Application of Wavelet Transform in Dynamic Illumination

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Abstract: - Many importance sampling techniques for direct lighting concentrate on either sampling the light source or the BRDF, or the product distribution of both. First, we present a generalized factor method to simplify multiple function integral into triple function product issues. Then we introduce an optimal wavelet product representation to reduce computation by a strategy for hierarchically sampling a wavelet tree, which starts at the coarsest resolution and recursively moves down to finer resolutions.

Keywords: - Monte Carlo Integral; Direct Illumination; Wavelet Product; Importance Sampling

1 Introduction

Image based lighting is being used for rendering. Image based lighting offers a number of advantages over simple lighting techniques such as directional or point lights. The use of a good sampling strategy for illumination is critical when integrating image-based lighting, such as environment maps, into a rendering system. This is because direct illumination in the form of high dynamic range (HDR) environment maps can have high frequency detail. The problem of efficient sampling of the illumination is compounded when the scene contains materials with high frequency BRDFs. High fidelity images based on a whole range of reflection phenomena described by the *rendering equation* [J. T. Kajiya 1986][1] often take hours or days to compute.

$$L_o(\boldsymbol{x},\vec{\omega}_o) = L_e(\boldsymbol{x},\vec{\omega}_o) + \int_{\Omega} f_r(\boldsymbol{x},\vec{\omega}_i,\vec{\omega}_o) L_i(\boldsymbol{x},\vec{\omega}_i)\cos(\theta_i)\, d\vec{\omega}_i. \quad (1)$$

The performance can be improved if we incorporate knowledge about the function being integrated into the sampling process. The idea is to concentrate samples to parts of the function where it is likely to be large. This technique is called *importance sampling*, and can vastly reduce the variance in Monte Carlo techniques[2].

Several researchers have recently worked on this problem, by either combining samples drawn independently according to the importance of the illumination and the BRDF [3] [4], or more recently, by drawing samples from the product distribution of the illumination and the BRDF [5]. These approaches produce high quality images with a small number of samples in unoccluded regions.

Recently, Clarberg et al. [6] presented an algorithm called Wavelet Importance Sampling (WaIS) that samples products of wavelet functions. Their algorithm uses a property of wavelets that allows a wavelet product to be evaluated in a top-down fashion. However, WaIS addressed only aspects of direct illumination and static ones.

In this paper, we represented an improvement of WaIS for real-time rendering with dynamic objects under global illumination.

2 General integral of function product

2.1 Multi-function product integral

Given n distinct objects in a dynamic scene, the exitant radiance B at a surface point x along view direction θ due to distant environment lighting L is the product integral over all incident directions sampled at a surrounding cubemap Ω [Sun, W. et al.][7]:

$$B(\boldsymbol{x},\vec{\omega}_o) = \int_{\Omega} L(\vec{\omega}_i)O_1(x,\vec{\omega}_i)\prod_{j=1}^{n} O_j(x,\vec{\omega}_i)\, f_r\left(\boldsymbol{x},\vec{\omega}_i \leftrightarrow \vec{\omega}_o\right)(\vec{N}\cdot\vec{\omega}_i)d\vec{\omega}_i = \int_{\Omega} L(\vec{\omega}_i)\tilde{O}_1(x,\vec{\omega}_i)\prod_{j=1}^{n} \tilde{O}_j(x,\vec{\omega}_i)\, f_r\left(\boldsymbol{x},\vec{\omega}_i \leftrightarrow \vec{\omega}_o\right)d\vec{\omega}_i . \quad (2)$$

where $\vec{\omega}_i$ is the incident direction, $\vec{N}$ is the normal at x, f_r is the BRDF, O_1 is the *local visibility* at x due to self-occlusion. $O_i(2\leq i\leq n)$ is the *dynamic occlusion* at x occluded by the i^{th} neighboring object in the scene. In order to eliminate the dependance of the BRDF on the normal, the cosine term $(\vec{N}\cdot\vec{\omega}_i)$ is combined with the self visibility O_1 as $\tilde{O}_1$ as:

$$\tilde{O}_1(\boldsymbol{x},\vec{\omega}_i) = O_1(\boldsymbol{x},\vec{\omega}_i)(\vec{N}\cdot\vec{\omega}_i) \quad (3)$$

For a fixed vertex x and view direction $\vec{\omega}_o$, equation (2) can be simplified as:

$$B = \int L(\vec{\omega}_i)\tilde{O}_1(\vec{\omega}_i)\prod_{j=1}^{n}\tilde{O}_j(\vec{\omega}_i)\, f_r(\boldsymbol{x},\vec{\omega}_i)d\vec{\omega}_i \quad (4)$$

It is exactly the product integral of (n+2) functions:

$$B = \int \prod_{j=1}^{n+2} F_j(\vec{\omega}_i)\, d\vec{\omega}_i \quad (5)$$

2.2 Factoring for dynamic radiance transfer

For dynamic radiance transferring, an effective approach to accelerating the evaluation of equation (5) is stated as follows:

$$B = \int \left[\prod_{j=1}^{n+1} F_j(\vec{\omega}_i)\right]\cdot F_{n+2}(\vec{\omega}_i)d\vec{\omega}_i = \langle T, F_{n+2}(\vec{\omega}_i)\rangle \quad (6)$$

where the radiance transfer vector T is the product of n+1 functions as:

$$T = \prod_{j=1}^{n+1} F_j(\vec{\omega}_i) \quad (7)$$

If F_1, F_2, $\cdots$; F_{n+1} are fixed, in other words, only F_{n+2} varies (i.e., dynamic instead of static), radiance transfer vector T needs to be computed only once. Therefore, shading integral reduces to a simple double function product integral of T and F_{n+2}, which can be approximated by the wavelet importance sampling method. Here we assume that only one function in the shading integral varies. This assumption is reasonable for lighting design systems, where normally the designer adjusts only one variable at a time, and real-time feedback is highly appreciated. For example, the designer may experiment with different lighting effects by fixing view conditions and objects. The designer may also render the scene from different view conditions by fixing the lighting and the objects. Another popular operation is to fix the lighting and view conditions, and relocate a single object in the scene. As long as there is only one (note that it can be any one) varying parameter, this approach can be used to generate all-frequency shadows in real-time.

In equation (5), the product of n+2 functions is factored into the product of two sets, one with n+1 functions, and the other with only one function. More generally, this factorization has the following form:

$$B = \int \left[\prod_{j=1}^{k} F_j(\vec{\omega}_i)\right]\cdot\left[\prod_{j=k+1}^{n+2} F_j(\vec{\omega}_i)\right]d\vec{\omega}_i = \langle T_1, T_2\rangle \quad (8)$$

where $T_1 = \prod_{j=1}^{k} F_j(\vec{\omega}_i)$ and $T_2 = \prod_{j=k+1}^{n+2} F_j(\vec{\omega}_i)$.

As a result, the product of n+2 functions reduces to the double function product integral of two radiance transfer vectors.

3 Product importance sampling rendering with double functions

3.1 Direct rendering

The direct illumination is given by the integral:

$$L_{dir}(\boldsymbol{x},\vec{\omega}_o) = \int_{\Omega} f_r(\boldsymbol{x},\vec{\omega}_i,\vec{\omega}_o)L(\boldsymbol{x},\vec{\omega}_i)\, v(\boldsymbol{x},\vec{\omega}_i)cos(\theta_i)\, d\vec{\omega}_i . \quad (9)$$

where the incident radiance, $L(\boldsymbol{x},\vec{\omega}_i)$, is provided by light sources in the scene, and $v(\boldsymbol{x},\vec{\omega}_i)$ is the visibility of a light source in direction $\vec{\omega}_i$. In order to apply realistic lighting to a virtual scene, it is common to capture real lighting in a high-dynamic range environment map, and use that for L during rendering.

3.2 **Product importance sampling estimator**

A common approach to evaluate the direct lighting equation is to use Monte Carlo integration, which replaces the continuous integral with the average of N Monte Carlo samples.

Burke et al. [3] introduced a technique for rendering objects with complex materials illuminated by an environment map.

In their work, the aim is to perform importance sampling using the *product* of the incident light distribution and the BRDF as the importance function:

$$\mathrm{p}(\vec{\omega}_i) := \frac{f_r(\vec{\omega}_i, \vec{\omega}_o) L(\vec{\omega}_i) \cos(\theta_i)}{\int_\Omega f_r(\vec{\omega}_i, \vec{\omega}_o) L(\vec{\omega}_i) \cos(\theta_i)\, d\vec{\omega}_i} \qquad (10)$$

Observe that the normalization term in the denominator is the direct illumination integral with the visibility term $v(\vec{\omega}_i)$ omitted. In other words, this term is the exitant radiance in the absence of shadows. Burke et al. refer to it as *Lns* .radiance, no shadows. [1]:

$$L_{ns} := \int_\Omega f_r(\vec{\omega}_i, \vec{\omega}_o) L(\vec{\omega}_i) \cos(\theta_i)\, d\vec{\omega}_i . \qquad (11)$$

If sample directions $\vec{\omega}_i^{(j)} \sim p(\vec{\omega}_i), j = 1, \ldots, N$, are drawn according to the product distribution in Equation (10), then Equation (9) can be estimated as $L_{N.p}(\vec{\omega}_o)$, where

$$L_{N.p}(\vec{\omega}_o) = \frac{1}{N} \sum_{j=1}^{N} \frac{f_r\left(\vec{\omega}_i^{(j)}, \vec{\omega}_o\right) L\left(\vec{\omega}_i^{(j)}\right) v(\vec{\omega}_i^{(j)}) \cos(\theta_i)}{\mathrm{p}\left(\vec{\omega}_i^{(j)}\right)}$$

$$= \frac{L_{ns}}{N} \sum_{\mathrm{j}=1}^{\mathrm{N}} v\left(\vec{\omega}_i^{(j)}\right). \qquad (12)$$

$L_{N.p}(\vec{\omega}_o)$ is referred to as the *bidirectional estimator* for the direct illumination integral.

Fig. 1: *Dragon model in an indoor HDR EM. Left: Importance sampling from BDRF, 200 samples/pixel. Right: Bidirectional importance sampling.*

As showed in Figure 1, note that the variance of the bidirectional estimator for the reflected radiance is proportional to the variance in the visibility function. This is an improvement over sampling techniques that only consider either the illumination or the BRDF in the sampling process. This is because the variance of these techniques depends in addition on the variance in the function that they do not sample from, BRDF or illumination respectively.

4 Optimal Product importance sampling using wavelet

The efficient computation of the multi-function product integral and the product of multiple functions are focused now. Compared with the pixel domain representation, wavelets allow us to approximate signals at low distortion with a small number of significant coefficients. Haar bases have an interesting property that simplifies the computation as many of the integral coefficients are zero [Mallat, et al.][8].

4.1 2D Haar Bases

Nonstandard Haar wavelet transform [Stollnitz et al. 1996] [15] decomposes a $2^n \times 2^n$ image into a 2D signal with $2^n \times 2^n$ coefficients. Each coefficient corresponds to a basis function defined in the region <*j*, *k*, *l*> where *j* is the *scale* (0≤*j*<*n*), *k* and *l* are spatial translations (0≤*k*, *l* <2^j). In each region <*j*, *k*, *l*>, four normalized 2D Haar basis functions are defined:

$\phi^j_{t_1,t_2}$ *normalized Haar scaling basis function:*

$$\phi^j_{t_1,t_2}(x,y) = 2^j \phi^0(2^j x - t_1, 2^j y - t_2)$$

where ϕ^0 is the *mother scaling function*.

ψ^j_{kl} *normalized Haar wavelet basis function*. There are three types of wavelets defined in the region <*j*, *k*, *l*>:

$$\psi_m{}^j_{t_1,t_2}(x,y) = 2^j \psi_m{}^0(2^j x - t_1, 2^j y - t_2),$$

where $\psi_m{}^0, m = 1,2,3$, are three different mother wavelets, denoting the horizontal, vertical and diagonal differences.

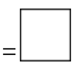
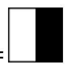
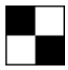
$\phi^0 =$  $\psi_1{}^0 =$  $\psi_2{}^0 =$  $\psi_3{}^0 =$ 

Fig.2: The mother scaling function and the three mother wavelet functions.

A two-dimensional image can be further expressed as a sum of the first scaling function plus the wavelet functions as:

$$F = F_{0,0}^{0}\phi_{0,0} + \sum_{k=0}^{l-1}\sum_{t}\sum_{m} F_{k,t}^{m}\psi_{k,t}^{m} = \sum_{i} F_i\psi_i \qquad (13)$$

Here, set vector $\mathbf{t} = (t_1, t_2)$, F is a two-dimensional image with $2^l \times 2^l$ pixels.

4.2 General 2D Wavelet Product

Given two functions expressed in an orthonormal basis, it is possible to multiply them together and get the product expanded in the same basis Ng et al. [9]

Let $G = \sum G_j\, \psi_j$ and $H = \sum H_k\, \psi_k$ be the two images represented in the Haar basis. The wavelet product, $F = \sum F_i\, \psi_i$, of G and H is then given by:

$$F = G \cdot H \Leftrightarrow \sum F_i\, \psi_i = \sum G_j\, \psi_j \cdot \sum H_k\, \psi_k \qquad (14)$$

By integrating against the i^{th} basis function, we can directly obtain the i^{th} coefficient for the wavelet representation of the product F as follows:

$$F_i = \iint F(x)\psi_i(x)dx = \iint \psi_i(x)G(x)H(x)dx$$

$$= \textstyle\sum_j \sum_k G_j H_k \iint \psi_i(x)\psi_j(x)\psi_k(x)dx = \sum_j \sum_k C_{ijk}\, G_j H_k \quad (15)$$

$$C_{ijk} = \iint \psi_i(x)\psi_j(x)\psi_k(x)dx \qquad (16)$$

Note that these equations are valid for any domain and suitable orthonormal basis, only the tripling coefficients will differ. Due to the compact support of the Haar basis functions, most of the tripling coefficients will be zero. The non-zero coefficients are given by the Haar tripling coefficient theorem by Ng R. [10]. The integral of three 2D Haar basis functions is non-zero if and only if one of the following three cases holds:

1. All three are the scaling function. In this case, $C_{ijk} = 1$.
2. All three functions occupy the same wavelet square, and all are of different wavelet types. $C_{ijk} = 2^l$, where the square is at level l.
3. Two are identical wavelets, and the third is either the scaling function or a wavelet that overlaps at a strictly coarser level. $C_{ijk} = \pm 2^l$, where the third function exists at level l.

In this application, where we are looking at a specific basis function, ψ_i, the theorem can be rewritten to make the different cases more clear:

1. ψ_i is the mother scaling function:
   (a) ψ_j and ψ_k are also the mother scaling function. $C_{ijk} = 1$.
   (b) ψ_j and ψ_k are identical wavelets (at any level). $C_{ijk} = 1$.
2. ψ_i is a wavelet function at level l:
   (a) All three functions occupy the same wavelet square and all are of different wavelet types. $C_{ijk} = 2^l$.
   (b) ψ_j and ψ_k are identical wavelets under the support of ψ_i and exist at a strictly finer level. $C_{ijk} = \pm 2^l$.
   (c) One of the wavelets is identical to ψ_i, and the other is either the mother scaling function or a wavelet that overlaps at a strictly coarser level. $C_{ijk} = \pm 2^{l'}$, where the coarser function exists at level l'.

4.3 Wavelet importance sampling

For simplicity, the image $F(\mathbf{x})$ is defined to cover the unit square. Consider a wavelet square $s = (l, \mathbf{t})$ at level l and translation t. The square has an area of $A(s) = 2^{-l} \times 2^{-l} = 2^{-2l}$. The average function value $F(s)$ over the square, can be found by integrating the function over s [6]. However, due to the constant and disjoint scaling functions, the average function value is given by the scaling coefficient for the square as follows:

$$F(s) = \iint_s F(x)dx = 2^l \iint \phi_{l,\mathbf{t}}^{0}(x)F(x)dx = 2^l F_{l,\mathbf{t}}^{0} \qquad (17)$$

$$I = \iint F(x)dx = F_{0,\mathbf{0}}^{0} \qquad (18)$$

Thus, the probability density of the square s, is given by:

$$p(s) = \frac{F(s)}{I} = 2^l \frac{F_{l,\mathbf{t}}^{0}}{F_{0,\mathbf{0}}^{0}} \qquad (19)$$

the probability of placing a sample at a coordinate x within the square s, should be equal to:

$$p(x \in s) = p(s)A(s) = 2^{-2l}\frac{F(s)}{I} = 2^{-l}\frac{F_{l,\mathbf{t}}^{0}}{F_{0,\mathbf{0}}^{0}} \qquad (20)$$

For recursive algorithms, it is useful to know the conditional probabilities for each child square, given that the parent square is sampled. Let s be the parent square at level l, and let s_i, $i = 1 \ldots 4$, be the four child squares at level $l+1$. The conditional probability for each of the four children can be expressed in the function values for the parent and child squares as:

$$p(x \in s_i | x \in s) = \frac{p(x \in s_i)}{p(x \in s)} = \frac{2^{-2(l+1)} F(s_i)/I}{2^{-2l} F(s)/I} = \frac{1}{4}\frac{F(s_i)}{F(s)} \qquad (21)$$

and similarly expressed in scaling coefficients as:

$$p(x \in s_i | x \in s) = \frac{p(x \in s_i)}{p(x \in s)} = \frac{2^{-(l+1)} F^0_{l+1,\boldsymbol{t}}/F^0_{0,\boldsymbol{0}}}{2^{-l} F^0_{l,\boldsymbol{t}}/F^0_{0,\boldsymbol{0}}} = \frac{1}{2}\frac{F^0_{l+1,\boldsymbol{t}}}{F^0_{l,\boldsymbol{t}}} \qquad (22)$$

4.4 Sampling of Wavelet Products

For a simple case, the importance function f(x) is a product of only two functions, f(x) =g(x)h(x). We store approximations of g(x) and h(x) as images, G and H respectively, expressed as Haar wavelets. Then coefficients for the product F = G H of the two wavelets can be computed using theory in 4.2.

In practice, as stated in last section, it is unnecessary to compute detail coefficients for the wavelet product, as only the scaling coefficients at each level are needed for sampling. So the general product in 4.2 could be simplified by direct product of only scaling coefficients. While replacing ψ_i with the specific scaling function $\phi_{l,\boldsymbol{t}}$, the scaling coefficient for the product is then given by:

$$F^0_{l,\boldsymbol{t}} = \iint F(x)\phi_{l,\boldsymbol{t}}(x)dx = \iint \phi_{l,\boldsymbol{t}}(x)G(x)H(x)dx$$

$$= \sum_j \sum_k C'_{ijk}\, G_j H_k \qquad (23)$$

where C'_{ijk} are modified tripling coefficients, defined as:

$$C'_{ijk} = \iint \phi_{l,\boldsymbol{t}}(x)\psi_j(x)\psi_k(x)dx \qquad (24)$$

It turns out that the C'_{ijk} for a scaling function at level l are non-zero if and only if one of the following two cases holds:

1. ψ_j and ψ_k are either the mother scaling function or wavelets at strictly coarser levels, l_j and l_k. $C'_{ijk} = \pm 2^{l_j + l_k - l}$.
2. ψ_j and ψ_k are identical wavelets under the support of $\phi_{l,\boldsymbol{t}}$, and exist at the same or finer levels. $C'_{ijk} = 2^l$.

the first case corresponds to a multiplication of the scaling coefficients for G and H at level l that overlap $\phi_{l,\boldsymbol{t}}$, scaled by 2^l, i.e., a multiplication of the scaling coefficients $G^0_{l,\boldsymbol{t}}$ and $H^0_{l,\boldsymbol{t}}$. Hence, scaling coefficients for the product as:

$$F^0_{l,\boldsymbol{t}} = 2^l G^0_{l,\boldsymbol{t}} H^0_{l,\boldsymbol{t}} + 2^l \sum_{l' \geq l, \boldsymbol{t}' \in \boldsymbol{t}, m} G^m_{l',\boldsymbol{t}'} H^m_{l',\boldsymbol{t}'} \qquad (25)$$

where the summation is over all wavelet coefficients that are under the support of $\phi_{l,\boldsymbol{t}}$. the scaling coefficients $G^0_{l,\boldsymbol{t}}$ and $H^0_{l,\boldsymbol{t}}$ can easily be computed separately for the two functions, using standard wavelet reconstruction from their respective wavelet coefficients.

This simplified way is much more efficient than the general one.Once the product F can be computed, the importance sampling probability computing is as same as above equations for single function case described in 4.3.

5 Application and results

In our application, the BRDF is given in local coordinates with respect to the reflection vector, while an environment map is commonly expressed in global coordinates. By rewriting the environment map as a four-dimensional function $L(\vec{\omega}, \vec{\omega}_r)$, where the direction $\vec{\omega}$ is given with respect to $\vec{\omega}_r$, the environment map is in the same local space as the BRDF.

By a change of variables, the BRDF can be transformed into a function that is more compact. There are many ways for such reparameterizations. In our application, we need a parameterization that is suitable for both the BRDF and for the environment map. The BRDF is centered about the reflection vector $\vec{\omega}_r = (\theta_r, \varphi_r)$, instead of around the surface normal $\vec{N}$.

In practice, both the BRDF and the environment map are tabulated as a sparse 2D set of 2D wavelet compressed images. The maps are stored at the resolution 64×64 or 128×128.

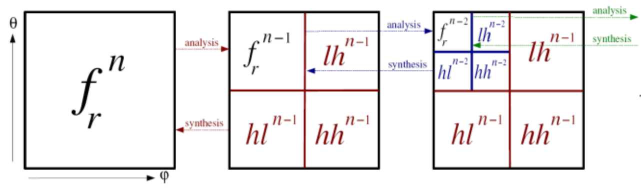


Figure 3: *2D Wavelet transform applied on each hemisphere of the original BRDF data.*

A non-standard approach of wavelet transform for f_r is employed here (Figure 3).

A ray tracing rendering result is implemented and showed as figure 4.

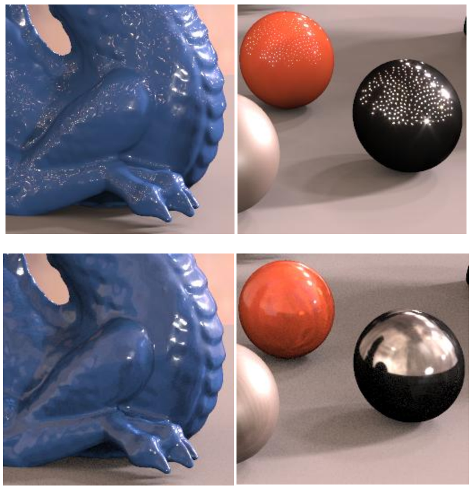


Fig. 4: *Top: Structred sampling results;Bottom: Wavelet sampling rendering results*

6 Conclusions and future work

Wavelet product importance sampling is an efficient way for static direct illumination with complex environment mapping. According to the feature of its product sampling of two functions, the factoring scheme we developed makes shading integral reduce to a simple double function product integral. Such way is suitable for dynamic global lighting situations with multiple objects where normally only one variable is adjusted at a time, and real-time feedback is highly appreciated.

A GPU enabled pipeline is also used to accelerate the real-time rendering and worked well in practice.

Wavelet representations of BRDF and EM provide novel approaches for complex rendering. The way we proposed here can be used in other domains where the efficient computation and real-time generation are critical such as game, animation, and simulation.

References

[1] James T. Kajiya. The Rendering Equation. *Computer Graphics (Proceedings of ACM SIGGRAPH* 86), 1986, pp. 143–150.

[2] Malvin H. Kalos and Paula A. Whitlock. *Monte Carlo Methods, Volume 1: Basics*. Wiley-Interscience, New York, 1986.

[3] Burke D., Ghosh A., Heidrich W.. Bidirectional importance sampling *for* direct illumination. *Rendering Techniques 2005 Eurographics Symposium on Rendering* (Aire-la-Ville, Switzerland) , 2005, Eurographics Association, pp. 139–146.

[4] Agarwal S., Ramamoor R., Belongie S., and Jensen H.W.. Structured importance sampling of environment maps. *SIGGRAPH 03*, 2003. pp.605–612.

[5] Kollig T., Keller A.. Efficient multidimensional sampling. *Computer Graphics Forum 21*, 3, 2002, pp.557–563.

[6] Clarberg P., Jarosz W., Akenine-Moller T., and Jensen H. W.. Wavelet importance sampling: Efficiently evaluating products of complex functions. *ACM Transactions on Graphics (SIGGRAPH '05) 24*, 3, 2005, pp. 1166–1175

[7] Sun, W., and Mukherjee, A.. Generalized wavelet product integral for rendering dynamic glossy objects. *ACM Transactions on Graphics (SIGGRAPH 2006) 25*, 3, 2006, pp.477–487.

[8] Mallat S. G.. A Theory for Multiresolution Signal Decomposition: The Wavelet Representation. IEEE Transactions on Pattern Analysis and Machine Intelligence, 11,7, 1989,pp. 674–693.

[9] Ng, R., Ramamoorthi, R., and Hanrahan, P.. All frequency shadows using non-linear wavelet lighting approximation. ACM TOG (SIGGRAPH 2003) 22, 3, 376–381.

[10] Ng R., Ramamoorthi R., and Hanrahan P.. Triple Product Wavelet Integrals for All-Frequency Relighting. *ACM Transactions on Graphics*, 23,3, 2004 ,pp. 477–487.