A Nonlinear Dynamic Model for Compound Planetary Gear Sets

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Abstract: - A discrete nonlinear torsional vibration model of a compound planetary set comprises *d* number of decks is proposed in this study. The model includes all possible power flow configurations, any number of planets in any spacing arrangement and any planet mesh phasing configurations. It also includes time variation of gear mesh stiffnesses as well as clearance (backlash) non-linearities. The nonlinear model equations of motion were obtained by assembling equations of motions of single separate decks according to central elements connections. Equations of motions were solved using HBM in conjunction with inverse Fourier Transform. Dynamic gear mesh forces for first, second and fourth gear ratios were obtained.

Keywords: Compound deck, Planetary, nonlinear, HBM, gear ratio.

1. Introduction

Planetary gear sets are widely used in many machine applications. Planetary gear sets have several advantages over counter-shaft gear systems, including their higher transmitted power density, compactness, ability to achieve multiple speed ratios through different power flow arrangements, and lower gear noise. In addition, axi-symmetric orientation of the planet gears in the gear set creates negligible radial bearings forces.

Planetary gear train dynamics has been a topic of interest to powertrain researchers for two main reasons. First, the forces generated at the meshes of the planetary gear set under high-speed dynamic conditions are typically larger than the quasi-static forces transmitted by the same meshes, influencing the fatigue lives of the gears and planet bearings. Secondly, these dynamic mesh forces are transmitted through to the surrounding structures causing structure-borne noise. Therefore, a dynamic model of a planetary gear pair should aid a designer in quantifying the impact of dynamics on gear set durability and noise, and also help finding ways of reducing the dynamic response amplitudes.

The great majority of the planetary gear dynamic models developed to date for single-stage sets were linear with time-invariant mesh stiffness, so that modal analysis techniques were employed. Models by Antony [1] predicted the free and forced vibrations. Kahraman [2] employed a purely torsional model for all possible power flow configuration of simple, double-planet and complex-compound planetary gear sets such as long-planet systems. Toda and Botman[3] examined the planet mesh phasing influence using two-dimensional time-variant models. They showed that planet mesh forces can be reduced or even neutralized by planet mesh phasing. A later model by Kahraman [4] expanded these formulations to a three-dimensional one with the axial and rotational motions of the gears included so that dynamic response of planetary gear sets using helical gears can be predicted.

The experimental and theoretical studies on single and multiple mesh spur gear dynamics [5-7] clearly indicate that spur gears should be modeled as nonlinear systems having periodically varying parameters.

There are a very few published nonlinear time-varying planetary gear set dynamic models. One such model by Kahraman [5] used a two-dimensional formulation with both mesh stiffness fluctuations and gear backlash included. Tao and Hai Yan [8] investigated the frequency response of nonlinear planetary set with multiple clearances by only focusing on a single power flow configuration in which the ring gear was held stationary. Another recent study by Alshyyab and Kahraman[9] developed a torsional single stage

nonlinear model, equations of motion were solved semi-analytically using a multi-term harmonic balance and a numerical integration method.
To the best knowledge of the authors, multi-deck dynamics is not explored yet. The aims of this study are proposing a purely nonlinear torsional model for a compound or what so called multi-deck gear sets. A multi-term harmonic balance solution will be used to find steady-state dynamic mesh forces for all power flow configuration.

2. Dynamic Model

A schematic or what so called stick diagram of a generic compound planetary set shown in Fig. 1a is composed of a number of single-planet-single-stage-decks, and clutches (not shown) or permanent connections, either constraining central members (sun, ring or carrier) to act as reaction members, rigidly connecting central members or leaving them to rotate freely. Connections, rather than meshes, between members belong to the same deck are not allowed. Connections of central members are chosen here arbitrary to keep generality of the model. Splitting model of Fig.1a into its single deck components as shown in Fig 1b enables writing the equations of motion of the model utilizing recent torsional nonlinear single stage planetary model proposed by alshyyab and kahraman[9]. Any single deck (the *n*th

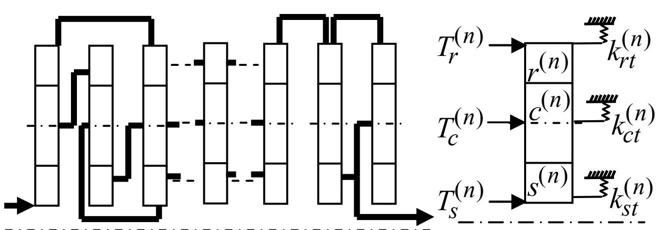

Fig. 1 Stick diagram, a) Multi deck compound planetary gear set. b) the *n*th deck external applied torques and torsional constraints

one) comprises a constraint stiffnesses $k_{it}^{(n)}$ ($i = s,r,c$), number $n_p^{(n)}$ of planet gears $pi^{(n)}$ ($i = 1,2,\ldots,n_p^{(n)}$),and three central members, sun ($s^{(n)}$), ring ($r^{(n)}$) and carrier ($c^{(n)}$). The use of the superscript (*n*) or (,*n*) hereafter is to indicate quantity or parameter pertains to deck-*n*. Planet gears are held by a rigid carrier through rigid planet bearings and are free to rotate with respect to their carrier. Central elements' support bearings are rigid as well.
The *n*th deck dynamic model is shown in Fig. 2. It employs a number of simplifying assumptions. (i) Each gear body is assumed to be rigid and the flexibilities of the gear teeth at each gear mesh interface are modeled by a spring having periodically time-varying stiffness acting along the gear line of action. The mesh stiffness is subject to a clearance element representing gear backlash. (ii) Each gear and the planet carrier were assumed to move only in the torsional $\theta^{(n)}$ direction only. (iii) Viscous gear mesh damping elements are introduced to represent energy dissipation of the system. (iv) Gears and carriers were assumed to be free of any eccentricities or run-out and roundness errors. The central members $s^{(n)}$, $r^{(n)}$ and $c^{(n)}$ are constrained by torsional linear springs of stiffness magnitudes $\bar{k}_{st}^{(n)}$, $\bar{k}_{rt}^{(n)}$ and $\bar{k}_{ct}^{(n)}$, respectively. The magnitudes of these stiffness constraints can be chosen accordingly to simulate certain power flow arrangements. Each gear body i ($i = s^{(n)}, r^{(n)}, c^{(n)}, p1^{(n)}, \cdots, pn_p^{(n)}$) is modeled as a rigid disk of polar mass moment of inertia $I_i^{(n)}$, radius $r_i^{(n)}$ and torsional displacement $\theta_i^{(n)}$. Here $\theta_i^{(n)}$ is the vibrational component of the displacement defined from the nominal rotation of the gear. Planets are located at radius $r_c^{(n)}$ at arbitrary spacing angles $\Phi_i^{(n)}$ defined positive in counter-clockwise direction. External torques $T_i^{(n)}$ ($i = s^{(n)}, r^{(n)}, c^{(n)}$) are applied to the central members to represent input or output torque values.
The mesh of gear j ($s^{(n)}$ or $r^{(n)}$) with a planet $pi^{(n)}$ is represented by a periodically time-varying stiffness element $\bar{k}_{jpi}^{(n)}(t)$ subjected to a piecewise linear backlash function $\bar{g}_{jpi}^{(n)}$ that includes a clearance (backlash) of amplitude $2\bar{b}_{jpi}^{(n)}$. Accordingly, the dynamic model of each single deck with $n_p^{(n)}$ planets includes $2n_p^{(n)}$ clearances. A periodic displacement function of $\bar{e}_{jpi}^{(n)}(t)$ is applied along the line of action to account for intentional gear tooth profile modifications, surface wear, and tooth manufacturing errors. Loses of gear contacts are represented by constant viscous damper

coefficient $\bar{c}_{jpi}^{(n)}$. Here, it is assumed that all planets $pi^{(n)}$ and their respective meshes with gear j ($s^{(n)}$ or $r^{(n)}$)

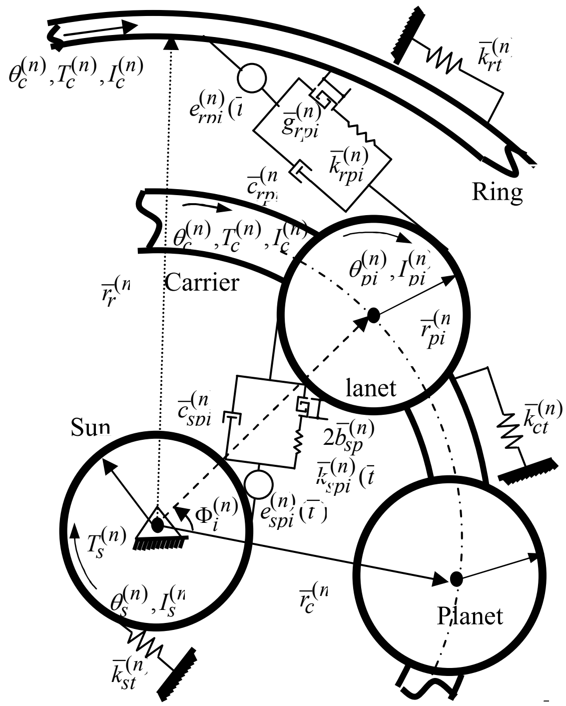

Fig. 2 Dynamic model of a single-stage

are identical so that $\bar{k}_{jpi}^{(n)}(t)$, $\bar{e}_{jpi}^{(n)}(t)$, $\bar{b}_{jpi}^{(n)}$, $\bar{g}_{jpi}^{(n)}$ and $\bar{c}_{jpi}^{(n)}$ are the same for each $jpi^{(n)}$ mesh, except the phase angles of $\bar{k}_{jpi}^{(n)}(t)$ and $\bar{e}_{jpi}^{(n)}(t)$ which differ due to planet phasing conditions.

3. Equations of Motions

Equations of motion for the compound planetary set of Fig. 1a can be written by utilizing a recent work of Alshyyab and Kahraman [9] for single-planet-single-stage planetary set

Non-dimensional equations of motion for a ($3+n_p^{(n)}$) dof nonlinear torsional single deck model shown in Fig 2 are, ($i\in[1,2,..,n_p^{(n)}]$, $j\in[s,r]$)

$$\begin{aligned}&\ddot{\theta}_{cpi}^{(n)}(t)+\ddot{\theta}_c^{(n)}(t)+2\zeta_{sc}^{(n)}\omega_{sc}^{(n)}\dot{p}_{spi}^{(n)}(t)-2\zeta_{rc}^{(n)}\omega_{rc}^{(n)}\dot{p}_{rpi}^{(n)}(t)\\&+[\omega_{sc}^{(n)}]^2\kappa_{spi}^{(n)}(t)g_{spi}^{(n)}(t)-[\omega_{rc}^{(n)}]^2\kappa_{rpi}^{(n)}(t)g_{rpi}^{(n)}(t)=0\end{aligned}\tag{1}$$

$$\begin{aligned}&\ddot{\theta}_j^{(n)}(t)+2\zeta_{jj}^{(n)}\omega_{jj}^{(n)}\sum_{i=1}^{n_p}\dot{p}_{jpi}^{(n)}(t)+[\omega_j^{(n)}]^2\theta_j^{(n)}(t)\\&+[\omega_{jj}^{(n)}]^2\sum_{i=1}^{n_p}\kappa_{jpi}^{(n)}g_{jpi}^{(n)}(t)=f_j^{(n)}(t)\end{aligned}\tag{2}$$

$$\begin{aligned}&\ddot{\theta}_c^{(n)}(t)+\frac{I_p^{(n)}}{I_{ce}^{(n)}}\sum_{i=1}^{n_p}\ddot{\theta}_{cpi}^{(n)}(t)-2\zeta_{sc}^{(n)}\omega_{sc}^{(n)}\sum_{i=1}^{n_p}\dot{p}_{spi}^{(n)}(t)\\&-2\zeta_{rc}^{(n)}\omega_{rc}^{(n)}\sum_{i=1}^{n_p}\dot{p}_{rpi}^{(n)}(t)-[\omega_{sc}^{(n)}]^2\sum_{i=1}^{n_p}\kappa_{spi}^{(n)}(t)g_{spi}(t)\\&+[\omega_c^{(n)}]^2\theta_c^{(n)}(t)-[\omega_{rc}^{(n)}]^2\sum_{i=1}^{n_p}\kappa_{rpi}^{(n)}(t)g_{rpi}^{(n)}(t)=f_c^{(n)}(t)\end{aligned}\tag{3}$$

Here, the dot sign means derivative with respect to time. In these equations $I_{ce}^{(n)}=I_c^{(n)}+n_p^{(n)}(I_p^{(n)}+\bar{r}_c^2 m_p^{(n)})$, $m_p^{(n)}$ is the equivalent mass moment of inertia of deck-n carrier assembly, $m_{pi}^{(n)}$ and $m_p^{(n)}$ are the mass of planet $p_i^{(n)}$ and the summation of masses of $p_i^{(n)}$ planets of deck-n. Absolute rotations $\theta_s^{(n)}$, $\theta_r^{(n)}$, $\theta_c^{(n)}$, and the relative rotations of planets with respect to the carrier $\theta_{cpi}^{(n)}(t)=\theta_{pi}^{(n)}(t)-\theta_c^{(n)}(t)$ are used as the coordinates. In these equations, $p_{jpi}^{(d)}(t)$ are the relative gear mesh displacements along the line of action which is defined as [9],

$$p_{jpi}^{(n)}(t)=\delta_j r_p^{(n)}\theta_{cpi}^{(n)}(t)+r_j^{(n)}[\theta_j^{(n)}(t)-\theta_c^{(n)}(t)]-e_{jpi}^{(n)}(t)\ ,\tag{4}$$

$$\delta_j=\begin{cases}1 & j=s,\\-1 & j=r,\end{cases}\tag{5}$$

the piecewise-linear displacement functions are defined as,

$$g_{jpi}^{(n)}[p_{jpi}^{(n)}(t)]=\begin{cases}p_{jpi}^{(n)}(t)-b_{jp}^{(n)}, & p_{jpi}^{(n)}(t)>b_{jp}^{(n)},\\0, & |\,p_{jpi}^{(n)}(t)\,|<b_{jp}^{(n)},\\p_{jpi}^{(n)}(t)+b_{jp}^{(n)}, & p_{jpi}^{(n)}(t)<-b_{jp}^{(n)},\end{cases}\tag{6}$$

In Eq. 1, non-dimensional stiffness is given by

$$\kappa_{jpi}^{(n)}(t)=k_{jpi}^{(n)}(\bar{t})/\bar{k}_{jp}^{(n)}=1+\hat{k}_{jp}^{(n)}(\bar{t})/\bar{k}_{jp}^{(n)},\tag{7}$$

where $\bar{k}_{jp}^{(n)}$ and $\hat{k}_{jpi}^{(n)}(\bar{t})$ are the mean and alternating components of $jpi^{(n)}$ mesh stiffness. The Other non-dimensional parameters and quantities of equations (1-6) were obtained by non-dimensionalizing using a characteristic length and frequency $b_a=0.01mm$ and $\omega_a=1000Hz$, respectively, these dimensionless parameters are given by Alshyyab and Kahraman [9]. Equations of motion for a compound planetary set composed of a number of single-planet decks can be obtained by assembling equations of motions of its split decks (Eq.1), with the fact that assembled equations for two connected members should be divided the same inertia term during the course of nondimensionalization.

3.1 Steady-state Response Using the Multi-term Harmonic Balance Method

The parametric excitations $\kappa_{jpi}^{(n)}(t)$ ($j = s, r,\ \ n = 1,2$) in Eq. 1 are periodically time-varying. These excitations have a dimensional fundamental frequency $\Lambda^{(n)} = m_n\Lambda = \Omega_{cp}^{(n)} Z_p^{(n)} / \omega_a$, where Λ is independent variable, $\Omega_{cp}^{(n)}$ is the angular velocity of the planets with respect to their carrier, $Z_p^{(n)}$ is the number of planet teeth and m_n is an integer to accommodate commensurate frequencies of the first and the second deck meshes. Stiffnesses $\kappa_{jpi}^{(n)}(t)$ can be written in Fourier series form as,($j \in [s,r]$, $n \in [1,2]$, $i \in [1,2,\cdots,n_p]$)

$$\kappa_{jpi}^{(n)}(t) = 1 + \sum_{h=m_n}^{m_n H} [\kappa_{2h}^{(jpi,n)} \cos(h\Lambda t) + \kappa_{2h+1}^{(jpi,n)} \sin(h\Lambda t)]\,. \quad (8)$$

Where, the increment of h in the last equation is m_n. H is the number of harmonic terms sufficient to describe the periodic stiffness function. Amplitudes and phasing of the harmonic terms are given by Alshyyab and Kahraman [7].

The applied external torques where assumed constants. Periodic solutions of equations (1) is assumed as, ($s,\ \ c, r, cp1, cp2, \ldots, cpn_p$,)

$$\theta_a^{(n)}(t) = u_1^{(a,n)} + \sum_{n=1}^{N} [u_{2n}^{(a,n)} \cos(\Lambda \frac{n}{\eta} t) + u_{2n+1}^{(a,n)} \sin(\Lambda \frac{n}{\eta} t)] \quad ,(9)$$

Where again superscript n refers to deck-n, η is sub-harmonic index, $u_j^{(a,n)}$ is the amplitude of the solution harmonic contents, and N is the number of harmonic terms considered in the assumed solution. The same way, the mesh displacement functions $p_{jpi}^{(n)}(t)$ are written as

$$p_{jpi}^{(n)} = P_1^{(jpi,n)} + \sum_{n=1}^{N} [P_{2n}^{(jpi,n)} \cos(\Lambda \frac{n}{\eta} t) + P_{2n+1}^{(jpi,n)} \sin(\Lambda \frac{n}{\eta} t)] \,,(10)$$

with

$$P_1^{(jpi,n)} = \delta_j r_p u_1^{(cpi,n)} + r_j (u_1^{(j,n)} - u_1^{(c,n)}) - E_1^{(jpi,n)} \,,(11)$$

$$P_{2n}^{(jpi,n)} = \delta_j r_p u_{2n}^{(cpi,n)} + r_j (u_{2n}^{(j,n)} - u_{2n}^{(c,n)}) - E_{2(n/\eta)}^{(jpi,n)} \,,(12)$$

$$P_{2n+1}^{(jpi,n)} = \delta_j r_p u_{2n+1}^{(cpi,n)} + r_j (u_{2n+1}^{(j,n)} - u_{2n+1}^{(c,n)}) - E_{2(n/\eta)+1}^{(jpi,n)} \,.(13)$$

where $\delta_j = 1$ for $j = s$ and $\delta_j = -1$ for $j = r$. Before enforcing a harmonic balance, the clearance functions $g_{jpi}^{(n)}(t)$ should be written in the same form as well,

$$g_{jpi}^{(n)} = v_1^{(jpi,n)} + \sum_{n=1}^{N} [v_{2n}^{(jpi,n)} \cos(\Lambda \frac{n}{\eta} t) + v_{2n+1}^{(jpi,n)} \sin(\Lambda \frac{n}{\eta} t)] \,.(14)$$

By dividing the solution period T into Q subintervals ($\Delta T = T / Q$), and using Inverse Discrete Fourier Transforms, the amplitudes $v_k^{(jpi,n)}$ can be obtained as [5],

$$v_1^{(jpi,n)} = \frac{1}{Q} \sum_{q=1}^{Q-1} g_{jpi_q}^{(n)} \,,\ v_{2n}^{(jpi,n)} = \frac{2}{Q} \sum_{q=1}^{Q-1} g_{jpi_q}^{(n)} \cos(\frac{2\pi nq}{Q}) \,,$$

$$v_{2n+1}^{(jpi,n)} = \frac{2}{Q} \sum_{q=1}^{Q-1} g_{jpi_q}^{(n)} \sin(\frac{2\pi nq}{Q}) \,, \quad (15)$$

where $g_{jpi_q}^{(n)} = g_{jpi}^{(n)}(q\Delta t)$.

Substituting equations (4-10) into equation (1), and applying a harmonic balance, a set of nonlinear algebraic equations is obtained in the form $\mathbf{S}(u_i^{(a,n)}, \Lambda) = \mathbf{0}$, where the elements of vector **S** are given by alshyyab and Kahraman [9].

4. Two-deck Planetary Set

As a case study, Two-deck planetary set shown in Fig.3 will be considered. Parameters of this set are given in Table 1. The set is composed of clutches, rigid connections and two single-planet-single-stage decks. Connections $c^{(1)}r^{(2)}$ and $c^{(2)}r^{(1)}$ link rigidly carrier of first deck $c^{(1)}$ with ring of the second deck $r^{(2)}$ and ring of first deck $r^{(1)}$ with carrier of second deck $c^{(2)}$, respectively. All aforementioned connections are permanent. The output torque T_o is connected to the $c^{(2)}r^{(1)}$ permanently. Therefore, power flow configurations of first, second, third and fourth gear ratios can be obtained by changing central members assignments as input, reaction or free to rotate members via proper sequence of clutch release and activation. Input/output speed of ratio 2.82 suitable for first gear drive can be obtained by activation of clutches C13R and C12 which assign the sun of the first deck $s^{(1)}$ as an input member and rigidly constraint the sun $s^{(2)}$ of the second deck as a reaction member, respectively. The second gear drive of 1.55 input/output gear ratio is achieved by maintaining clutch C12 activated, releasing of clutch C13R and activation of clutch

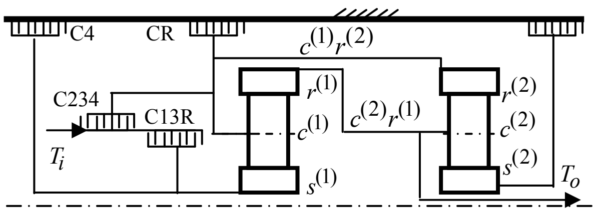

Fig. 3 Two deck planetary set.

C234 to transfer the input torque T_i to $c^{(1)}r^{(2)}$ connection. The reaction member is the difference between the second and fourth gear drive arrangements. Releasing clutch C12 and activation of clutch C4, sun of the first deck $s^{(1)}$ would be

n	$Z_s^{(n)}$	$Z_r^{(n)}$	module	Press. angle
1	30	70	1.5	21.3°
2	44	80	1.5	21.3°

Table 1 Parameters of first and second deck n=1, 2.

reaction element and the input/output speed ratio would be reduced to 0.7, which is suitable for the fourth gear drive. For the third gear drive the input/output speed ratio is 1.0, it can be achieved by activation of clutches C234 and C13R while all other clutches are released, this configuration locks the assembly as one block with all its members have the same angular speed, hence, there is no parametric excitations.

5. Results and Discussion

The parameters of the two deck study case of this work are given in Table 1. Each deck is formed by four equally spaced planets ($n_p^{(n)} = 4$). The gear parameters are such that the involute contact ratios of sun-planet and ring-planet meshes are 1.55, 1.81, and 1.58, 1.78 for first and second deck meshes, respectively. These contact ratios are estimated theoretically. The half backlash values for both meshes are chosen as $\bar{b}_{jp} = 0.0\ mm$, that is the linear case analysis is considered only. The characteristic length is $b_a = 0.1\ mm$, and the characteristic frequency needed for nondimensionalization is chosen as $\omega_a = 1000$ Hz. Table 2, summarizes external applied torques and constraint stiffnesses for each member of first and second decks for the first, second and fourth gear drive ratio configuration, where reaction members are constraint by large torsional stiffnesses, and other free to rotate members are constraint with small stiffnesses.

	1st Gear Ratio			2ed Gear Ratio			4th Gear Ratio		
n	$s^{(n)}$	$r^{(n)}$	$c^{(n)}$	$s^{(n)}$	$r^{(n)}$	$c^{(n)}$	$s^{(n)}$	$r^{(n)}$	$c^{(n)}$
External Torque, $T_i^{(n)}$, $N.m$									
1	750	1750	-2500	0	0	0	-225	-525	750
2	1375	2500	-3875	412.5	750	-1162.5	0	0	0
Torsional Constraint, $k_{it}^{(n)}$, $N.m/Rad.$									
1	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{+8}	10^{-1}	10^{-1}
2	10^{-1}	10^{-1}	10^{+8}	10^{-1}	10^{-1}	10^{+8}	10^{-1}	10^{-1}	10^{-1}
Mean sun/planet stiffness	$\bar{k}_{spi}^{(1)} = 2.2(10)^8$, $\bar{k}_{spi}^{(2)} = 2.5(10)^8$ N/m								
Mean ring/planet stiffness	$\bar{k}_{rpi}^{(1)} = 2.8(10)^8$, $\bar{k}_{rpi}^{(2)} = 3.4(10)^8$ N/m								

Table 2 Planetary set parameters. n is the deck

Mean listed applied torques were estimated statically for an input torque of 750 N.m.

Values of 0.02 damping ratio is used to find the dimensionless characteristic damping given by alshyyab and kahraman [9]. As steady stated dynamic mesh forces are key parameters for design guide lines, dimensionl dynamic mesh forces along the lines of action are computed using formulas given by alshyyab and Kahraman[9]. In the any gear drive arrangement, rigid connections $c^{(1)}r^{(2)}$ and $c^{(2)}r^{(1)}$ imply that $\theta_c^{(1)} = \theta_r^{(2)}$ and $\theta_r^{(1)} = \theta_c^{(2)}$, respectively. Hence, the algebraic equation for the first and second decks as given by Alshyyab and Kahraman [9] can be assembled accordingly to get a new set of nonlinear algebraic equation

$$\mathbf{S=0.} \tag{16}$$

The last equation is solved iteratively for first, second and fourth gear ratios for equally spaced planets. Figure 5 is period-1 root mean square forces (*rms*) for both deck meshes. The first and second columns of Fig. 5 are the sun/planet and ring/planet period-1 ($\eta = 1$) mesh forces for first, second and fourth gear ratios, respectively. Harmonic and superharmonic resonances are shown in these figures. For all kinematics configurations of first, second and fourth gear ratios, when the mean transmitted load values though a mesh are nonzero valued, the corresponding dynamic forces are of considerable magnitudes or even they are many times larger than the mean transmitted load in the resonant regions. If the mean transmitted load values are zero, the corresponding dynamic force response is almost zero, except regions of resonances. Values of mean mesh transmitted loads are given in table 2.

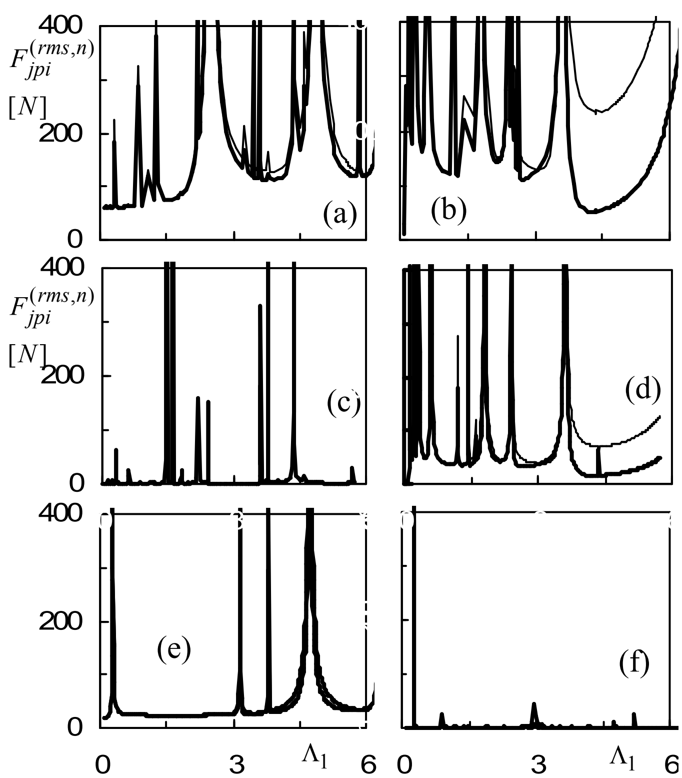

Fig. 5, *rms* mesh force response. (a), (c) and (d) for deck-1 meshes. (b), (d) and (f) for deck-2 meshes. (-) sun/planet meshes, (~) ring/planet meshes.

6. Conclusions

In this work, a torsional nonlinear dynamic model for general compound planetary set was developed. Equations of motion for the special linear case of the model for two deck planetary set were solved for first, second and fourth gear ratios. The solutions were obtained using the harmonic balance method in conjugate of inverse Fourier Transform.

The authors' on going work is to find period-1 and the higher periodic solutions for the nonlinear case as well as demonstrate a detailed parametric study of the system parameters.

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