Transient Heat Conduction in 3D Fuse Modeled by Conservative Averaging Method

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Abstract: – Three-dimensional mathematical model of the automotive fuse is considered in this paper. Initially, partial differential equations of the transient heat conduction are given to describe heat-up process in the fuse. Original method of conservative averaging is used to analytically approximate these equations with system of three ordinary differential equations.

Key-Words: – heat conduction, quasi-linear, transient process, three-dimensional, analytical reduction, conservative averaging.

1 Introduction

Usually, mathematical modeling of the fuse is implemented by making one dimensional assumptions [1]-[4]. In this paper, we use original method of conservative averaging to transform initial 3D statement of the problem to the statement of new type that consists of three ordinary differential equations. Approximate analytical 3D solution is obtainable from the solution of the transformed problem. Conservative averaging method is theoretically well founded for linear partial differential equations [6]-[9]. Here (as in [10], [11]) we investigate quasi-linear problem.

2 Mathematical Statement of Original 3D Problem

We start with accurate formulation of the three-dimensional mathematical model of the transient heat conduction problem for typical car fuse (Fig.1).

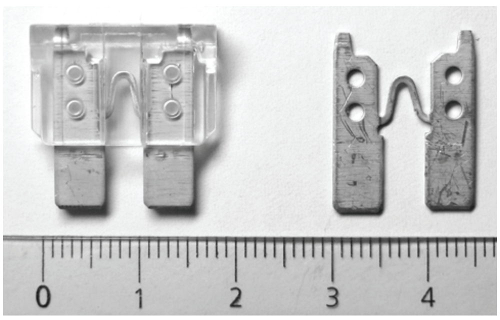

Fig.1. Example of automotive fuse with and without plastic shell

We seemingly straighten out the fuse and use geometry of the model as shown in Fig.2.

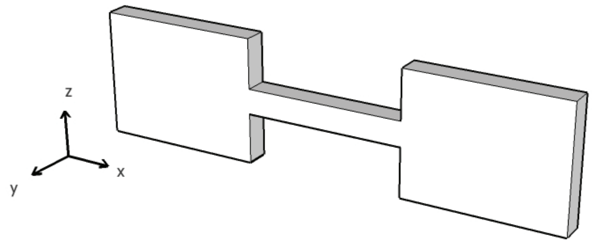

Fig.2

Because of the symmetry, it is enough to use only the shaded part of the model (Fig.3 and Fig.4)

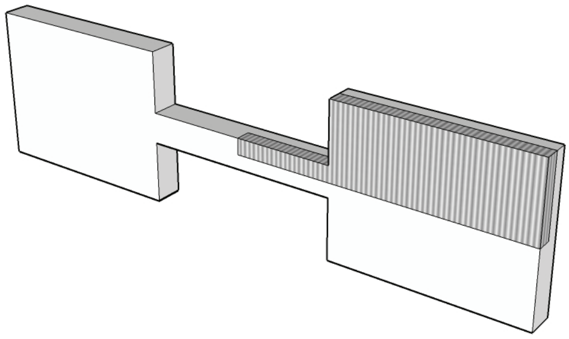
Fig.3

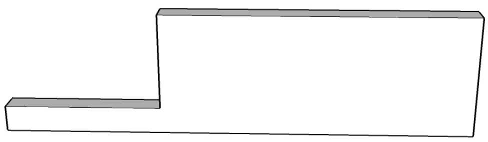
Fig.4

Let us treat this domain as two connected sub-domains $\overline{G}_0$ and $\overline{G}_1$:

$\bar{G}_0 = \{(x,y,z) \mid x \in [0,l], y \in [0,b], z \in [0,h]\}$,

$\bar{G}_1 = \{(x,y,z) \mid x \in [l,l+L], y \in [0,b], z \in [0,H]\}$.

If temperature in domain G_i is denoted as function $U_i(x,y,z,t)$, then differential equation for the heat transfer is

$$\frac{\partial}{\partial t}(c\rho U_i) = \frac{\partial}{\partial x}\left(k\frac{\partial U_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial U_i}{\partial y}\right) + \\ + \frac{\partial}{\partial z}\left(k\frac{\partial U_i}{\partial z}\right) + F_i(x,y,z,t,U_i), \tag{1}$$

$$(x,y,z) \in G_i, \quad t > 0, \quad i = \overline{0,1}.$$

Source function F_i (heat produced by electrical current) can be approximated with linear function:

$$F_i(x,y,z,t,U_i) = C_i\left(1 + \alpha\left(U_i - \Theta\right)\right), \tag{2}$$

where $C_0 = \dfrac{\rho_{ref} I^2}{h^2 b^2}$, $C_1 = \dfrac{\rho_{ref} I^2}{H^2 b^2}$.

Parameter ρ_{ref} is resistivity of the material at some reference temperature, α is temperature coefficient at the same reference temperature; I – electrical current. Heat capacity c and density ρ depend on temperature. Parameter k is heat conductivity coefficient. $\Theta = \Theta(t)$ – temperature of environment.

Besides main equations (1), we add symmetry conditions:

$$\left.\frac{\partial U_0}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial U_0}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial U_0}{\partial z}\right|_{z=0} = 0, \tag{3}$$

$$\left.\frac{\partial U_1}{\partial x}\right|_{x=l+L} = 0, \quad \left.\frac{\partial U_1}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial U_1}{\partial z}\right|_{z=0} = 0 \tag{4}$$

and heat exchange conditions on the outer surfaces

$$\left.\left(k\frac{\partial U_0}{\partial y} + h_y U_0\right)\right|_{y=b} = h_y \Theta(t), \\ \left.\left(k\frac{\partial U_1}{\partial y} + h_y U_1\right)\right|_{y=b} = h_y \Theta(t), \tag{5}$$

$$\left.\left(k\frac{\partial U_0}{\partial z} + h_z U_0\right)\right|_{z=h} = h_z \Theta(t), \\ \left.\left(k\frac{\partial U_1}{\partial z} + h_z U_1\right)\right|_{z=H} = h_z \Theta(t), \tag{6}$$

$$\left.\left(-k\frac{\partial U_1}{\partial x} + h_z U_1\right)\right|_{x=l+0, z\in[h,H]} = h_z \Theta(t), \tag{7}$$

where h_y, h_z are heat exchange coefficients for the surfaces in corresponding direction.

We also add conjugation conditions, i.e. continuity of the temperature and fluxes between both parts of the fuse:

$$U_0\big|_{x=l-0} = U_1\big|_{x=l+0}, \quad \left.\frac{\partial U_0}{\partial x}\right|_{x=l-0} = \left.\frac{\partial U_1}{\partial x}\right|_{x=l+0}, \tag{8}$$

$y \in [0,b], \quad z \in [0,h]$.

Finally we add initial conditions:

$$U_0\big|_{t=0} = U_1\big|_{t=0} = U^0 = const. \tag{9}$$

3 Conservative Averaging Method

3.1 The First Conservative Averaging Procedure

We introduce the integral average value of the functions $U_i(x,y,z,t)$ in the *y*-direction:

$$V_i(x,z,t) = \frac{1}{b}\int_0^b U_i(x,y,z,t)\,dy. \tag{10}$$

In praxis, firstly, the thickness b is very small in comparison with the width of the fuse. Secondly, material of the fuse (metal) has high heat conductivity coefficient. These features allow us to use the simplest form of conservative averaging method – the approximation by the constant. Procedure of the analytical transformations is given in papers [6]-[13] in more detail. Shortly, we integrate main equation (1) over the segment $y \in [0,b]$ and then we use boundary conditions (5) and linear representation of the source function (2). Finally, we take into account integral equality (10) and obtain:

$$\frac{\partial}{\partial t}(c\rho V_i) = \frac{\partial}{\partial x}\left(k\frac{\partial V_i}{\partial x}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial V_i}{\partial z}\right) - \\ -\frac{h_y}{b}(V_i - \Theta) + C_i\left(1 + \alpha\left(V_i - \Theta\right)\right), \qquad i = \overline{0,1}. \tag{11}$$

Because of the linearity, the additional boundary conditions (BC) of the new problem are the same as in the statement of the original problem (1)-(9):

$$\left.\frac{\partial V_0}{\partial x}\right|_{x=0} = \left.\frac{\partial V_1}{\partial x}\right|_{x=l+L} = 0, \quad \left.\frac{\partial V_i}{\partial z}\right|_{z=0} = 0, \quad i = \overline{0,1} \tag{12}$$

$$\left.\left(k\frac{\partial V_0}{\partial z} + h_z V_0\right)\right|_{z=h} = h_z \Theta(t), \\ \left.\left(k\frac{\partial V_1}{\partial z} + h_z V_1\right)\right|_{z=H} = h_z \Theta(t), \tag{13}$$

$$\left.\left(-k\frac{\partial V_1}{\partial x} + h_z V_1\right)\right|_{x=l+0, z\in[h,H]} = h_z \Theta(t), \tag{14}$$

We also add conjugation conditions at $z \in [0,h]$:

$$V_0\big|_{x=l-0} = V_1\big|_{x=l+0}, \quad \left.\frac{\partial V_0}{\partial x}\right|_{x=l-0} = \left.\frac{\partial V_1}{\partial x}\right|_{x=l+0}, \tag{15}$$

and initial conditions:

$$V_0\big|_{t=0} = V_1\big|_{t=0} = U^0 = const\,. \tag{16}$$

3.2 The Second Step of the Conservative Averaging

As the next step we will make conservative averaging in the *x*-direction. In this method we introduce averaged value function over domain G_0. We define two separate functions for the domain G_1 – one for interval $z \in (0,h)$ and the other one for interval $z \in (h,H)$ because of different conditions on the line $x = l$:

$$\begin{gathered} W_0(z,t) = \frac{1}{l}\int_0^l V_0(x,z,t)dx, \ z \in (0,h), \\ W_1(z,t) = \frac{1}{L}\int_l^{l+L} V_1(x,z,t)dx, \ z \in (0,h), \\ W_2(z,t) = \frac{1}{L}\int_l^{l+L} V_1(x,z,t)dx, \ z \in (h,H). \end{gathered} \tag{17}$$

In this case, we use exponential approximation in the following form:

$$V_0(x,z,t) = W_0(z,t) + p_0(z,t) \times \\ \times\left[\cosh\left(\frac{x}{l}\right) - \sinh(1)\right], \tag{18}$$

$$V_1(x,z,t) = W_i(z,t) + p_i(z,t) \times \\ \times\left[\cosh\left(\frac{x-l-L}{L}\right) - \sinh(1)\right], \quad i = \overline{1,2} \tag{19}$$

Equalities (18), (19) are chosen in such way that they fulfill integral equalities (17) (conservation of the heat energy) and BC (12) at $x = 0$ and $x = l + L$. We use conjugation conditions (15) to find unknown functions p_0, p_1 and afterwards obtain functions V_0, V_1:

$$V_0(x,z,t) = W_0(z,t) + Bl\left(W_1(z,t) - W_0(z,t)\right) \times \\ \times\left[\cosh\left(\frac{x}{l}\right) - \sinh(1)\right], \quad B = \frac{e}{l+L}, \tag{20}$$

$$V_1(x,z,t) = W_1(z,t) + BL\left(W_0(z,t) - W_1(z,t)\right) \times \\ \times\left[\cosh\left(\frac{x-l-L}{L}\right) - \sinh(1)\right], \quad z \in (0,h) \tag{21}$$

We find function p_2 and representation of function V_1 in interval $z \in (h,H)$ from expression (19) by means of BC (14):

$$V_1(x,z,t) = W_2(z,t) + D\left(\Theta(t) - W_2(z,t)\right) \times \\ \times\left[\cosh\left(\frac{x-l-L}{L}\right) - \sinh(1)\right], \quad z \in (h,H) \tag{22}$$

$$D = \frac{2eLh_z}{k\left(e^2-1\right) + 2Lh_z}.$$

On the line $z = h$ discontinuity for the temperature field could appear. Such kind of discontinuities was considered in papers [13], [14].

We integrate differential equations (11) of the first step of averaging in order to obtain equations for the second step. We use representations (20), (21), (22) of functions V_0, V_1 and integral equalities (17) to make approximate analytical reduction of 2D system to 1D system of partial differential equations.

$$\frac{\partial}{\partial t}(\bar{c}\bar{\rho}W_0) = \frac{\partial}{\partial z}\left(k\frac{\partial W_0}{\partial z}\right) + \frac{C}{l}\left(W_1 - W_0\right) - \\ -\frac{h_y}{b}\left(W_0 - \Theta\right) + C_0\left(1 + \alpha\left(W_0 - \Theta\right)\right), \tag{23}$$

$$\frac{\partial}{\partial t}(\bar{\bar{c}}\bar{\bar{\rho}}W_1) = \frac{\partial}{\partial z}\left(k\frac{\partial W_1}{\partial z}\right) + \frac{C}{L}\left(W_0 - W_1\right) - \\ -\frac{h_y}{b}\left(W_1 - \Theta\right) + C_1\left(1 + \alpha\left(W_1 - \Theta\right)\right), \tag{24}$$

$$\frac{\partial}{\partial t}\left(\bar{\bar{c}}\bar{\bar{\rho}}W_2\right) = \frac{\partial}{\partial z}\left(k\frac{\partial W_2}{\partial z}\right) - D_1\left(W_2 - \Theta\right) - \\ -\frac{h_y}{b}\left(W_2 - \Theta\right) + C_1\left(1 + \alpha\left(W_2 - \Theta\right)\right), \tag{25}$$

$$C = kB\sinh(1), \ D_1 = \frac{k\sinh(1)}{L^2}D.$$

As we mentioned in the introduction, here we consider the quasi-linear problem. This problem differs significantly from the problem considered in our papers [10], [11] in the point that in the earlier statement we have made the averaging procedure over the sub-domain with linear differential equation. That is why we must explain deeper the averaging procedure for the left hand side of the equation (23) (procedure for the equations (24), (25) can be realized in the same way). In this paper we use *enthalpy* form of the heat equation (see, e.g. [5], chapter 7). This form is substantially more suitable for the use of the mean value theorem:

$$\frac{1}{h}\int_0^h \frac{\partial}{\partial t}\left[c(V_0)\rho(V_0)V_0\right]dz =$$
$$=\frac{\partial}{\partial t}\left[c(\bar{V}_0)\rho(\bar{V}_0)\frac{1}{h}\int_0^h V_o dz\right]=$$
$$=\frac{\partial}{\partial t}\left[\bar{c}\,\bar{\rho}W_0\right], \qquad \bar{c}\,\bar{\rho}=c(\bar{V}_0)\rho(\bar{V}_0),\ \bar{V}_0=V_0\left(\bar{x},z,t\right).$$

It is possible to choose the mean value more or less freely. We propose to use the corresponding middle point, i.e. $\bar{x}=l/2$, $\bar{\bar{x}}=L/2$.

Again, boundary and initial conditions are the same as in the original problem because of the linearity:

$$\left.\frac{\partial W_0}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial W_1}{\partial z}\right|_{z=0}=0, \tag{26}$$

$$\left.\left(k\frac{\partial W_0}{\partial z}+h_z W_0\right)\right|_{z=h}=h_z\Theta(t),$$
$$\left.\left(k\frac{\partial W_2}{\partial z}+h_z W_2\right)\right|_{z=H}=h_z\Theta(t), \tag{27}$$

$$W_0|_{t=0}=W_1|_{t=0}=W_2|_{t=0}=U^0=const\,. \tag{28}$$

We also ask for continuity of the averaged temperature and fluxes on the line $z=h$. That gives additional conjugation conditions:

$$W_1|_{z=h-0}=W_2|_{z=h+0},\ \left.\frac{\partial W_1}{\partial x}\right|_{z=h-0}=\left.\frac{\partial W_2}{\partial x}\right|_{z=h+0} \tag{29}$$

3.3 The Third Step of the Conservative Averaging

As the last step we will make conservative averaging procedure in the *z*-direction. We introduce three new functions for this purpose:

$$u_0(t)=\frac{1}{h}\int_0^h W_0(z,t)dz,$$
$$u_1(t)=\frac{1}{h}\int_0^h W_1(z,t)dz, \tag{30}$$
$$u_2(t)=\frac{1}{H-h}\int_h^H W_2(z,t)dz.$$

We use exponential approximation in the form

$$W_0(z,t)=u_0(t)+q_0(t)\left[\cosh\left(\frac{z}{h}\right)-\sinh\left(1\right)\right],$$
$$W_1(z,t)=u_1(t)+q_1(t)\left[\cosh\left(\frac{z}{h}\right)-\sinh\left(1\right)\right], \tag{31}$$
$$W_2(z,t)=u_2(t)+q_2(t)\left[\cosh\left(\frac{z-h}{H-h}\right)-\sinh\left(1\right)\right]+$$
$$+q_3(t)\left[\sinh\left(\frac{z-h}{H-h}\right)-\cosh\left(1\right)+1\right].$$

By this representation we fulfill the integral equalities (conservation of the heat energy (30)) and the symmetry conditions (26) at $z=0$. In order to find four unknown parameters in the representation (31), we have BC (27) and conjugation conditions (29). This gives:

$$W_0(z,t)=u_0(t)+e_0\left(u_0(t)-\Theta\right)\left[\cosh\left(\frac{z}{h}\right)-\sinh\left(1\right)\right],$$
$$W_1(z,t)=u_1+$$
$$+\left[e_1\left(u_1-u_2\right)+e_2\left(u_2-\Theta\right)\right]\left[\cosh\left(\frac{z}{h}\right)-\sinh\left(1\right)\right],$$
$$W_2(z,t)=u_2+ \tag{32}$$
$$+\left[e_3\left(u_1-u_2\right)+e_4\left(u_2-\Theta\right)\right]\left[\cosh\left(\frac{z-h}{H-h}\right)-\sinh\left(1\right)\right]+$$
$$+\left[e_5\left(u_1-u_2\right)+e_6\left(u_2-\Theta\right)\right]\left[\sinh\left(\frac{z-h}{H-h}\right)-\cosh\left(1\right)+1\right],$$

where constants e_i are:

$$e_0=\frac{-ehh_z}{ek\sinh(1)+hh_z},\ e_1=\frac{-ee_7}{e_8e_9},\ e_2=\frac{e\left(\sinh(1)-1\right)e_{10}}{e_8e_9},$$
$$e_3=\frac{e_8e_{11}}{e_8e_9+e_7},\ e_4=\frac{-e_{10}\left(1+e_8\left(\cosh(1)-1\right)\right)}{e_8e_9+e_7},\ e_5=\frac{-e_7e_8}{e_8e_9+e_7},$$
$$e_6=\frac{e_8e_{10}\left(\sinh(1)-1\right)}{e_8e_9+e_7},\ e_7=k\left(e^2-1\right)+2h_z\left(H-h\right),$$
$$e_8=\frac{\sinh(1)}{2e^2}\frac{\left(H-h\right)}{h},\ e_9=e_7\left(\cosh(1)-1\right)-e_{11}\left(\sinh(1)-1\right),$$
$$e_{10}=2eh_z\left(H-h\right),\ e_{11}=k\left(e^2-1\right)+2h_z\left(H-h\right)$$

After integration of equations (23)-(25), we finally obtain system of ordinary differential equations:

$$\frac{d}{dt}(\tilde{c}\tilde{\rho}u_0)=\frac{C}{l}\left(u_1-u_0\right)+C_0\left(1+\alpha u_0\right)-E_0\left(u_0-\Theta\right), \tag{33}$$

$$\frac{d}{dt}(\tilde{\tilde{c}}\tilde{\tilde{\rho}}u_1)=\frac{C}{L}\left(u_0-u_1\right)-\frac{h_y}{b}\left(u_1-\Theta\right)+C_1\left(1+\alpha\left(u_1-\Theta\right)\right)+$$
$$+\frac{k\sinh(1)}{h^2}\left[e_1\left(u_1-u_2\right)+e_2\left(u_2-\Theta\right)\right], \tag{34}$$

$$\frac{d}{dt}(\hat{c}\hat{\rho}u_1)=C_2\left(u_0-\Theta\right)-\frac{h_y}{b}\left(u_2-\Theta\right)+$$
$$+C_1\left(1+\alpha\left(u_2-\Theta\right)\right)-E_1\left(u_1-u_2\right)-E_2\left(u_2-\Theta\right), \tag{35}$$

Here constants E_0,E_1,E_2 and coefficients c,ρ:

$$E_0=\frac{h_y}{b}+\frac{h_z}{h}\left(1+\frac{e_0}{e}\right),$$

$$E_1 = \frac{h_z\left(e_3 + e_5\left(e-1\right)\right)}{H-h} + \frac{ke_5}{\left(H-h\right)^2},$$

$$E_2 = \frac{h_z\left(e + e_4 + e_6\left(e-1\right)\right)}{H-h} + \frac{ke_6}{\left(H-h\right)^2},$$

$$\tilde{c} = \overline{c}\left(W_0(\tilde{z},t)\right),\ \tilde{\tilde{c}} = \overline{\overline{c}}\left(W_1(\tilde{z},t)\right),\ \hat{c} = \overline{\overline{c}}\left(W_1(\hat{z},t)\right),$$

$$\tilde{\rho} = \overline{\rho}\left(W_0(\tilde{z},t)\right),\ \tilde{\tilde{\rho}} = \overline{\overline{\rho}}\left(W_1(\tilde{z},t)\right),\ \hat{\rho} = \overline{\overline{\rho}}\left(W_1(\hat{z},t)\right),$$

$$\tilde{z} = h/2,\ \hat{z} = (H+h)/2.$$

This system of three ordinary differential equations must be supplemented with initial conditions:

$$u_0\big|_{t=0} = u_1\big|_{t=0} = U^0. \tag{36}$$

3.4 Simplified Averaged System of Ordinary Differential Equations

The main goal of this mathematical model is to predict time before melting of the material in the thinnest sub-domain G_0 because of inadmissible strong current. According to expression (2), density of the electrical current is H^2 / h^2 times bigger in this sub-domain. This reason allows us to propose another model besides the first one. As the second step of the averaging, we use the simplest approximation in the z-direction – approximation by constant.

We introduce averaged values:

$$\begin{aligned} w_0(x,t) &= \frac{1}{h}\int_0^h V_0(x,z,t)dz, \\ w_1(x,t) &= \frac{1}{H}\int_0^H V_1(x,z,t)dz. \end{aligned} \tag{37}$$

We assume that temperature is constant in z-direction because it changes only slightly in comparison with x-direction:

$$\begin{aligned} w_0(x,t) &= V_0(x,z,t), \\ w_1(x,t) &= V_1(x,z,t). \end{aligned} \tag{38}$$

Integration of the differential equations (11) immediately gives system of two 1D partial differential equations:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(c(w_0)\rho(w_0)w_0\right) = \frac{\partial}{\partial x}\left(k\frac{\partial w_0}{\partial x}\right) - \\ &-\left(\frac{h_y}{b} + \frac{h_z}{h}\right)\left(w_0 - \Theta\right) + C_0\left(1 + \alpha\left(w_0 - \Theta\right)\right), \\ &\frac{\partial}{\partial t}\left(c(w_1)\rho(w_1)w_1\right) = \frac{\partial}{\partial x}\left(k\frac{\partial w_1}{\partial x}\right) - \\ &-\left(\frac{h_y}{b} + \frac{h_z}{H}\right)\left(w_1 - \Theta\right) + C_1\left(1 + \alpha\left(w_1 - \Theta\right)\right). \end{aligned} \tag{39}$$

Boundary conditions remain the same:

$$\left.\frac{\partial w_0}{\partial x}\right|_{x=0} = \left.\frac{\partial w_1}{\partial x}\right|_{x=l+L} = 0. \tag{40}$$

The second conjugation condition changes substantially because of convective heat losses over the surface $\{x = l, z \in [h, h+H]\}$:

$$\begin{aligned} &w_0\big|_{x=l-0} = w_1\big|_{x=l+0}, \\ &hk\left.\frac{\partial w_0}{\partial x}\right|_{x=l-0} = \left.\left[Hk\frac{\partial w_1}{\partial x} - h_z\left(H-h\right)\left(w_1 - \Theta\right)\right]\right|_{x=l+0}. \end{aligned} \tag{41}$$

Integration of boundary condition and conjugation conditions was made to obtain previous equation.

By the way, such type of the second conjugation condition was used in paper [4]. The initial conditions remain the same:

$$w_0\big|_{t=0} = w_1\big|_{t=0} = U^0. \tag{42}$$

As the last step, we will apply the conservative averaging method in x-direction. We will use exponential approximation as the form used earlier:

$$\begin{aligned} w_0(x,t) &= u_0(t) + p_0(t)\left[\cosh\left(\frac{x}{l}\right) - \sinh\left(1\right)\right], \\ w_1(x,t) &= u_1(t) + p_1(t)\left[\cosh\left(\frac{x-l-L}{L}\right) - \sinh\left(1\right)\right] \end{aligned} \tag{43}$$

We have introduced the average integral values again:

$$\begin{aligned} u_0(t) &= \frac{1}{l}\int_0^l w_0(x,t)dx, \\ u_1(t) &= \frac{1}{L}\int_l^{l+L} w_1(x,t)dx. \end{aligned} \tag{44}$$

We obtain parameters $p_0(t)$, $p_1(t)$ of the representations (43) from the conjugation conditions (41):

$$p_1(t) = e\left[u_0(t) - u_1(t)\right] + p_0(t),$$

$$p_0(t) = e\frac{\left(g_1 + g_2\right)\left(u_1 - u_0\right) - g_2\left(u_1 - \Theta\right)}{g},$$

i.e.,

$$p_1(t) = e\frac{g_0\left(u_0 - u_1\right) - g_2\left(u_1 - \Theta\right)}{g}.$$

Here

$$g_0 = k\frac{h}{l}\left(e^2 - 1\right),\ g_1 = k\frac{H}{L}\left(e^2 - 1\right),$$

$$g_2 = 2h_z\left(H-h\right),\ g = g_0 + g_1 + g_2.$$

We integrate partial differential equations (39) and obtain system of ordinary differential equations finally:

$$\frac{d}{dt}\left(\bar{c}\bar{\rho}u_0\right)=\frac{G}{l^2}\left[\left(g_1+g_2\right)\left(u_1-u_0\right)-g_2\left(u_1-\Theta\right)\right]- \\ -\left(\frac{h_y}{b}+\frac{h_z}{h}\right)\left(u_0-\Theta\right)+C_0\left(1+\alpha\left(u_0-\Theta\right)\right), \qquad (45)$$

$$\frac{d}{dt}\left(\bar{\bar{c}}\bar{\bar{\rho}}u_1\right)=\frac{G}{L^2}\left[g_0\left(u_0-u_1\right)-g_2\left(u_1-\Theta\right)\right]- \\ -\left(\frac{h_y}{b}+\frac{h_z}{H}\right)\left(u_1-\Theta\right)+C_1\left(1+\alpha\left(u_1-\Theta\right)\right). \qquad (46)$$

Here

$$\bar{c}=c\left(w_0(\bar{x},t)\right),\ \bar{\rho}=\rho\left(w_0(\bar{x},t)\right),\ \bar{x}=l/2,$$

$$\bar{\bar{c}}=c\left(w_1(\bar{\bar{x}},t)\right),\ \bar{\bar{\rho}}=\rho\left(w_1(\bar{\bar{x}},t)\right),\ \bar{\bar{x}}=L/2,$$

$$G=\frac{k\left(e^2-1\right)}{2g}.$$

It remains to add the initial conditions for the completeness of the full statement of the 0-D problem:

$$u_0\big|_{t=0}=u_1\big|_{t=0}=U^0. \qquad (47)$$

4 Conclusions

We have approximated 3D problem and reduced its solution to the solution of the time-dependent non-linear system of two or three ordinary differential equations. Reduction was realized in two different ways by different assumptions. Both systems have similar structure, but different coefficients. The systems of ordinary differential equations are solvable with standard techniques. Approximate analytical 3D solution could be easily obtained from the solution of the transformed problem afterwards.

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