Three life insurance model research--Family unite the insurance model

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Abstract: - First, the family condition in the future is analyzed. In analyzing about muliti-life function, introducing the situation of three element, especially while considering the death order, consider a person dies before other two people (state of jointly surviving). Second, two kinds of association situations and two models are considered here. The first place is the couple and only children; the second place is couple own. At the last, the actuarial present value on the life insurance and annuity are calculated.

Key-Words: - life insurance, actuarial, survival function, life table, multi-element life function

1 Introduction

In a term life insurance policy, usually only insured one person, also possible for multi-person life insurance, such as couple, family, for elder parents, and etc. Usually we can divided by two group by how many insured people every policy, individual policy and multi-person policy, also called connectable life insurance[1]. Usually annually fund is for individual policy. For this paper, we study for multi-person policy, and study target on future family. For life-insurance, expect for group policy, mostly are for ordinary person. It has a huge potential market for future family, though it is very necessary we need pay attention on it, and develop products for such demands.

2 An analysis for a future home

Let think about a future family members, a husband, a wife, four elder parents, and one kid. The financial status, a couple has a decent income, enough for care about elder parents, growing a kid, and paying bills. Four elder parents have or partial has own endowment insurance, they basically do not need income from its child, financial dependently. And they have certain mount for medical insurance, expect from social security, they has lack information of commercial insurance. Young kid is still in study, financially not able to live independently, and need a quite spending on it. Another point, this couple has highly educated and has decent income, marriage age is elder than average from 20 century in China. Usually they married after 30, has own house and car, but instill financing. So they have quite a bit pressure, and their parents also have quite bit age.

Let study individual thoughts for each family member, for each elder person, they have been retired, quite old in age, and health condition is critical, health insurance and term life insurance are mainly focused, in health insurance we cover on critical sickness, hospitalization cover, and accident insurance. Based on social security, they already have some pension, and also can cover certain mount of medical expense, so they can offset these expense in short term. For growing kids, education and growing are two main concerns. Once they left college, they will ability to independently. Although, these expense are pressure on the couple. In case, couple in accident, the kids education and growing will be major effected. They take care elders and grow and with educate kid, and paying bills for the couple, so they need more security.

We are put death condition of family in consideration and analyze that death has influence upon family[2]. To introduce death in the model, one is anyone in death by couple. another case is both couple dead, on third case, the couple get old, and kid dead in accidentally. We are not put these elder parents death in consideration. We analyze that three death conditions have influence on family. Two cases we are study with, one is husband in death second wife in death. These two cases are different. Let assume, the husband dead how effect the whole family financial situation. From income structure, psychological impact for whole family, and sexual difference, the husband death will be major impact for take care a family, such as take care elder parents and grow kid. If happen in wife, these problems are also effects. However, which one will be more effect than others? Let study psychological impact for wife or husband, due to variable research, lost wife is more psychological impact than lost husband. The husband get more possibility become psychopathic because of lost loved one. So, in our model we have more insurance coverage for lost wife. Another case is both couple dead. In this case, the whole family suddenly down to terrible situation. Elder could death earlier because of lost loved kid, and young kid could live negatively because of loosing his/her parents. Thus, our proposal is elder parents could live normally, and young kid could still in normal education, and grow positively. For approach this goal, our coverage payment will be quite big. On third case, the couple get old, and kid dead in accidentally. If kid still quite young, the possibility get another kid will be quite high, if young kid become adult, the possibility for another one will be quite low. We assume that exceeding 18 age in death has more negative influence on family than under 18 age. In our life table and physiology, the possibility of a dead kid will be very low, even though younger than 5 years old will be quite high.

Then we create a allied insurance with husband, wife, and kid. We associated insurance with husband and wife. We set statement, any one of them dead, the state changed, they are able to claim the coverage. We give different mount base on the model we assumed. In second case, we set statement they only can claim only if couple are both dead, the payment coverage such as take care elders, grow kid and with education, and payment for after death. At last, we talk about whole family dead that mean, husband, wife, and kid.

3 Introducing multi-life function

From two chapter, we consider two conditions, husband, wife, kid and couple. Let assume three objects (x), (y), (z) means husband, wife and kid. For a allied insurance model, we set condition no change as long as all three in live, only condition change when one of them dead. For a couple policy the condition no change until both of them dead (husband and wife). For better understanding we give two definitions[1,3-5].

Definition 1: For a condition has N life objects, No i object has x_i years old. The future life call $T(x_i)(i=1,2,...,N)$. If we set condition for the first object dead define a condition termination. And record this condition as $(x_1, x_2, \ldots, x_N)$, then the condition maintain time as :

$$T(x_1, x_2, \ldots, x_N)=\min\{T(x_1),T(x_2),...,T(x_N)\}$$

Then we called minimum life from, or union life form.

Definition 2: For a condition has N life objects, No i object has x_i years old. The future lifetime call $T(x_i)(i=1,2,\ldots,N)$. If we set condition for the last object dead define a condition termination. And record this condition as $(\overline{x_1, x_2, \ldots, x_N})$, then the condition maintain time as:

$$T(\overline{x_1, x_2, \ldots, x_N}) = \max\{T(x_1), T(x_2), \ldots, T(x_N)\}$$

Then we called maximum life form, or the last life form.

Thought, for union life form the condition remain when all life object remains, for the last life form, the condition only change when no life objects exist any more.

When we talk about T(x),T(y), T(z), we assume each person independently, but in fact, they are associated together. Anyone of them death could effect others life time.

3.1 The probably distribution of future life of three union life condition

T(xyz) Distribution function:

$$\begin{aligned} F_{T(xyz)}(t) &= P\{T(xyz) \le t\} \\ &= P\{\min[T(x),T(y),T(z) \le t\} \\ &= 1-P\{T(x)>t\} \bullet P\{T(y)>t\} \bullet P\{T(z)>t\} \\ &= 1-{}_tp_x \bullet {}_tp_y \bullet {}_tp_z \end{aligned} \quad (1)$$

Then the probability of union live condition (xyz) at least t years:

$${}_tp_{xyz} = P\{T(xyz)>t\}=1-F_{T(xyz)}(t)={}_tp_x \bullet {}_tp_y \bullet {}_tp_z \quad (2)$$

T (xyz) probability density function is:

$$\begin{aligned} f_{T(xyz)}(t) &= \frac{d}{dt}F_{T(xyz)}(t) \\ &= \frac{d}{dt}(1-{}_tp_x \bullet {}_tp_y \bullet {}_tp_z) \\ &= -(-{}_tp_x u_{x+t})\,{}_tp_y \bullet {}_tp_z - (-{}_tp_y u_{y+t})\,{}_tp_x \bullet {}_tp_z - (-{}_tp_z u_{z+t})\,{}_tp_x \bullet {}_tp_y \\ &= {}_tp_x \bullet {}_tp_y \bullet {}_tp_z(u_{x+t}+u_{y+t}+u_{z+t}) \end{aligned} \quad (3)$$

3.2 The probably distribution of future life of two life condition

$T\{\overline{xy}\}$ distribution function[6-7] :

$$\begin{aligned} F_{T(\overline{xy})}(t) &= P\{T(x) \le t\} \bullet P\{T(y) \le t\} \\ &= F_{T(x)}(t) \bullet F_{T(y)}(t) = {}_tq_x \bullet {}_tq_y = (1-{}_tp_x)(1-{}_tp_y) \\ &= 1-{}_tp_x-{}_tp_y+{}_tp_x \bullet {}_tp_y \end{aligned} \quad (4)$$

Then the probability of union live condition $(\overline{xy})$ at least t years:

$$\begin{aligned} {}_tp_{\overline{xy}} &= P\{T(\overline{XY}) \ge t\} = 1-F_{T(\overline{XY})}(t) \\ &= {}_tp_x + {}_tp_y - {}_tp_x \bullet {}_tp_y \end{aligned} \quad (5)$$

$T(\overline{XY})$ probability density function is:

$$f_{T(\overline{xy})}(t)=\frac{d}{dt}F_{T(\overline{xy})}(t)$$
$$=\frac{d}{dt}(1_t-p_x-{}_tp_y+p_x\bullet{}_tp_y)$$
$$={}_tp_x\bullet u_{x+t}+{}_tp_y\bullet u_{y+t}-(u_{x+t}+u_{y+t})\,{}_tp_x\bullet{}_tp_y$$
(6)

3.3 A probability of union conditions

Let us talk about three objects, that (x), (y),(z), any one of dead and others remain life. The probability presented by ${}_nq^1_{xyz}$ .Mean (x) is dead before (y),(z) and within n years. Then,

$$q^1_{xyz}=\int_0^n\int_0^\infty {}_sp_y\,{}_sp_z(u_{y+s}+u_{z+s})\bullet{}_tp_x u_{x+t}dsdt$$
$$=\int_0^n {}_tp_x u_{x+t}[\int_0^\infty {}_sp_y\,{}_sp_z(u_{y+s}+u_{z+s})ds]dt \quad (7)$$
$$=\int_0^n {}_tp_x u_{x+t}(1-{}_tp_y\,{}_tp_z)dt$$
$$=\int_0^n {}_tp_x u_{x+t}dt-\int_0^n {}_tp_x\,{}_tp_y\,{}_tp_z u_{x+t}dt$$

The probability presented by ${}_nq^1_{yxz}$. Mean (y) is dead before (x), (z) and within n years.Then,

$${}_nq^1_{yxz}=\int_0^n {}_tp_y u_{y+t}dt-\int_0^n {}_tp_y\,{}_tp_x\,{}_tp_z u_{y+t}dt$$
(8)

The probability presented by ${}_nq^1_{zxy}$. Mean (z) is dead before (x), (y) and within n years.Then,

$${}_nq^1_{zxy}=\int_0^n {}_tp_z u_{z+t}dt-\int_0^n {}_tp_z\,{}_tp_x\,{}_tp_y u_{z+t}dt \quad (9)$$

4 Analysis of Policy Requirement

4.1 Insurance target

A family, male 30 years or older, Female 28 years or older, good health condition, a couple are able to work and has a 5 years old or older child. Also accept a couple without child.

4.2 Responsibility

For Term-life insurance: husband, wife and kid, any of them dead, we pay for lost. The payment ratio base on we talked about before. M1,M2, M3, and M2>M1>M3.

For couple both dead, we pay for M4 and M4>(M1+M2+M3), that we can take care good for elder parents and child for couple both lost.

For annuity: kid able to get payment after 10, and every year get M5, that enough for pay education expense. After 18, he/she get M6 annually. M6>M5. That moment, kids should be in college, the education expense will higher than before, and payment stops at 25 years old. At that moment, he should in master degree or in work. For couple, anyone is more than 60 years old, they can get the payment until they dead.

4.3 Exception

These conditions are not in our coverage insured person dead by drug.Insured person dead by suicide. Insured person dead by drunk drive or driving without LC .Insured person dead by involve rebellions.

5 Valuation of Premium with Fixed Interest

5.1 Long-life insurance

For easier calculation we set payment as 1, then in three life union condition (xyz) terminates, pay off is 1;couple last live condition finishes,Payment set 1;in three life union condition $(\overset{1}{x}yz)$, $(\overset{1}{y}xz)$, $(\overset{1}{z}yx)$ finishes,Payment set 1.Then, these three single premium and on-level premium is $\overline{A}_{xyz}$, ${}_hp_1$, $\overline{A}_{\overline{xy}}$, ${}_hp_2$, $\overline{A}^1_{xyz}$, ${}_hp_{3x}$, $\overline{A}^1_{yxz}$, ${}_hp_{3y}$, $\overline{A}^1_{zxy}$, ${}_hp_{3z}$, then[8-10],

$$\overline{A}_{xyz}=E(v^{T(xyz)})=\int_0^\infty v^t\bullet{}_tp_x\,{}_tp_y\,{}_tp_z(\mu_{x+t}+\mu_{y+t}+\mu_{z+t})dt \quad (10)$$

$$\overline{A}_{\overline{xy}}=\int_0^\infty v^t[{}_tp_x\mu_{x+t}+{}_tp_y\,{}_t\mu_{y+t}-{}_tp_x\,{}_tp_y(\mu_{x+t}+\mu_{y+t})]dt$$
$$=\overline{A}_{\bar{x}}+\overline{A}_{\bar{y}}-\overline{A}_{xy} \quad (11)$$

$$\overline{A}^1_{zxy}=\int_0^\infty v^t\bullet{}_tp_x\mu_{x+t}(1-{}_tp_y\,{}_tp_z)dt$$
$$=\int_0^\infty v^t\bullet{}_tp_x\mu_{x+t}dt-\int_0^\infty v^t\bullet{}_tp_x\,{}_tp_y\,{}_tp_z\mu_{x+t}dt$$
$$=\overline{A}_x-\int_0^\infty v^t\bullet{}_tp_x\,{}_tp_y\,{}_tp_z\mu_{x+t}dt$$
(12)

$$\overline{A}^1_{yxz}=\int_0^\infty v^t\bullet{}_tp_y\mu_{y+t}(1-{}_tp_x\,{}_tp_z)dt$$
$$=\int_0^\infty v^t\bullet{}_tp_y\mu_{y+t}dt-\int_0^\infty v^t\bullet{}_tp_y\,{}_tp_x\,{}_tp_z\mu_{y+t}dt \quad (1$$
$$=\overline{A}_y-\int_0^\infty v^t\bullet{}_tp_y\,{}_tp_x\,{}_tp_z\mu_{y+t}dt$$
3)

$$\overline{A}^1_{zxy}=\int_0^\infty v^t\bullet{}_tp_z\mu_{z+t}(1-{}_tp_x\,{}_tp_y)dt$$
$$=\int_0^\infty v^t\bullet{}_tp_z\mu_{z+t}dt-\int_0^\infty v^t\bullet{}_tp_z\,{}_tp_x\,{}_tp_y\mu_{z+t}dt \quad (1$$
$$=\overline{A}_z-\int_0^\infty v^t\bullet{}_tp_z\,{}_tp_x\,{}_tp_y\mu_{z+t}dt$$

4)On-level premium is in h year when (xyz)set as

initial mount. No K□0≤K ≤h□year payment is 1 the today value is v^k , and actuarial present value is $\ddot{a}_{xyz:\overline{h|}}$, then,

$$\ddot{a}_{xyz:\overline{h|}} = \sum_{k=0}^{h-1} v^k \bullet {}_k p_{xyz} = \sum_{k=0}^{h-1} v^k \bullet {}_k p_x \, {}_k p_y \, {}_k p_z \quad (15)$$

Based on level premium, actuarial present value of premium is equal to payment after death precision calculated value, then,

$$\overline{A}_{xyz} = {}_h p_1 \bullet \ddot{a}_{xyz:\overline{h|}} \quad (16)$$

$$\overline{A}_{\overline{xy}} = {}_h p_2 \bullet \ddot{a}_{xyz:\overline{h|}} \quad (17)$$

$$\overline{A}^1_{xyz} = {}_h p_{3x} \bullet \ddot{a}_{xyz:\overline{h|}} \quad (18)$$

$$\overline{A}^1_{yxz} = {}_h p_{3y} \bullet \ddot{a}_{xyz:\overline{h|}} \quad (19)$$

$$\overline{A}^1_{zxy} = {}_h p_{3z} \bullet \ddot{a}_{xyz:\overline{h|}} \quad (20)$$

Then,

$${}_h p_1 = \overline{A}_{xyz} \,/\, \ddot{a}_{xyz:\overline{h|}} \quad (21)$$

$${}_h p_2 = \overline{A}_{\overline{xy}} \,/\, \ddot{a}_{xyz:\overline{h|}} \quad (22)$$

$${}_h p_{3x} = \overline{A}^1_{xyz} \,/\, \ddot{a}_{xyz:\overline{h|}} \quad (23)$$

$${}_h p_{3y} = \overline{A}^1_{yzx} \,/\, \ddot{a}_{xyz:\overline{h|}} \quad (24)$$

$${}_h p_{3z} = \overline{A}^1_{zxy} \,/\, \ddot{a}_{xyz:\overline{h|}} \quad (25)$$

For n years both sides policies, the single premiums is,

$$\overline{A}_{u:\overline{n|}} = \int_0^n v^t \, {}_t p_u \mu_{x+t} dt + v^n \, {}_n p_x \quad (26)$$

(u) means common condition, could be (xyz),also could be $(\overline{xy})$.

5.2 Survive annuity

5.2.1 Life time survive annuity

when the condition continues, every policy initially pay 1 yuan for life time annual payment $Y = \ddot{a}_{\overline{K+1|}}$,the precise value as ,

$$\ddot{a}_{xyz} = E(Y) = \sum_{k=0}^{\infty} \ddot{a}_{\overline{K+1|}} \, {}_{k\backslash} q_{xyz} = \sum_{k=0}^{\infty} v^k \, {}_k p_{xyz} = \sum_{k=0}^{\infty} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \quad (27)$$

$$\ddot{a}_{\overline{xy}} = \sum_{k=0}^{\infty} v^k \, {}_k p_{\overline{xy}} = \sum_{k=0}^{\infty} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (28)$$

5.2.2 N years fixed survive annuity

$$\ddot{a}_{xyz:\overline{n|}} = E(Y) = \sum_{k=0}^{n-1} \ddot{a}_{\overline{K+1|}} \, {}_{k\backslash} q_{xyz} + \ddot{a}_{\overline{n|}} \, {}_n p_u = \sum_{k=0}^{n-1} v^k \, {}_k p_{xyz} = \sum_{k=0}^{n-1} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \quad (29)$$

$$\ddot{a}_{\overline{xy}:\overline{n|}} = \sum_{k=0}^{n-1} v^k \, {}_k p_{\overline{xy}} = \sum_{k=0}^{n-1} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (30)$$

5.2.3 Life time survive annuity of N years deferred period

$${}_{n\backslash}\ddot{a}_{xyz} = E(Y) = \sum_{k=n}^{\infty} \ddot{a}_{\overline{K+1|}} \, {}_{k\backslash} q_{xyz} = \sum_{k=n}^{\infty} v^k \, {}_k p_{xyz} = \sum_{k=0}^{\infty} v^{k+n} \, {}_{k+n} p_x \, {}_{k+n} p_y \, {}_{k+n} p_z \quad (31)$$

$${}_{n\backslash}\ddot{a}_{\overline{xy}} = \sum_{k=n}^{\infty} v^k [{}_k p_x (1 - {}_k p_y) + {}_k p_y (1 - {}_k p_x)] = \sum_{k=0}^{\infty} v^{k+n} [{}_{k+n} p_x (1 - {}_{k+n} p_y) + {}_{k+n} p_y (1 - {}_{k+n} p_x)] \quad (32)$$

5.2.4 N-M years survive annuity of M years deferred period

$${}_{m/}\ddot{a}_{xyz:\overline{n-m|}} = E(Y) = \sum_{k=m}^{n-1} v^k \, {}_k p_{xyz} = \sum_{k=m}^{n-1} v^k \, {}_k p_x \, {}_k p_y \, {}_k p_z \quad (33)$$

$${}_{m/}\ddot{a}_{\overline{xy}:\overline{n-m|}} = \sum_{k=m}^{n-1} v^k \, {}_k p_{\overline{xy}} = \sum_{k=m}^{n-1} v^k ({}_k p_x + {}_k p_y - {}_k p_{xy}) \quad (34)$$

6 Conclusion

For this paper, we study for multi-person policy, and study target on future family. First,the family condition in the future is analysed. In analysing about muliti-life function,introducing the situation of three element,especially while considering the death order,consider a person dies before other two people,but these two people are a kind of state of jointly surviving. Second,two kinds of association situations and two models are considered here. The first place is the couple and only children,the second place is couple own. At the last,the actuarial present value on the life insurance and annuity are calculated.

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