eMathTeacher Tool for Learning Mamdani's Direct Method*

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Abstract: - In this paper a tutorial for learning Fuzzy Inference Systems, designed under eMathTeacher's philosophy, is presented. An eMathTeacher [12] is an eLearning on line self assessment tool that help users to active learning maths concepts and algorithms by themselves, correcting their mistakes and providing them with clues to find the right solution. The tool presented here is an example of this new concept on Computer Aided Instruction (CAI) resources and has been implemented as a set of Java applets and designed as an auxiliary instrument for active eLearning Mamdani's Fuzzy Inference method. Its characteristics of visualization, simplicity and interactivity, make this tutorial a great value pedagogical instrument.

Key-Words: - eMathTeacher, eLearning, active learning, interactive Java applets, Fuzzy Inference Systems, Fuzzy Logic, Computer Assisted Instruction (CAI).

1 Introduction

Fuzzy Logic has undergone a huge improvement during the last decade as Fuzzy Logic methods can be used to solve problems that are difficult to address with traditional techniques. Industries are implementing control systems and knowledge management by means of Fuzzy Inference techniques and using them for different purposes. Applications of Fuzzy logic can be found in different fields such as modeling the dynamics of spread of epidemics [5], automated cow status monitoring [4], Motor speed control system [3], human resources management [2], calculus of Incurred But Not Referred (IBNR) loss reserves in insurance companies [1], Database design [17] or expert systems for aiding students and specialists [18].

Education systems cannot keep out of the changes new technologies yield in our society and technological advances should generate a substantial change in our didactical methodologies. Thus, Web based learning technologies play a very important role in the modern education process. Some investigations have shown that computer assisted instruction has been more effective than traditional methodologies [19] and visualization technologies provide a very positive aid to the learning task [16].

Furthermore, mathematical concepts and algorithmic procedures as well, are often difficult for the learners to understand. However, comprehensive research is required to determine the best methodology to be applied to the design and development of computer-assisted training, as well as the efficiency of the teaching/learning processes based on this particular method of instruction [6].

It is widely accepted that visualization enhances comprehension, engagement and satisfaction among students. Visualization technologies can be used to illustrate many mathematical concepts.

A good graphic interface environment would surely become helpful for a better understanding of the concepts or how the methods may be implemented.

We have been working in this field developing Interactive Java Tutorials both for being used by teachers on classroom lectures and students when learning by themselves. Those tutorials are available in our Department website [21, 22 and 24].

In this paper we present a new set of java applets designed for self learning Mamdani's Fuzzy Inference method. We also analyze which are the main requirements these tools should accomplish to be useful for students, and present the advantages they provide for learning processes.

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2 eMathTeacher tools

An eMathTeacher [12] is an eLearning tool that works as a virtual math teacher. In other words: they are on line self assessment tools that help students to active learning math concepts and algorithms by themselves, correcting their mistakes and providing students with clues to find the right solution. They can also be used as complementary material for bLearning [11] both for being used by teachers on classroom lectures and by students when learning by themselves. However, the most important characteristic of an eMathTeacher is the feasibility of being used for practicing with math methods and algorithms while the system guides the user towards the right answer.

2.1 Minimum requirements for an eMathTeacher

These are the minimum conditions we consider an eMathTeacher must fulfil:

- Step by step inquiring: for every process step, the student should provide the solution while the application waits in a stand by mode, expecting the user's input.
- Step by step evaluation: just after the user's entry, the eMathTeacher evaluates it, providing a tip for finding the proper answer if it is wrong or executing it otherwise.
- Visualization of the step's changes.
- Easy to use: the students' effort would be focused on learning the topic, not the tool.
- Flexible and powerful: allowing the user to introduce and modify the example and to repeat the process if desired.
- Nice and friendly graphic environment, helping insight.
- Clear presentation: focusing the student's attention on the essential concepts instead of getting spread on minor ideas.
- Platform independency: reaching the widest possible audience and keeping right performance when the platform is updated.
- Full time and place availability, allowing users' utilization when and where needed.

2.2 eMathTeacher tools on active learning

The authors are designing and developing several tools, whose purpose is to enhance students' comprehension and learning of their given subjects. Some of them are already available [21, 22, 24] in Applied Mathematics Department (U. Politécnica of Madrid) website[1]. Those tools have been designed under eMathTeacher philosophy.

Computer Aided Instruction may be a significant aid for learning as shown by a comparison study between a control section and a computer-based studio section of calculus-based Introductory Physics performed in Arcadia University (Canada) [10]. It showed that the two groups were statistically different, with the studio class outperforming the traditional lecture class in all cases. Collectively, their results clearly proved that the studio model was significantly more effective than the lecture model.

Recent approaches in the aim have shown that visualization aided interactive tools provide a very positive aid to the learning task [9, 13, 16]. Visualization characteristics of eMathTeacher tools play an important role, increasing insight –an image is worth thousand words–, engagement, memorization and satisfaction for the students.

Visualization is a very important feature when learning maths, but the main characteristic of the eMathTeacher tools are their full interactivity. The major objective when pursuing this feature has been the student's learning task. This way, as will be described in section 3, the user must execute the method's step while the tool evaluates and corrects the process.

eMathTeacher tools do enhance teacher's lectures, but their best feature lies on the possibilities they reveal when used by the students themselves. In Deakin University (Australia) [15], it was demonstrated that students who used interactive tools learned 60% faster, and after 30 days the knowledge kept was from 25% to 50% higher than those who did not use them.

Graphical and dynamical visualizations are more appealing for learners than exercises or text books, but, if the students are not required to give some kind of answers or predict what is happening next, they might adopt a passive attitude that is not beneficial at all and might even be prejudicial to their training. It has been verified that learners spend much more study time when visualization is involved, but those who are actively engaged have consistently outperformed the other ones who passively viewed them [6]. That is why, during the execution, the program should in some way ask the user which must be the next step to be done, not just show it. Here lays the main difference between an eMathTeacher, like the one presented in this paper, and a simple demonstrative visualization tool.

[1] http://www.dma.fi.upm.es/java

The main contributions of eMathTeachers for visualization and learning of maths concepts are:

- A change in the students' method of learning, providing them with adapted powerful tools.
- Enhancement of concepts and processes comprehension by the use of graphical visualizations as a way to facilitate theory's insight.
- Improvement of experimentation opportunities, increasing interactivity.
- Augment of students' understanding, engagement and satisfaction.

3 Mamdani's Method eMathTeacher

Keeping the features described in section 2 in mind, we have designed and implemented a tutorial [23] for learning Fuzzy Inference Systems (FIS) that has been designed under eMathTeacher's philosophy.

There are many tutorials for learning FIS. Most of them are demonstrative [25, 14], are not placed in an open website [7] or/and are implemented on expensive programs [26].

3.1 Fuzzy inference systems

Fuzzy inference is the process of transforming an input space into an output space by means of fuzzy logic. It is a method that interprets values in an input vector and, based on if-then rules, assigns values to an output vector. Fuzzy inference systems try to formalize the reasoning of human language using multivalued logic to solve a decision problem.

One of the advantages of using fuzzy (multivalued) instead of classic bivalued logic is its flexibility and simplicity when implementing mathematical formalizations as it is based on natural language and it can work with imprecise data.

This is why fuzzy inference systems have been successfully applied in fields such as automatic control, data classification, decision analysis, expert systems, and computer vision.

3.2 Mamdani's inference method

Mamdani's direct method is the easiest, oldest and most popular fuzzy inference methodology. It was proposed in 1974 by Ebrahim Mamdani [8] when attempting to design a control system formalizing the expert's knowledge by means of synthesizing a set of linguistic control rules. Mamdani's work was based on fuzzy algorithms for complex systems and decision processes presented by Zadeh in [20]. This Mamdani's method has a simple structure of min-max operations and is, therefore, very common in applications.

The fuzzy inference process consists of four steps: evaluating the antecedent for each rule, obtaining a conclusion for each rule, aggregating all conclusions, and finally defuzzifying.

3.2.1. Evaluating the antecedents

This step consists of two parts. First, we take the inputs (always numerical values) and determine their membership degree, i.e. the degree to which they belong to each of the given fuzzy sets via membership functions (input fuzzification). The membership degree is always a value belonging to [0, 1] interval.

Then, if the antecedent of a given rule has more than one part, a fuzzy operator (t-norm, t-conorm, negation or modifier) is applied to obtain a single value that represents the result of the antecedent for that rule.

3.2.2. Obtaining conclusions

This step, for each rule, applies an implication operator to the single value of the antecedent obtained in step one, and to the consequent of the rule (a fuzzy set), obtaining a new fuzzy set which is the conclusion of the rule.

Two of the most commonly used operators are the minimum, which truncates the consequent membership function, and the product, which scales it.

3.2.3. Aggregating all conclusions

In this step, the outputs of the rules, obtained in step two, are combined into a single fuzzy set using an aggregation operator. The operator implemented in this tool is the maximum although some other, such as the sum or the probabilistic or, are also commonly used.

3.2.4. Defuzzifying

When we try to find a solution to a decision problem, we want to obtain a number as output and not a fuzzy set. Therefore, we need to transform the fuzzy set obtained in step three into a single number. This process is called deffuzification, and it is supported in this tool via the centroid method, which returns the centre of the area under the curve defined by the membership function of the obtained fuzzy set.

3.3 Tool's description

The tool contains a theoretical part, which is split into three chapters. First, the motivation behind the use of fuzzy logic in contrast to classical logic is presented with the help of an easy example. The second chapter contains definitions of fuzzy inference systems and if-then rules, whereas the third chapter focuses on Mamdani's inference method.

The second part of the tool consists of a fully interactive example comprising a series of applets, each one implementing one of the four steps of Mamdani's method explained in section 3.2. These applets meet all the requirements of an eMathTeacher, providing full interactivity, a user friendly environment and a clear presentation.

The fuzzy inference system presented in this example is a road traffic controller, whose main purpose is to minimize the waiting time of the cars in a crossroad, as well as the length of the line of vehicles. This controller is defined as follows: two input variables (rate of car arrivals in a green light, and length of the cars line in a red light) in the range 0 – 30 cars; and one output variable (duration of the traffic light's green phase) in the range 5 – 50 seconds.

This example uses two rules: the first one (with two antecedents and one consequent) will be processed by the user, while the second one will be provided (processed) by the system to be aggregated with the first one in the third step.

3.3.1. Applet 1: Evaluating the antecedents

This applet has been designed to practice the first step of Mamdani's inference direct method.

The user must choose the numerical values of the input variables, within the permitted range, and the membership function of both antecedents. Then, the user must evaluate the antecedents (with the selected membership functions and numerical values), select an OR operator and finally apply this operator to obtain a single value that represents the result of the antecedent.

The application then checks if the results entered by the user are correct. In case one or more of the results are incorrect the application shows an error message indicating that one of the variables is out of range or suggesting a theory review.

3.3.2. Applet 2: Obtaining conclusions

In this applet, the user completes the processing for the first rule, obtaining the conclusion of this rule.

The user must enter the membership degree of the antecedent, obtained as a result in the previous applet, select the implication method and choose between the two membership functions associated to the consequent of the rule.

A drawing panel is provided (see figure 1), so that the user can draw the resulting fuzzy set of applying the implication method to the membership function of the consequent and the membership degree of the antecedent; this fuzzy set is the conclusion of the rule. The application then checks if the drawing entered by the user is correct. In case the result is incorrect the application shows an error message.

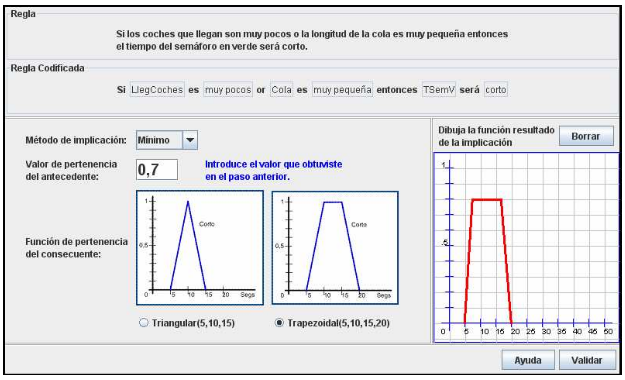

Figure 1: Applet for obtaining conclusions

3.3.3. Applet 3: Aggregating conclusions

In this applet, the user will aggregate the conclusion of the first rule, which has been processed in the two previous applets, together with a second rule, whose associated conclusion fuzzy set is also given in the example.

The user must enter the membership degree of the antecedent obtained in the first applet, and both the implication method and the membership function of the consequent selected in the second applet. The application will then draw the conclusion fuzzy set for the first rule.

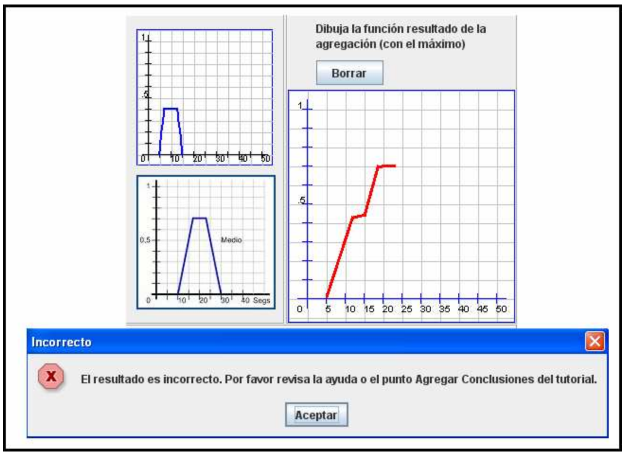

Figure 2: Applet for aggregating conclusions showing an error message.

A drawing panel is provided (see figure 2), so that the user can draw the resulting fuzzy set of applying the aggregation method (maximum) to both fuzzy sets. This set represents the final conclusion of the fuzzy inference system. The application then checks if the drawing entered by the user is correct. In other case the application shows an error message.

3.3.4. Applet 4: Defuzzifying

This applet differs on the previous ones in the sense that it does not provide the same degree of interactivity, instead it shows the calculations made to defuzzify the conclusion fuzzy set via the centroid method.

The user must remember and enter the membership degree of the antecedent obtained in the first applet, and both the implication method and the membership function of the consequent selected in the second applet. The application will then draw the conclusion fuzzy set based on the aforementioned parameters and calculate the final result of the inference process with the centroid method.

4 Conclusions and future work

The tutorial presented in this paper is an example of eMathTeacher as it meets all the minimum requirements and has been designed for supporting active learning. Being fully interactive, easy to use, intuitive and visual are the characteristics kept in mind during design and implementation phases. Actually, these qualities have demonstrated to help achieving increased insight and engagement for the students when working by themselves. In our experience, eMathTeacher tools are very good aids for learning as they improve comprehension, memorization and satisfaction for the students.

We have described the didactical benefits of eMathTeachers; however, as we are implementing them by means of Java applets, there are some features we lack and are currently trying to add. One of the Java applets' handicaps is that, due to security reasons, they are not allowed to write in the client's hard disk. It means that users cannot save their work for later review. The problem has been solved in some particular cases, but it is not completely satisfactory as it requires the user's permission and it disagrees with the minimum requirements for an eMathTeacher tool. We are considering the use of Java Web Start or signed applets to avoid this problem.

We are also thinking of creating new eMathTeachers implementing other Fuzzy Inference Systems, such as Sugeno's method and working on the implementation of algorithms from other fields of mathematics such as Karnaugh maps and Quine McCluskey's algorithm for simplification of Boolean functions or Gauss method for solving linear equation systems.

eMathTeacher active learning tools introduce a new concept in mathematics Computer Aided Instruction and represent a revolution in this field. These tools pursue a new goal on CAI, so as to acting as genuine virtual trainers extending teacher's hand through the Web, promoting active learning and offering the enhanced insight and appeal provided by graphical and dynamical tools.

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