Interactive tools for Calculus e-learning

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Abstract: - Nowadays, TICs (technologies for informatics and communication) provide a very positive aid to the learning tasks. Interactive tutorials provides access to a big amount of information in a multi-sequential way, by using specifically designed Java applets suitable for the learning of a specific knowledge. Due to these facts, the use of these tools has many advantages for learning mathematics. In this sense, we present here an interactive tool for differential calculus with a theoretical environment.

Key-Words: - Interactive Tutorials, Java Applets, World Wide Web, Derivative, Calculus

1 Introduction

The reading by exploration or navigation of a hypertext is multi-sequential and interactive. The reader makes visual sweepings and searches of fragments of interest. It is recommendable to use textual or graphical tools that appear in the screen and that allow the user to identify and to distinguish the contents of the hypertext. Navigation has replaced linear reading, the information is a space to travel, a path to explore.

The work that we present here is the elaboration of an Interactive Tutorial, including several Java applets, to be used by teachers on the classroom lectures and by the students when learning by themselves. This web tool concerns the notion of derivative of a function in a point and its applications, with special attention to applications and Rolle and Lagrange mean value theorems. This is part of the syllabus of the course "Calculus", taught in the first semester of Computer Science at the Polytechnic University of Madrid. The tutorial includes definitions, theorems and the most important results, along with some practical applications. We have designed several interactive applets that allow the reader to experience a high degree of interactivity, offering him the freedom to generate its own examples. These applets are made using programming standards for the World Wide Web, like Java or JavaScript.

The tutorial presented in this paper is just part of a more general project currently developing which covers most of the areas of Calculus.

We have been working since many years ago, developing interactive tools to help the teacher to present and display, in an animated and interactive way, many mathematical notions and algorithms in the classroom. These tools can be also used by the students through the web page of the Department to experiment, in a virtual way, with the contents of the various courses taught by us.

2 Objectives

The main objective of this work is to develop interactive tools for Calculus (in particular, the current paper focus on the subject "Derivative of a function") to be used both by teachers on classroom lectures and by students when learning by

themselves. To make it as friendly and attractive for students as possible, we have given special attention to the following properties of these tools:

- A graphical interface for the hypertext which can be easily handled by the user. It allows the visualization of the contents and the organization of the information in an immediate way through pull-down menus. One of our goals is that the different applications which are presented in the tutorial can be easily and quickly found within each section.
- Implementation of didactic applets for the most relevant concepts and applications of differential calculus. These applets are immersed in a theoretical framework. Therefore, as we pretended, the user can interact with the tutorial, so that its use is more attractive and interesting.
- Accessibility from the web page of the department, as additional documentation for the course "Calculus". This will be also integrated in a b-learning moodle context.
- Design of the applets using programming standards for the World Wide Web, to avoid incompatibilities.
- Facility to include new functionalities and algorithms in the future, if desired.

3 Description of the interactive tutorial

This interactive tutorial focuses in practical aspects. Numerous practical examples are included, mainly in the form of interactive applications for the Web. These applications have been implemented using technologies characteristic of the Web, in the form of Java applets or as dynamical web pages using Javascript.

The previous figure shows the page from which the reader can access the different sections that we describe next. It is available at http://www.dma.fi.upm.es/java/calculo/derivacion

The theoretical part begins with an illustration, made using flash technology, of the derivative as an instant speed, followed (in a different page) by the formal definitions of derivative of a function in a point, side derivatives, higher degree derivatives and the relation between continuity and differentiability of a function.

The geometrical interpretation of the derivative of a function as the slope of the tangent of its graph is the goal of the first applet in the tutorial.

We show the most common particular cases of non-differentiability: non-continuity points, angles, continuity points with infinite oscillation, …

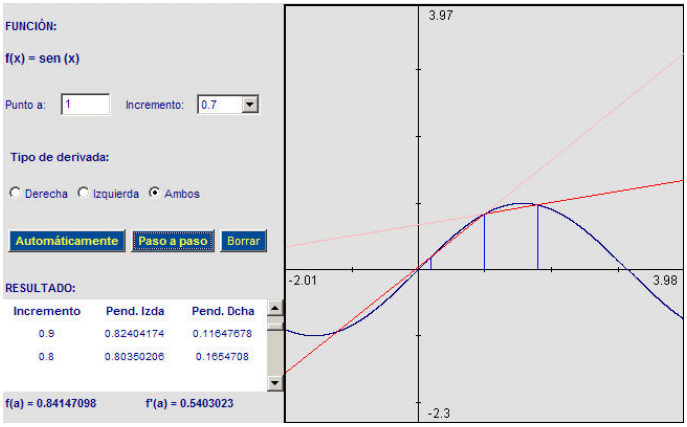
We then develop applets to experiment with the main applications of differentiability: Rolle and Lagrange mean value theorems, and the relations between the increase or decrease of a function and the sign of its first derivative.

The last page includes some self-evaluation tests.

We describe next some of the applets.

3.1 The "geometrical interpretation" applet

This application, written in dynamic HTML with Javascript, shows how the derivative of a function in a point is the slope of the tangent to the graph of the function at the given point.

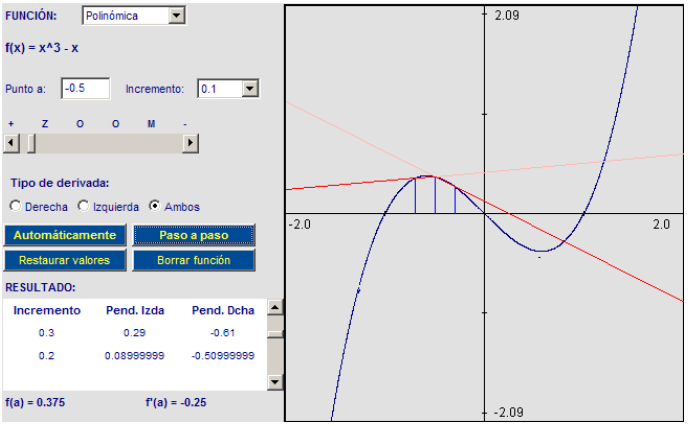


Since the goal of this first applet is mainly didactic, the function is fixed. When the user chooses a point (with the mouse or writing its coordinate) the applet displays a sequence of secant lines approaching the tangent line. At the same time, the slopes of these secant lines are computed. The student can see how the values of these slopes converge to the derivative of the function.

Finally, the applet gives the student the option to zoon into the point to see how the tangent line is a local approximation of the function.

3.2 The general applet

Once the student is familiar with the notion and geometrical interpretation of the derivative, he may further experiment with a more general and versatile applet, where he can choose the (trigonometric or polynomial) function to differentiate, as well as the point and the approximation rate.



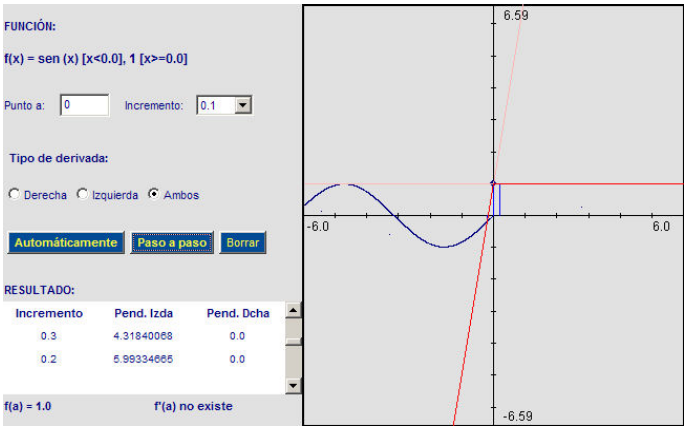
All the applets in our tutorial have the possibility of a step by step visualization, zooming in and out, and moving the graph with the mouse.

3.3 Applets for particular cases

Since the families of allowable functions for the above applet does not include pathological cases, we have developed several applets which illustrate some special cases.

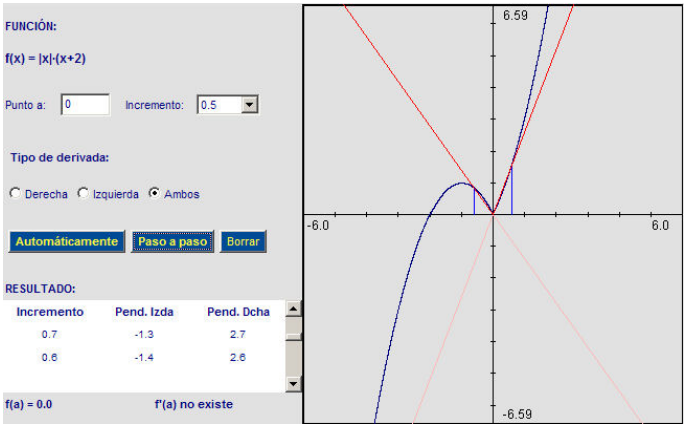
3.3.1 Discontinuous functions

We show how in the case of a non-continuous function with a jump discontinuity, one of the side derivatives is infinite. This is shown geometrically when the sequences of secant lines is displayed.



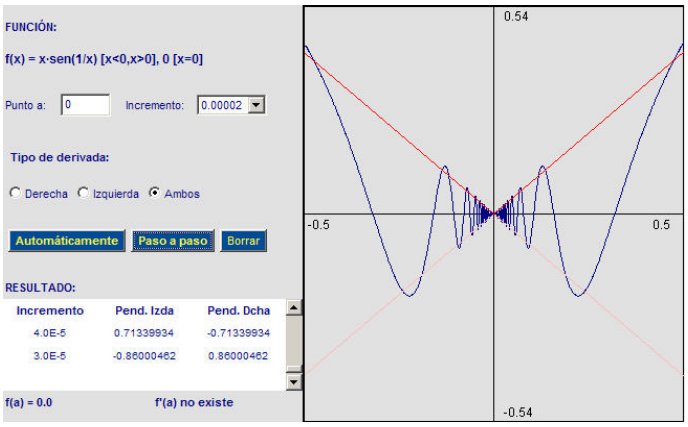
3.3.2 Continuous non differentiable function

The next applet illustrates the case of a continuous function with no derivative, and therefore no tangent, in an angle point with finite (but different) side derivatives. The applet shows, in a dynamical and geometrical way, the non-existence of the tangent line in such a point.



3.3.3 Continuous function without side derivatives

Although less frequent that the previous case, there exist functions where the lack of differentiability is due to the non existence of any of the side derivatives.



3.3.4 (Non trivially) derivable function

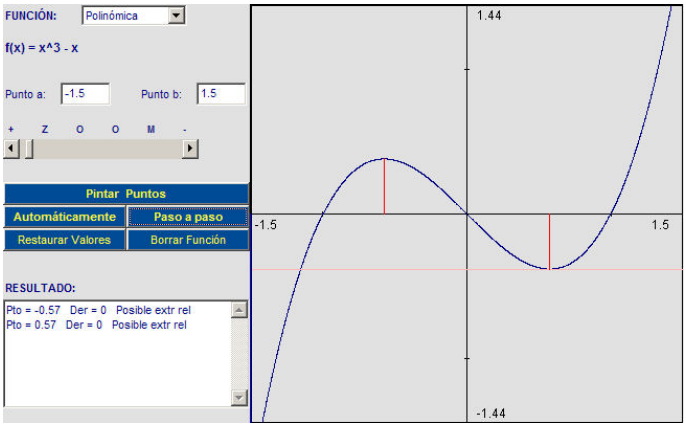
Finally, we show an example closely related with the previous one, but in this case the function is differentiable at zero. Although the function $x^2\sin(1/x)$ oscillates, it does it between two functions, x^2 and $-x^2$, both of them with null derivative at zero, hence the derivative of our function at zero must be also zero.

3.4 Applications

The last part of the tutorial is devoted to the main applications of differential calculus to the study of functions.

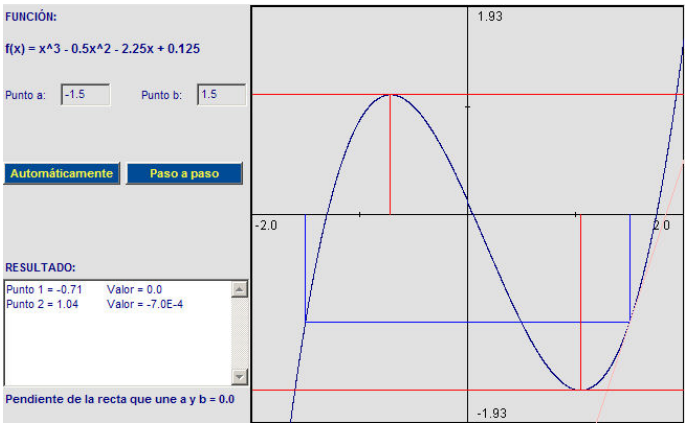
3.4.1 Relative extrema

If a function has a relative extremum at a point at which the functions is differentiable, then its derivative must change its sign when passing through this point. This is the key fact used in the next applet to search for possible extrema of a function. The geometric idea behind this result is made clear in the applet.



3.4.2 Rolle Theorem

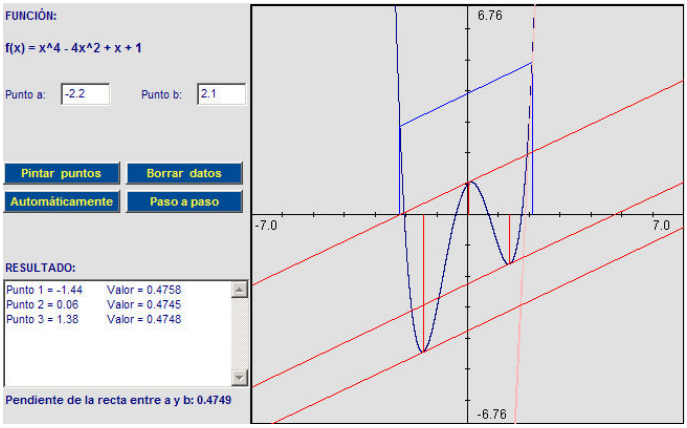
A related result is Rolle Theorem. This fundamental result states that if a differentiable function takes the same value at two points, then the derivative cancels at some intermediate point. Again, we use an applet to explain this result and to search graphically for points at which the derivative vanishes.



3.4.3 Lagrange mean value theorem

As a consequence of Rolle theorem we obtain one of the most important results in Infinitesimal Calculus: Lagrange mean value theorem. The relation between both results is revealed by a new applet.

This applet works searching the points whose existence is guaranteed by Lagrange mean value, by evaluating the derivative at many points along the interval. When the slope gets over the slope searched, it goes back and makes a search until it finds the point with the required derivative.

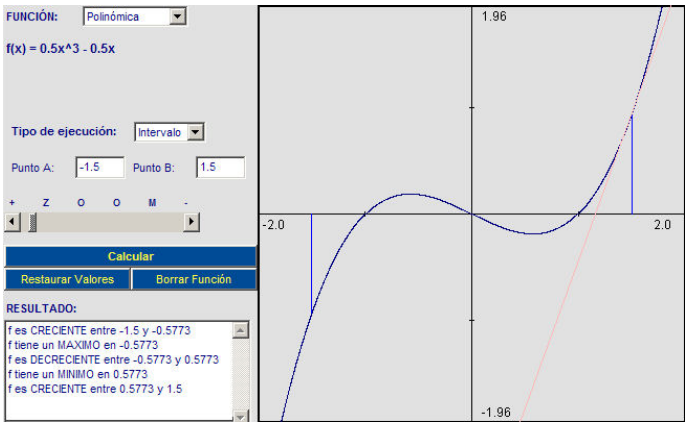


3.4.4 Increase and decrease of functions

One of the consequences of Lagrange mean value theorem is the necessary and sufficient conditions for a function to be increasing or decreasing in an interval. This result is used to construct an applet that determines the intervals at which a function is increasing or decreasing.

4 Conclusions and future work

The didactical benefits of this interactive tutorial for Calculus, according to our experience in teaching these mathematical concepts, are:



- It helps the student to understand the concept and the geometrical interpretation of the derivative of a function in a point, as well as the fundamental role Lagrange mean value theorem plays in Calculus.
- It helps the teachers in their lectures by navigating through the examples and the applications implemented along the tutorial.
- They offer the student the opportunity to experiment, increasing interactivity.

In general, interactive tutorials including Java applets are very good aids for learning mathematics, as they

improve comprehension, engagement, memorization and the satisfaction of the students, as well as the interest and motivation amongst pupils when the teacher makes use of them.

Finally, as a future work, we have mentioned that we are currently developing similar tutorials for other parts of Calculus, in an attempt to cover as much as possible of this discipline. We would like also to increase the family of allowable functions to include rational, exponential and logarithmic functions and basic combinations of them.

Moreover, we are also working in other of Mathematics related to Computer Science. For example, we have also made tutorials for Discrete Mathematics, Dynamical Systems, Fractal Geometry and Image Processing.

We intend also to elaborate interactive books for these subjects, including this kind of applets and different kinds of tests of an adaptive nature, i.e., than can be dynamically generated according to the knowledge of the students.

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