + (h_{ij}^{1-}\gamma + h_{ij}^{2-}\beta)\alpha_2^j]/\alpha_1^i, \; [\sum_{j=1}^{n}(h_{ij}^{1-}\gamma + h_{ij}^{2-}\beta)\alpha_1^j + (h_{ij}^{1+}\gamma + h_{ij}^{2+}\beta)]/\alpha_2^j \right\} \qquad (32)$$

with $\tilde{\alpha} = [\alpha_1 \;\; \alpha_2]^T$.

In the symmetrical case $\rho_1(i,j) = \rho_2(i,j) = \tilde{\alpha}\beta^i\gamma^j$, we can deduce the following result.

Corollary 3.3: Suppose that (E,A) is regular and impulse-free, the system (17) is CWEAS if and only one of the following conditions holds:

(i) $$\beta\gamma\alpha \geq (|H_1|\gamma + |H_2|\beta)\alpha \qquad (33)$$

(ii) $$1 > \beta\gamma \geq \left\{ \sum_{j=1}^{n}(|h_{ij}^1|\gamma + |h_{ij}^2|\beta)\frac{\alpha^j}{\alpha^i} \right\} \qquad (34)$$

matrices H_1 and H_2 are defined by (29).

Remark 6. We can extend the results of this section to the Roesser 2D model given in Kaczorek (1987), Lewis (1987-1992) by

$$\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix}\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\begin{bmatrix} x_{i,j}^h \\ x_{i,1}^v \end{bmatrix} \qquad (35)$$

with the boundary conditions

$$x^h(0,j) \text{ and } x^v(i,0) \quad \text{for } i,j = 0,1,$$

Several techniques may be used to show that the implicit Roesser and implicit FM model are equivalent (Kacsorek, 1989). Indeed, in the Roesser model define

$$F_1 = \begin{bmatrix} E_1 & 0 \\ E_2 & 0 \end{bmatrix}, \; F_2 = \begin{bmatrix} 0 & E_3 \\ 0 & E_4 \end{bmatrix} \qquad (36)$$

and similar quantities with respect to $A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$.

Then (35) may be written as

$$F_1\, x(i+1,j) + F_2\, x(i,j+1) = Ax(i,j) \qquad (37)$$

Consequently all the results derived in this section still hold for the Roesser model (35) on taking into account the relation (36).

Following the same ideas, those results can easily be extended to the following general 2D system model:

$$Ex(i+1,j+1) = A_0 x(i,j) + A_1 x(i,j+1) + A_2 x(i+1,j) \quad (38)$$

3. Conclusion

In this paper, we have given an extension of the concept of componentwise asymptotic (exponential) stability for singular 1D and 2D discrete linear singular systems. Necessary and sufficient conditions for componentwise asymptotic (exponential) stability have been given. The results for the symmetrical case have been obtained as a particular case.

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