Slip Modeling in Timber-Framed Walls with Wood-Based or Fibre-Plaster Sheathing Boards

M. PREMROV, P. DOBRILA, B.S. BEDENIK, I. ŠPACAPAN
Faculty of Civil Engineering
University of Maribor
Smetanova ulica 17, SI-2000 Maribor
SLOVENIA
http://www.fg.uni-mb.si

Abstract: - The paper provides mathematical modelling for prefabricated timber-framed walls composed of a timber frame and two different types of sheathing boards. Since by wood-based boards (WBB) the tensile strength is similar to the compressive one, there are practically no cracks appearing in the boards. On the other hand, in case of fibre-plaster sheathing boards (FPB) the tensile strength is approximately 10-times lower than the compressive one and therefore cracks in the tensile diagonal board's direction usually appear. Based on analysis of experimental research results [1] special approximate mathematical models have been developed. The models enable simultaneously to consider the flexibility of mechanical fasteners in the connecting areas, as well as possible cracks appearing in the tensile area of the sheathing boards.

Key-Words: - Timber structures, Walls, Slip, CFRP, Modelling

1 Introduction

In addition to the important applications of timber in bridges, railroad infrastructure and many other applications, there is an increasing tendency worldwide toward building multi-level prefabricated timber structures with timber-framed walls as the main bearing capacity prefabricated elements. As the influence of the horizontal load increases with the height of the structure, the wall's load-carrying capacity becomes critical when taller structures are subjected to heavy horizontal forces, particularly by structures located in seismic and windy areas.

In the presented research the treated walls consist of solid timber frame coated by sheets of board-material fixed by mechanical fasteners to one or both sides of the timber frame (Fig. 1). There are many types of panel sheet products available which may have some structural capacity such as wood-based materials (plywood, OSB, hardboard, particleboard, etc.) or plaster and fibre-plaster boards (FPB), made from gypsum, recently the most frequently used in Central Europe. One of the most important reasons for an increased application of these types of gypsum products is namely their relatively good fire protection. For example, single gypsum sheathed board of 15mm thickness assures 45 minutes of fire protection. Additionally, gypsum is a healthy natural material and is consequently particularly desired for residential buildings. On the other hand, from a structural point of view the tensile strength of FPB is very low, approximately 10-times lower than the compressive one, and can not be compared with the overall strength of the timber frame at all.

In this research we will limit our attention to modelling of the walls with wood-based or FPB sheathing boards. A special attention will be focused to the comparision of the numerical results, including forming of cracks, destruction force and slip in the timber frame-board connecting area.

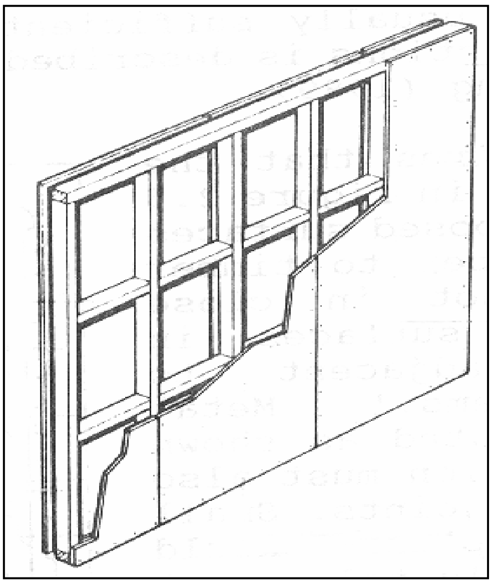

Fig.1. Composition of the wall.

2 Modelling of walls with FPB

We will limit our analysis to the horizontal force (F_H) influence acting at the top of the wall. By employing FPB as a coating material a horizontal load shifts a part of the force over the mechanical fasteners to the FPB and the wall acts like a deep composite beam with a semi rigid connecting area between the timber frame and FPB [2]. For design purposes a simplified design method for mechanically jointed beams according to Annex B of Eurocode 5 [3] is widely used. Expression of the so called »γ-method« is based on the differential equation for the partial composite action with the following fundamental assumptions [4, 5]:

a) Bernoulli`s hypothesis is valid for each sub-component,
b) slip stiffness is constant along the element,
c) material behaviour of all sub-components is linear elastic.

The effective bending stiffness $(EI_y)_{eff}$ of mechanically jointed beams which empirically considers the flexibility of fasteners via coefficient γ_y, taken from Eurocode 5 [3], can be written in the form of:

$$(EI_y)_{eff} = \sum_{i=1}^{n} E_i \cdot \left(I_{yi} + \gamma_{yi} \cdot A_i \cdot a_i^2\right) = \sum_{i=1}^{n_{timber}} \left(E_i \cdot I_{yi} + E_i \cdot \gamma_{yi} \cdot A_i \cdot a_i^2\right)_{timb.} + \sum_{j=1}^{n_{board}} \left(E_i \cdot I_{yi}\right)_{FPB} \quad (1)$$

where n is the total number of elements in the considered cross-section and a_i is a distance between global y-axis of the whole cross-section and local y_i-axis of the i-th element with a cross-section A_i.

2.1 Modelling of fasteners flexibility

We can mention from Eq. (1) that the bending stiffness strongly depends on the stiffness coefficient of the fasteners (γ_{yi}). It can be defined via the fastener spacing (s) and the slip modulus per shear plane per fastener (K) using Eurocode 5 [3] in the form of:

$$\gamma_y = \frac{1}{1+k_y}; \qquad k_y = \frac{\pi^2 \cdot A_{t1} \cdot E_t \cdot s}{L_{eff}^2 \cdot K} \quad (2)$$

In the proposed mathematical model the value of the modulus (K) varies according to the lateral force (F_1) acting on one fastener, which can be determinated according to the shear force (V_z), the spacing between fasteners (s), the effective shear stiffness $(ES_y)_{ef}$ in the connecting plane and on the effective bending stiffness $(EI_y)_{eff}$ of the cross-section in the following form:

$$F_1 = \frac{(ES_y)_{eff}}{(EI_y)_{eff}} \cdot \frac{s}{2} \cdot V_z \quad (3)$$

The value of K directly depends on the slip (Δ) in the timber frame - FPB connecting area. As long as behaviour of fasteners is almost elastic (Fig. 2a) the value of K is maximal ($K=K_{ser}$) and it is constant (Fig. 2b). The value of K_{ser} depends on national codes. With an increasing part of plasticity the value of K decreases. We propose the *three-linear interpolation diagram* to simulate the behaviour of fasteners depending on the value of F_1 (Fig. 2b).

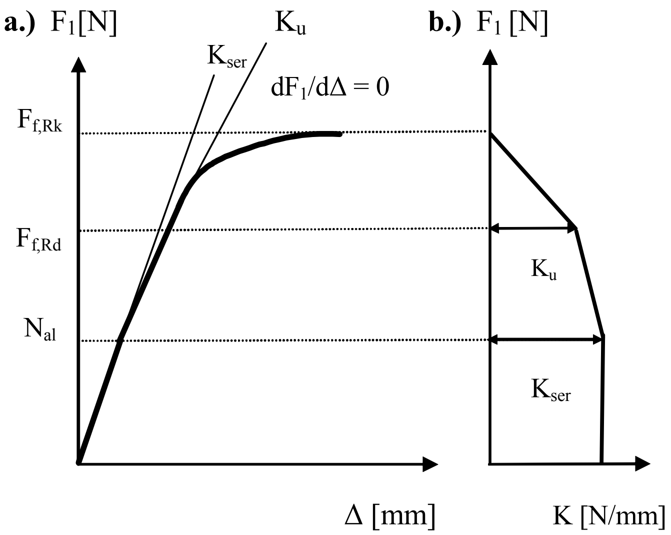

Fig. 2. a.) F_1–Δ diagram, b.) Three–linear diagram for K.

It is important to determine three fundamental diagram points:

$$F_1 \le N_{al} \quad \Rightarrow \quad K = K_{ser} \quad (4a)$$

$$F_1 = F_{f,Rd} \quad \Rightarrow \quad K = K_u = \frac{2}{3} \cdot K_{ser} \quad (4b)$$

$$F_1 = F_{f,Rk} \quad \Rightarrow \quad K = 0 \quad (4c)$$

N_{al} …. allowable lateral load-bearing capacity per shear plane per fastener,
$F_{f,Rd}$ … design lateral load-bearing capacity per shear plane per fastener,
$F_{f,Rk}$ … characteristic lateral load-bearing capacity per shear plane per fastener.

For intermediate values of F_1 linear interpolation is used according to Fig. 2b. The slip (Δ) in the timber frame - FPB connecting area under the force F_H is calculated in the form of:

$$\Delta = \frac{F_1}{K} \quad (5)$$

2.2 Modelling of cracks in FPB

The tensile strength (f_{bt}) of fibre-plaster sheeting material is very low. Consequently, cracks in tensile area of FPB usually appear. The horizontal force forming the first tensile crack ($F_{H,cr}$) in FPB is defined according to the bending stress criteria in a form as:

$$F_{H,cr} = \frac{2 \cdot f_{bt} \cdot (EI_y)_{eff}}{E_b \cdot b \cdot h_d} \quad (6)$$

Four major assumptions are considered in the presented modelling of the cracked cross-section [5]:

a.) The tensile area of the fibreboards is neglected after the first crack formation.

b.) The stiffness coefficient of the fasteners in the tensile connecting area (γ_{yt}) is assumed to be constant and equal to the value by appearing the first crack.

c.) The stiffness coefficient of the fasteners in the compressed connecting area (γ_{yc}) is not constant and depends on the lateral force acting on one fastener, as it is declared in Eq. (4).

d.) The normal stress distribution is assumed to be linear. This simplification can be used only by assumption that behaviour of timber frame in tension is almost elastic until failure and that the compressive normal stress in timber and in FPB is under the belonging yield point.

Position (x_{II}) of a new neutral axis (y_{II}) is computed according to the presented computational scheme (Fig. 3) by respecting the equilibrium criteria in x-direction:

$$F_{cb} + F_{ct} - F_{tt} = 0 \quad (7)$$

$$F_{tt} = \sigma^c_{tt} \cdot A_{t1}$$
$$F_{ct} = \sigma^c_{ct} \cdot A_{t1} \quad (8)$$
$$F_{cb} = \frac{\sigma^c_{cb} \cdot x_{II} \cdot 2t}{2} = \sigma^c_{cb} \cdot x_{II} \cdot t$$

$$x_{II} = \frac{-C_2 + \sqrt{C_2^2 - 4C_1 \cdot C_3}}{2C_1};$$
$$C_1 = t,$$
$$C_2 = A_{t1} \cdot n \cdot (\gamma_{yc} + \gamma_{yt}), \qquad n = \frac{E_t}{E_b} \quad (9)$$
$$C_3 = -n \cdot A_{t1} \cdot \left[\gamma_{yt} \cdot \left(b - \frac{a}{2}\right) + \gamma_{yc} \cdot \frac{a}{2}\right]$$

Because of crack appearing the effective bending stiffness $(EI_y)_{eff}$ is now decreased according to Eq. (1):

$$(EI_y)^{II}_{eff} = E_b I_b + E_t I_t =$$
$$E_b \cdot 2t \cdot \frac{x_{II}^3}{3} +$$
$$+ E_t \cdot \left\{ \frac{2a^3 \cdot c}{12} + \frac{d^3 \cdot c}{12} + A_{t1} \cdot \left(\gamma_{yc} \cdot z_{cII}^2 + \gamma_{yt} \cdot z_{tII}^2\right) \right\}$$
$$z_{cII} = x_{II} - \frac{a}{2}; \quad z_{tII} = b - \frac{a}{2} - x_{II} \quad (10)$$

If we declare as a characteristic destruction condition the case when the tensile normal stress in timber ($\sigma_{tt,max}$) achieves the characteristic tensile timber strength ($f_{t,0,k}$), the characteristic horizontal destruction force ($F_{H,k}$) is computed in the following form:

$$F_{H,k} = \frac{f_{t,0,k} \cdot (EI_y)^{II}_{eff}}{E_t \cdot \left(\gamma_{yt} \cdot z_{tII} + \frac{a}{2}\right) \cdot h} \quad (11)$$

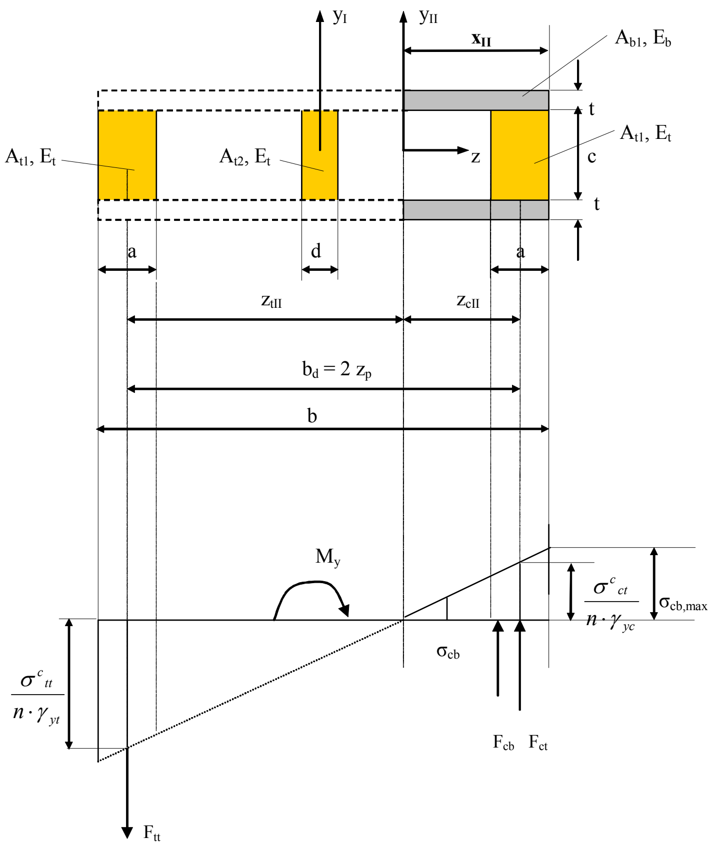

Fig. 3: Mathematical model for the cross-section with a tensile crack in FPB.

In engineering design it is important to reinforce the FPB if there is any possibility of cracks appearing. Experimental study of CFRP strip reinforcing can be found in [6]. Semi-analytical mathematical models with the fictive enlarged thickness of the boards are proposed in [5] or [7] and will not be presented or discussed in this paper.

3 Modelling of walls with wood-based sheathing boards

Of course we can model the walls with wood-based sheathing boards with the described procedure of the composite model described in Section 2, respecting an important fact that the tensile strength of the wood-based boards is similar to the compressive one. Consequently, it can be aspect that there is now crack appearing in the tensile area of the boards. Therefore, it can be predicted that the stresses in the fasteners reach their yielding point before any cracks in the boards are formed.

Consequently, two simplified computational methods are given in Eurocode 5 [3] in order to determine the load-carrying capacity of the wall diaphragm. The first simplified analysis – *Method A*, is identical to the «*Lower bound plastic method*«, presented by Källsner and Lam [8]. This method defines the wall's characteristic shear resistance ($F_{v,k}$) as a sum of the fasteners' lateral capacity ($F_{f,Rk}$) in the shear loaded edges in the form of:

$$F_{v,k} = \sum F_{f,Rk} \cdot \frac{b_i}{s} \cdot c_i \quad (12)$$

where b_i is the wall panel width and s is a fastener spacing. The parameter c_i is empirical described in the form of:

$$c_i = \begin{cases} 1 & \text{for } b_i \geq b_0 \\ \dfrac{b_i}{b_0} & \text{for } b_i \leq b_0 \end{cases} \qquad \text{where } b_0 = h/2 \quad (13)$$

The second simplified analysis – *Method B* is applicable to walls made from sheets of wood-based panel products only, fastened to a timber frame. The fastening of the sheets to the timber frame should either be by nails or screws, and the fasteners should be equally spaced around the perimeter of the sheet. According to Method A the sheathing material factor (k_n), the fastener spacing factor (k_s), the vertical load factor ($k_{i,q}$) and the dimension factors for the panel (k_d) are included in the design procedure in the form of:

$$F_{v,k} = \sum F_{f,Rk} \cdot \frac{b_i}{s_0} \cdot c_i \cdot k_d \cdot k_{i,q} \cdot k_s \cdot k_n \quad (14)$$

where coefficient s_0 depends on the fastener diameter (d) and wood characteristic density (ρ_k) in the form of:

$$s_0 = \frac{9700 \cdot d}{\rho_k} \quad (15)$$

It should be underlined that the both simplified mehods can be applicable only for wood-based panels where the tensile strength is relatively high and the elements tend to fail because of fastener yielding and not because of crack appearing in the sheathing boards.

4 Numerical example

4.1 Geometrical and material properties

Numerical analysis is performed for the panel wall of actual dimensions *h=263.5 cm* and *b=125 cm*, composed of timber studs (*2x9x9cm* and *1x4.4x9cm*) and timber girders (*2x8x9cm*). The boards of the thickness *t=15 mm* are fixed to the timber frame using staples of *Φ1.53 mm* and length *l = 35 mm* at an average spacing of *s = 75 mm* (Fig. 4). Examples with two different material types (FPB and plywood) of the sheathing boards will be separately analysed and compared.

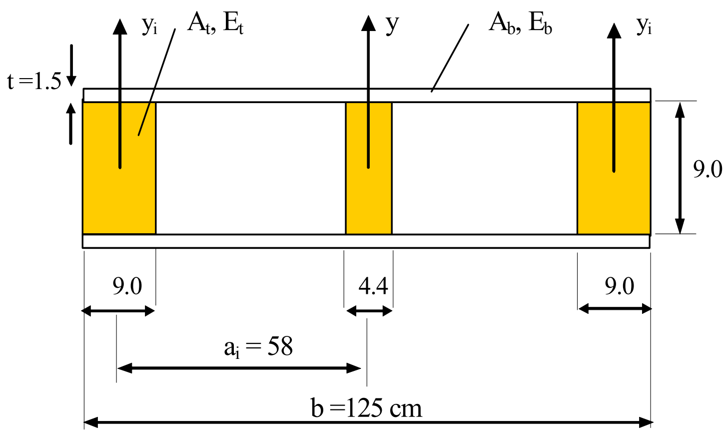

Fig. 4. Cross-section of the test sample.

Material properties for the timber of quality C22 are taken from EN338 [9], for the FPB Knauff plasterboards from [10] and the plywood boards from [11]. All material properties are listed in Table 1.

Table 1. Properties of used materials.

	Timber C22	**FPB Knauf**	**Swedian (S) Plywood***
$\mathbf{E_{0,m}}$ [N/mm^2]	10000	3000	9200
$\mathbf{f_{m,k}}$ [N/mm^2]	22.0	4.0	23.0
$\mathbf{f_{t,0,k}}$ [N/mm^2]	13.0	2.5	15.0
$\mathbf{f_{c,0,k}}$ [N/mm^2]	20.0	20.0	15.0
$\boldsymbol{\rho_k}$ [kg/m^3]	340	1050	410
$\boldsymbol{\rho_m}$ [kg/m^3]	410	1050	410

* *The values are given for 12mm typical thickness of the board.*

4.2 Geometrical and material properties

a) Since the computational model according to Fig. 3 is considered, $h_d = 254.5\ cm$ and $b_d = 116\ cm.$

b) Lateral load-bearing capacity of the staples: According to Eurocode 5 [3] using Johansen expressions we obtain for the characteristic lateral load-carrying capacity for the staples ($F_{f,Rk}$) and for the belonging designed value ($F_{f,Rd}$) respecting $k_{mod} = 0.9$:

FPB:

$F_{f,Rk} = 659.69\ N$

$F_{f,Rd} = 456.71\ N$

WBB (plywood):

$F_{f,Rk} = 516.74\ N$

$F_{f,Rd} = 357.74\ N$

Allowable lateral load-bearing capacity per shear plane per fastener (N_{al}) is not declared in Eurocode 5 [3] thus it can be obtained for the both types of the boards using Brüninghoff [12]: $N_{al} = 203.03\ N$.

c) Slip modulus (K_{ser}) for the staples is computed using Eurocode 5 [3]:

FPB:

$$\rho_m = \sqrt{\rho_b \cdot \rho_t} = \sqrt{1050 \cdot 410} = 656.12\ kg/m^3$$

$$K_{ser}^{(FPB)} = \frac{\rho_m^{1.5} \cdot d^{0.8}}{80} = \frac{656.12^{1.5} \cdot 1.53^{0.8}}{80} = 295.215\ N/mm \quad (16)$$

WBB:

$$\rho_m = \sqrt{\rho_b \cdot \rho_t} = \sqrt{410 \cdot 410} = 410\ kg/m^3$$

$$K_{ser}^{(WBB)} = \frac{\rho_m^{1.5} \cdot d^{0.8}}{80} = \frac{410^{1.5} \cdot 1.53^{0.8}}{80} = 145.827\ N/mm \quad (17)$$

d.) The stiffness coefficient γ_{yi} before any cracks appearing in the boards is obtained using Eq. (2):

$$k_{yi}^{(FPB)} = \frac{\pi^2 \cdot A_{t1} \cdot E_t \cdot s}{L_{eff}^2 \cdot 2 \cdot K_{ser}^{(FPB)}} = \frac{\pi^2 \cdot 9^2 \cdot 1000 \cdot 7.5}{(2 \cdot 254.5)^2 \cdot 2 \cdot 2.952} = 3.920$$

$$k_{yi}^{(WBB)} = \frac{\pi^2 \cdot A_{t1} \cdot E_t \cdot s}{L_{eff}^2 \cdot 2 \cdot K_{ser}^{(WBB)}} = \frac{\pi^2 \cdot 9^2 \cdot 1000 \cdot 7.5}{(2 \cdot 254.5)^2 \cdot 2 \cdot 1.458} = 7.934 \quad (18)$$

$$\gamma_{yi}^{(FPB)} = \frac{1}{1 + k_{yi}^{(FPB)}} = \frac{1}{1 + 3.920} = 0.203$$

$$\gamma_{yi}^{(WBB)} = \frac{1}{1 + k_{yi}^{(WBB)}} = \frac{1}{1 + 7.934} = 0.112 \quad (19)$$

e.) The effective bending stiffness $(EI_y)_{eff}$ of the un-cracked cross-section is calculated using Eq.(1):

$$(EI_y)_{eff}^{(FPB)} = 300 \cdot \frac{2 \cdot 1.50 \cdot 125^3}{12} + \\ + 1000 \cdot \left(\frac{2 \cdot 9^4}{12} + \frac{4.4^3 \cdot 9}{12} + 2 \cdot 9 \cdot 9 \cdot 58^2 \cdot 0.203 \right) = \\ = 2.584 \cdot 10^8 \ kNcm^2$$

$$(EI_y)_{eff}^{(WBB)} = 920 \cdot \frac{2 \cdot 1.50 \cdot 125^3}{12} + \\ + 1000 \cdot \left(\frac{2 \cdot 9^4}{12} + \frac{4.4^3 \cdot 9}{12} + 2 \cdot 9 \cdot 9 \cdot 58^2 \cdot 0.112 \right) = \\ = 5.114 \cdot 10^8 \ kNcm^2 \tag{20}$$

f.) The horizontal force ($F_{H,cr}$) forming the first tensile crack in board is calculated using Eq.(6):

$$F_{H,cr}^{(FPB)} = \frac{2 \cdot 0.25 \cdot 2.583 \cdot 10^8}{300 \cdot 125 \cdot 254.5} = 13.53 \ kN$$

$$F_{H,cr}^{(WBB)} = \frac{2 \cdot 1.5 \cdot 5.114 \cdot 10^8}{920 \cdot 125 \cdot 254.5} = 52.42 \ kN \tag{21}$$

$$\Downarrow$$

$$F_{H,cr}^{(WBB)} >> F_{H,cr}^{(FPB)}$$

It is evident that $F_{H,cr}^{(WBB)}$ is higher than $F_{H,cr}^{(FPB)}$. Therefore, it can be concluded, that there is practically no possibility of any cracks forming by using plywood as a sheathing board.

g.) The characteristic horizontal load-carrying capacity ($F_{H,k}$) for FPB is calculated using Eq.(11), $x^{(FPB)}_{II}$ = $42.636\ cm$:

$$F_{H,k}^{(FPB)} = \frac{1.3 \cdot 1.575 \cdot 10^8}{1000 \cdot \left(0.150 \cdot 77.862 + \frac{9}{2} \right) \cdot 263.5} = 39.58 \ kN \tag{22}$$

Using the simplified Eurocode 5 [3] shear model in a form of Eq. (12) we obtain:

For FPB:

$$F_{v,k} = \sum F_{f,Rk} \cdot \frac{b_i}{s} \cdot c_i = 2 \cdot \frac{0.660 \cdot 125}{7.5} \cdot 1.0 = 21.99 \ kN \tag{23}$$

In this case $F_{v,k}$ means the force by which a full yielding of fasteners would appear. Of course, in case of FPB, where the tensile strength is very low, $F_{v,k}$ is much more higher than $F_{H,cr}$.

For WBB:

$$F_{v,k} = \sum F_{f,Rk} \cdot \frac{b_i}{s} \cdot c_i = 2 \cdot \frac{0.517 \cdot 125}{7.5} \cdot 1.0 = 17.22 \ kN \tag{24}$$

Table 2. Numerical results for lateral force acting on one fastener (F_1) and for slip (Δ) in the connecting area.

F_H [kN]	$F_1^{(FPB)}$ [N]	$F_1^{(WBB)}$ [N]	Δ_{FPB} [mm]	Δ_{WBB} [mm]
5.0	69.289	19.279	0.235	0.132
10.0	138.579	38.558	0.469	0.264
13.53 = $= F^{(FPB)}_{H,cr}$	187.497 $< N_{al}$	52.170	0.635	0.358
15.0	198.189	57.838	0.671	0.397
20.0	258.064	77.117	0.922	0.529
25.0	306.057	96.396	1.224	0.661
30.0	352.426	115.674	1.532	0.792
35.0	394.036	134.953	1.859	0.924
39.58 = $= F^{(FPB)}_{H,k}$	437.011 $< F_{f,Rd}$	152.613	2.138	1.045
52.42 = $F^{(WBB)}_{H,cr}$	/	202.12 ≈ N_{al}	/	1.384

4 Conclusion

It is obviously from the presented results that in case of FPB by first crack forming the force acting on one fastener (F_1) is strongly under its characteristic lateral load-carrying capacity ($F_{f,Rk}$). Therefore, our prediction that cracks in FPB appear before the stresses in the fasteners reach their yielding point, was completely correct. Force, forming the first tensile crack in the board, is namely strongly under the characteristic load-carrying capacity obtained with the shear model. Consequently, it is not recommended to use Eurocode 5 Method A to define the lateral load-carrying capacity of the wall.

On the other hand, by using plywood as a sheathing material, force forming the first tensile crack in the board is evidently higher than by FPB. The reason is in higher tensile strength of the board, which is in range of the strength of the timber frame. Consequently, the «Lower bound plastic method« (Eq.12 or Eq.14) can be used to determine the wall's load carrying capacity.

Acknowledgments

The presented research was partially supported by the Leonardo da Vinci project "Educational Materials for Designing and Testing of Timber Structures (TEMTIS)".

References:

[1] P. Dobrila, M. Premrov, Reinforcing Methods for Composite Timber Frame – Fibreboard Wall Panels. *Engineering Structures*, Vol.25, No.11, 2003, pp. 1369-1376.

[2] K.F. Faherty, T.G. Williamson, *Wood Engineering and Construction Handbook*, Mc Graw-Hill Publishing Company, 1989.

[3] CEN/TC 250/SC5 N173, *Eurocode 5: Design of Timber Structures, Part 1-1 General rules and rules for buildings*, Final draft prEN 1995-1-1, Brussels, 2003.

[4] A. Frangi, M. Fontana, Elasto-Plastic Model for Timber-Concrete Composite Beams with Ductile Connection. *Structural Engineering* International, Vol.13, No.1, pp. 47-57, 2003.

[5] M. Premrov, P. Dobrila, Modelling of Fastener Flexibility in CFRP Strengthened Timber-Framed Walls Using Modified γ – Method, *Engineering Structures* (in press), 2007.

[6] M. Premrov, P. Dobrila, B.S. Bedenik, Analysis of timber-framed walls coated with CFRP strips strengthened fibre-plaster boards, *International Journal of Solids and Structures*, Vol.41, No. 24/25, pp. 7035–7048, 2004.

[7] M. Premrov, P. Dobrila, Mathematical Modelling of Timber-Framed Walls Strengthened with CFRP Strips, *Applied Mathematical Modelling* (in press), 2007.

[8] B. Källsner, F. Lam, *Diaphragms and shear walls. Holzbauwerke: Grundlagen, Entwicklungen Ergänzungen nach Eurocode 5, Step 3*, Fachverlag Holz, Düsseldorf, pp. 15/1-17, 1995.

[9] European Committe for Standardization, *EN 338:2003 E: Structural timber – Strength classes*,Brussels, 2003.

[10] Knauf, *Gipsfaserplatten Vidivall/Vidifloor*, 2002.

[11] G. Steck, *Holzwerkstoffe – Sperrholz, Holzbauwerke: Bemessung und Baustoffe nach Eurocode 5, Step 1*, Fachverlag Holz, Düsseldorf, pp. 10/1-10, 1995.

[12] H. Brüninghoff, etal., *Eine Ausführliche Erläuterung zu DIN 1052, Teil 1 bis Teil 3.* Beuth – Kommentare, Beuth Bauverlag, Berlin, 1988.