2D Homogenisation procedure in masonry walls strengthened by FRP repointing technique

A. BARBIERI, A. CECCHI
Dipartimento di Costruzione dell'Architettura
Università IUAV di Venezia
Dorsoduro, 2206 - Venice
ITALY

Abstract: - In this work the effectiveness of FRP repointing technique for masonry wall strengthening is analysed by means of homogenisation procedure. Here a linear elastic analysis is performed that is significant under service loads. The masonry has been identified with a standard elastic continuum by means of a homogenisation method. Two homogenisation approaches are proposed: an analytical approach and a numerical approach, that allow to determine values of homogenised membrane moduli, for running bond texture, taking into account the effective micro-structure of masonry and considering the presence of FRP strengthening.

An extensive numerical analysis has been carried out to investigate the capacity of the homogenisation method to grasp the effect of geometrical and mechanical parameters in the analysis of masonry walls strengthened by CFRP (Carbon Fiber Reinforced Polymer) plates. The sensitivity of strain field to strengthening material is investigated on a meaningful case such as a masonry wall loaded by a horizontal displacement.

Key-Words: - Masonry, CFRP repointing, Homogenisation technique, Membrane moduli.

1 Introduction

The conservation of historical heritage is an important topic in Italy, due to the relevant amount of historical and monumental buildings in the country and the seismic events accoutred in the centuries. Hence, a branch of scientific research concerns the evaluation of suitable strengthening techniques able to preserve these structures for the future generations. In the last two decades new materials and techniques has been developed in the building market to strength existing structures and reinforce the new ones.

In the present paper a new technique -the FRP repointing- is validated by means of an homogenisation procedure. This technique consists of embedding continuous FRP rods or plates in the horizontal joint of wall by suitable paste; the horizontal joint is previously grooved, hence the masonry texture has to present continuous horizontal joints, as in the running bond courses. The efficiency of this technique is based on the good collaboration between FRP and filling paste, as this last and the blocks. The filling paste has to transfer the shear stresses between masonry and strengthening materials guaranteeing the anchorage and bonding along the interface. Few studies has been carried out on this technique [1, 2, 3, 4]. This technique is proposed to control the cracking phenomena in historical masonry structure. Generally these cracks appear on the point of the failure. Some study cases are in progress on bell towers and masonry columns [5, 6].

Cecchi et al. [7] developed a model for masonries in-plane loaded, reinforced with CFRP sheets. Here an analogous model is proposed following the same methodology for FRP repointing technique. The homogenisation approach links the masonry behaviour on the micro-level to the macro-level, to take into account global and local phenomena of masonry [8, 9], as FRP strengthening [7]. The homogenisation approach starts considering mechanical and geometrical properties of single masonry constituents (blocks and mortar joints) as of FRP strengthening material and identify an elementary cell, which regular repetition describe the body as a whole. In this way the field problem is led to the unit cell, reducing the computation effort and carrying out average values of mechanical properties.

In the present research the running bond texture is assumed as reference, because it often characterises the historical masonry, and CFRP plate is assumed as strengthening material of bed mortar joints. The CFRP plate is applied along the whole masonry thickness.

The mechanical and geometrical characteristics of masonry should be different and this aspect is influent on mechanical parameters. The analytical model proposed in [7], reviewed in the present research, is meaningful because it provides explicit equations of elastic membrane moduli. Moreover in the present research, the effective thickness of joint is taken into account. A comparison between the membrane moduli obtained with zero joint thickness and finite joint thickness is carried out. Furthermore, to evaluate the reliability of the 2D homogenised model, a 2D F.E. model has been performed. A numerical analysis has been carried out by comparing the strain filed at specific cross sections of an in-plane loaded masonry wall, which bed joints are strengthened by CFRP plate repointing technique.

2 Field problem for homogenisation procedure

Heterogeneous materials may be studied using homogenization techniques that permit the definition of an homogeneous body. A simplified model has been proposed by Cecchi and Rizzi [10] and implemented by Cecchi and Sab [11, 12] considering three perturbative parameters: 1) $\varepsilon = l/L$ ratio between l size of the cell and L dimension of the overall panel; 2) $\xi = E_m/E_b$, ratio between E_m Young modulus of mortar and E_b Young modulus of block; 3) $\psi = e/l$ ratio between e thickness of joint and l size of the characteristic module (cell).

In [11, 12] the mortar joint is considered as an interface without thickness and with a constitutive function, that is directly assigned as a linear function of the displacement jump across the joint. In the present research the effective joint thickness is considered, hence the effective dimension of masonry is considered in the following average operations. The constitutive function of vertical and horizontal joint is still defined as in [13]. Hence the constitutive function of joint is:

$$\mathbf{K} = \frac{1}{e}\left(\mu^M \mathbf{I} + (\mu^M + \lambda^M)(\mathbf{n} \otimes \mathbf{n})\right) \tag{1}$$

where e is the thickness of joint, $\mathbf{n}$ is the normal to the interface, λ^M and μ^M are the Lamé constants of mortar. The constitutive function (1) may be used correctly in the case of finite joint thickness because of the e value is defined.

If CFRP strengthened horizontal joint is considered, the constitutive function (1) becomes orthotropic, due to the fact that the joint behaviour in the horizontal and vertical directions is different . The research focuses on masonry walls loaded only in-plane and strengthened along the whole cross section, hence the homogenised membrane moduli $A_{\alpha\beta\gamma\delta}$ of strengthened masonry are evaluated (where the Grek index α, β=1,2).

The following auxiliary problem is solved on the elementary cell:

$$\begin{aligned} & div\boldsymbol{\sigma} = 0 \\ & \boldsymbol{\sigma} = \mathbf{a}(y)\ \boldsymbol{\varepsilon} \\ & \boldsymbol{\varepsilon} = \mathbf{E} + sym(grad\ \mathbf{u}^{per}) \\ & \boldsymbol{\sigma}\ \mathbf{e}_\alpha = anti - perodic\ on\ \partial Y_\alpha^\pm \\ & \mathbf{u}^{per}\ periodic\ on\ \partial Y_\alpha^\pm \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor; $\boldsymbol{\varepsilon}$ is the membrane strain tensor; $\mathbf{E}$ is the macroscopic membrane strain tensor and $\mathbf{u}^{per}$ is a periodic displacement field on ∂Y_α; $\mathbf{a}$ is the constitutive function defined as: $\mathbf{a}^B$ for $y \in$ block, $\mathbf{a}^J$ for $y \in$ joint, with $\mathbf{a}^B$ and $\mathbf{a}^J$ are respectively the block and the joint constitutive laws. The joint constitutive function changes in function of horizontal or vertical joint. All the materials are assumed isotropic.

The macroscopic tensors are related to the macroscopic displacement field ($U_1(x_1,x_2)$, $U_2(x_1,x_2)$) components as follows:

$$\mathbf{E}_{\alpha\beta} = \frac{1}{2}(\mathbf{U}_{\alpha,\beta} + \mathbf{U}_{\beta,\alpha}) \tag{3}$$

where the Greek index α, β=1,2. In particular, considering a 2D problem the analysis may be carried out in term of plane stress or plane strain, defining the lower and upper bound of the 3D solution.

Considering in-plane loading, the homogenised constitutive law of the panel, in the case of central symmetry, becomes:

$$\mathbf{N} = \langle \boldsymbol{\sigma} \rangle = \mathbf{A}^H \mathbf{E} \tag{4}$$

where $\mathbf{N}$ is the membrane tensor, $\mathbf{A}^H$ is the homogenised membrane modulus and $\langle \cdot \rangle$ is the average operator defined as:

$$\langle f \rangle = \frac{1}{t_i} \int_Y f(y_1, y_2) dy_1 dy_2 \tag{5}$$

where t_i is the dimension of Y; Y is the REV dimension, given by the sum of joint and block.

3 Basic assumptions

The masonry is made of UNI clay bricks (250x120x55 mm), whereas the mortar joint thickness is s_v=10 mm for head joint and s_h=10.

Let be ($\boldsymbol{x}$) a reference system for the global description of the masonry beam column, called $\Im$ the macroscopic configuration and let be ($\boldsymbol{y}$) a reference system for the elementary module Y-REV.

The Y module, as shown in figure 1 may be defined as:

$$Y = \left]-\frac{t_1}{2},\frac{t_1}{2}\right[\times \left]-\frac{t_2}{2},\frac{t_2}{2}\right[= \omega \tag{6}$$

where t_i are the 2 dimensions of Y, according to 2 axes directions; ω is the mean plane of masonry panel. The boundary of Y is defined as:

$$\partial Y = \partial Y_\alpha^+ \cup \partial Y_\alpha^- \tag{7}$$

As shown in figure 1, the dimensions of elementary cell are $t_1=b+e_v$ and $t_2=2a+2e_h$.

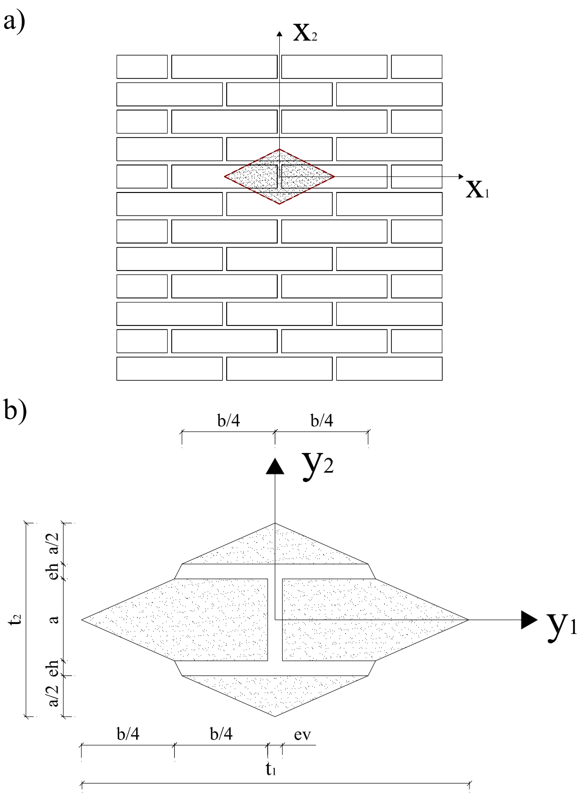

Figure 1 The elementary cell: Y-REV

3.1 Influence of joint thickness

The analytical model formulated by Cecchi and Sab [11] was implemented taking into account the joint thickness. A comparison between the two analytical formulations is carried out for the un-strengthened masonry.

Following the procedure of Cecchi and Sab [11], the homogenised moduli for elastic brick and joint are obtained in Y and not in Y_b. Hence, the effective joint thickness is taken in account in the mathematical procedure. The following equations, if compared with those of [11], show exactly the same structure:

$$A^H_{1111} = \frac{\frac{a}{a+e_h}\left(A^B_{1111}K' + \mathbf{B}\frac{e_h}{a}\right)\left(4K'\frac{e_h}{a+e_h} + \frac{b+e_v}{a+e_h}K''\frac{e_v}{a+e_h}\right)}{4\frac{e_h^2}{(a+e_h)^2}\frac{e_v}{b+e_v}\mathbf{B} + A^B_{1111}\frac{e_h}{a+e_h}\mathbf{C} + K'\mathbf{D}} \tag{8}$$

$$A^H_{2222} = K'\frac{A^B_{2222}K' + \mathbf{B}\frac{e_h}{b+e_v}}{\frac{e_h}{a+e_h}\frac{e_v}{b+e_v}\mathbf{B} + A^B_{2222}\left(\frac{e_h}{a+e_h} + \frac{e_v}{b+e_v}\right)K' + K'^2}$$

$$A^H_{1122} = \frac{A_{1122}K'\frac{a}{a+e_h}\left(4K'\frac{e_h}{a+e_h} + \frac{b+e_v}{a+e_h}K''\frac{e_v}{a+e_h}\right)}{4\frac{e_h^2}{(a+e_h)^2}\frac{e_v}{b+e_v}\mathbf{B} + A^B_{1111}\frac{e_h}{a+e_h}\mathbf{C} + K'\mathbf{D}}$$

$$A^H_{1212} = \frac{A_{1212}K''\left(K'\frac{e_v}{b+e_v} + 4\frac{a+e_h}{b+e_v}K''\frac{e_h}{b+e_v}\right)}{A^B_{1212}\frac{e_h}{a+e_h}\mathbf{F} + 2K''\mathbf{G}}$$

where

$$\mathbf{B} = \left(A^B_{1111}\right)^2 - \left(A^B_{1122}\right)^2$$

$$\mathbf{C} = 4K'\frac{e_h}{a+e_h} + \frac{b+e_v}{a+e_h}K''\frac{e_v}{a+e_h} + 4K'\frac{a}{a+e_h}\frac{e_v}{b+e_v}$$

$$\mathbf{D} = \frac{a}{a+e_h}\left(4K'\frac{e_h}{a+e_h} + \frac{b+e_v}{a+e_h}K''\frac{e_v}{a+e_h}\right)$$

$$\mathbf{F} = K'\frac{e_v}{a+e_h} + 4\frac{a+e_h}{b+e_v}K''\frac{e_h}{b+e_v} + 4\frac{(a+e_h)^2}{(b+e_v)^2}K''\frac{e_v}{b+e_v}$$

$$\mathbf{G} = K'\frac{e_v}{b+e_v} + 4\frac{a+e_h}{b+e_v}K''\frac{e_h}{b+e_v}$$

$K'=2\mu^M+\lambda^M$, $K''=\mu^M$ and μ^M, λ^M are the Lamè constants in plane strain hypothesis.

The differences lie in the ratios e_v/b and e_h/a which are substituted respectively by $e_v/(b+e_v)$ and $e_h/(a+e_h)$.

The diagrams for each membrane modulus $A_{\alpha\beta\gamma\delta}$ are plotted in figures 2, 3, 4 and 5, under plane strain (PE) and plane stress (PS) hypotheses, with joint of zero thickness and joint with finite thickness (g). The values are plotted versus ξ^{-1} ($=E^b/E^m$) and reported to the corresponding membrane modulus of homogeneous masonry made of block (E^b=90 GPa).

The investigation evinced that the bulk modulus A^H_{1111} does not feels the effect of joint thickness because the vertical joint is one time present along y_1 axis direction. The analogous consideration should be done for membrane modulus A^H_{1122} which is ν times the A^H_{1111} modulus. The joint thickness has different effect on A^H_{2222} and A^H_{1212} moduli: the homogenised elastic constants are bigger than those obtained applying expressions reported in [11]. The relevant effect of joint thickness is due to that the horizontal joint are present two times in y_2 axis direction.

The error done, considering a infinitesimal or finite joint thickness, in the evaluation of A^H_{1111} and A^H_{1122} starts from 0.03% for E^b/E^m=5 up to 0.19% for E^b/E^m=90; the error done in the evaluation of A^H_{2222} starts from 7% for E^b/E^m=5 up to 14% for E^b/E^m=90, whereas for A^H_{1212} starts from 5% for E^b/E^m=5 up to 13% for E^b/E^m=90.

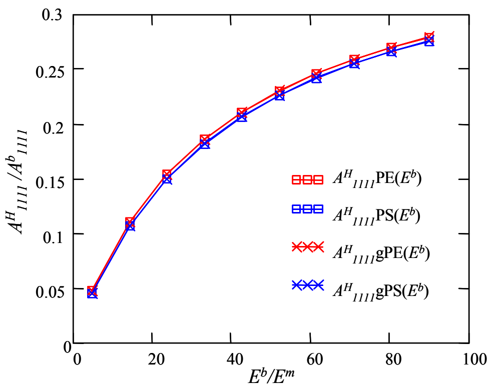

Figure 2 Trend of membrane modulus $A^H{}_{1111}$.

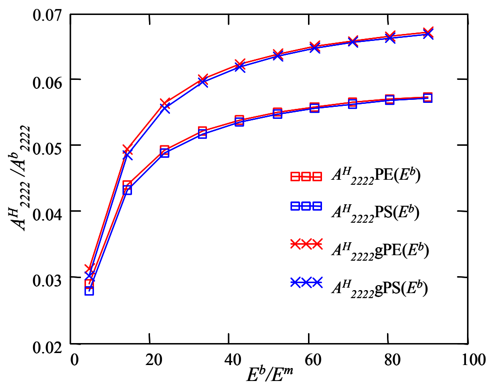

Figure 3 Trend of membrane modulus $A^H{}_{2222}$.

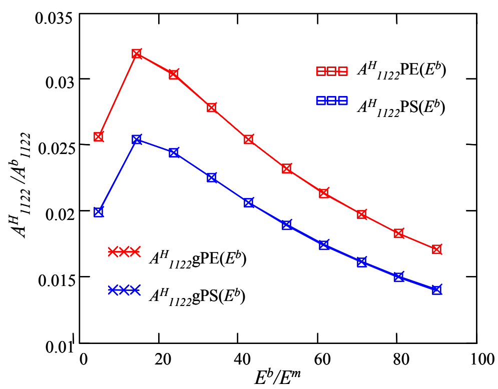

Figure 4 Trend of membrane modulus $A^H{}_{1122}$.

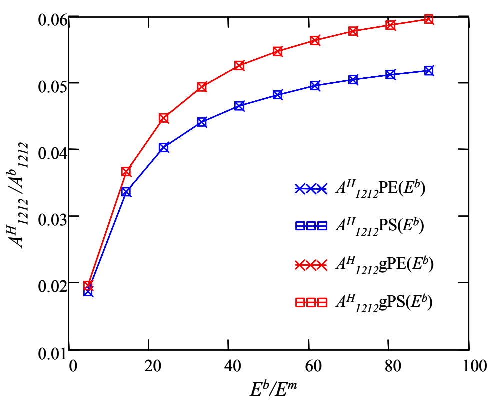

Figure 5 Trend of membrane modulus $A^H{}_{1212}$.

4 F.E.M. Homogenisation procedure

A numerical method was applied to evaluate the homogenised membrane moduli $A_{\alpha\beta\gamma\delta}$ of masonry panel un-strengthened and strengthened. The numerical analysis is carried out considering a 2D FEM model under in plane strain and plane hypothesis. The field problem of the elementary cell may be reported to the $Y/4$ due to the symmetry (Fig. 6). The periodic boundary conditions that must be imposed are:

$$\mathbf{u}_{\alpha\beta} = \mathbf{E}^{(\alpha\beta)} y_i + \mathbf{u}^{per}_{\alpha\beta}(\mathbf{y}) \qquad (9)$$

in particular three relevant cases are considered:

$$\mathbf{E}^{(11)} = (1);\ \mathbf{E}^{(22)} = (1);\ \mathbf{E}^{(12)} = \mathbf{E}^{(21)} = (1); \qquad (10)$$

hence:

- axial elongation along y_1 axis:

$$\mathbf{u}_{11} = \mathbf{E}^{(11)} y_1 + \mathbf{u}^{per}_{11}(\mathbf{y})$$

- axial elongation along y_2 axis:

$$\mathbf{u}_{22} = \mathbf{E}^{(22)} y_2 + \mathbf{u}^{per}_{22}(\mathbf{y})$$

- shear elongation along y_1 axis and y_2 axis:

$$\mathbf{u}_{12} = \mathbf{E}^{(12)} y_1 + \mathbf{E}^{(21)} y_2 + \mathbf{u}^{per}_{12}(\mathbf{y})$$

a)

b)

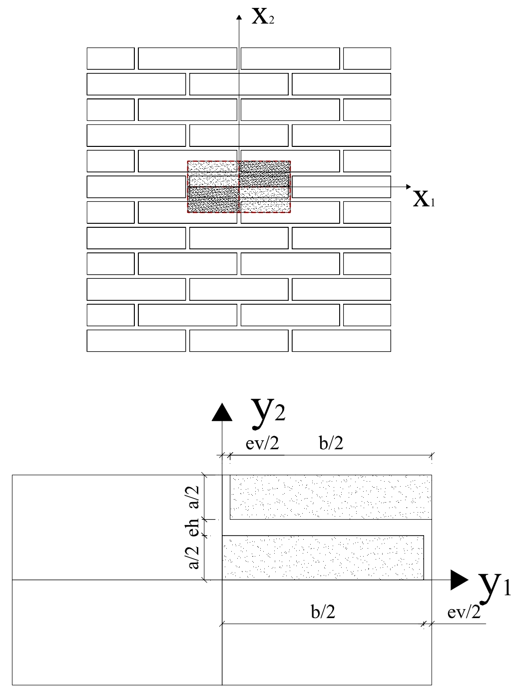

Figure 6 From the elementary cell Y (a) to $Y/4$ (b) for F.E.M. homogenisation procedure.

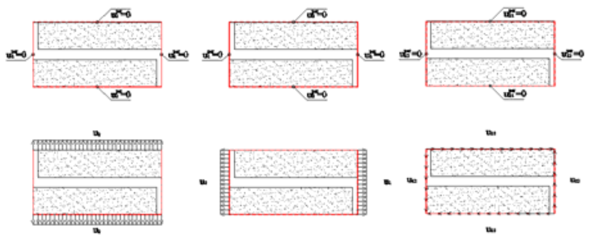

Figure 7 Displacement conditions on 2D elementary cell

The imposed suitable boundary conditions for u^{per} periodic on $\partial Y_\alpha^\pm$ are plotted in figure 7.
The $Y/4$ module is defined as follows:

$$Y/4 = \left]0,\frac{t_1}{2}\right[\times \left]0,\frac{t_2}{2}\right[\qquad (11)$$

The homogenised membrane moduli are evaluated by numerical model on the $Y/4$ solving field problem (3). Hence it is:

$$A_{\alpha\beta\gamma\delta} = \frac{1}{\omega}\int \sigma_{\alpha\beta\gamma\delta}\, dA \qquad (12)$$

The finite element model has been built for un-strangthened and strengthened masonry. The block, joint and strengthening materials are modelled by iso-parametric 4 node plate elements. The joint thickness for FRP repointing is composed by: mortar (4.4 mm), FRP (1.2 mm) and mortar (4.4 mm). The constitutive laws for mortar, brick and FRP are linear elastic and isotropic. The mechanical properties of materials in F.E. model are reported in table 1.

Young modulus [N/mm^2]	Poisson ratio
$E^b = 5000 \div 90000$	$\nu^b = 0.2$
$E^m = 1000$	$\nu^m = 0.2$
$E^{FRP} = 145\cdot 10^3;\ 210\cdot 10^3;\ 300\cdot 10^3$	$\nu^{FRP} = 0.4$

Table 1: Mechanical properties of materials.

5 Analytical and numerical results

The analytical results were compared with the numerical ones, considering the joint thickness. At the end, the only numerical analysis was carried on and the membrane moduli of un-strengthened and strengthened masonry were evaluated to show the efficiency of FRP repointing technique, considering different FRP longitudinal elastic modulus, as reported in table 1. The aim was to evaluate the increment in terms of membrane stiffness due to FRP repointing technique and the influence of this parameter.
In figure 8, the mesh of *Y/4* for strengthened masonry is reported.

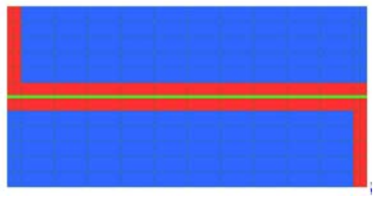

Figure 8 *Y*/4 elementary cell meshing of strengthened masonry.

The analytical and numerical models were compared considering the finite joint thickness. For each membrane moduli, the two models were plotted versus ξ^{-1} in figure 9, 10, 11 and 12.

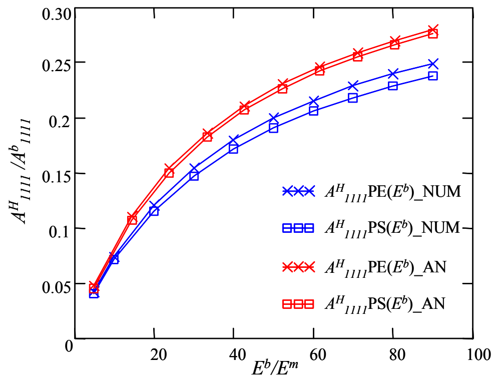

Figure 9 Trend of membrane modulus A^H_{1111}.

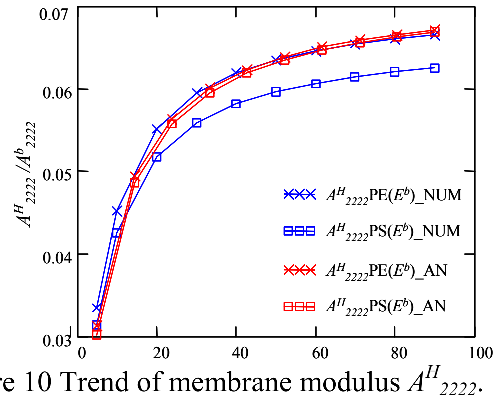

Figure 10 Trend of membrane modulus A^H_{2222}.

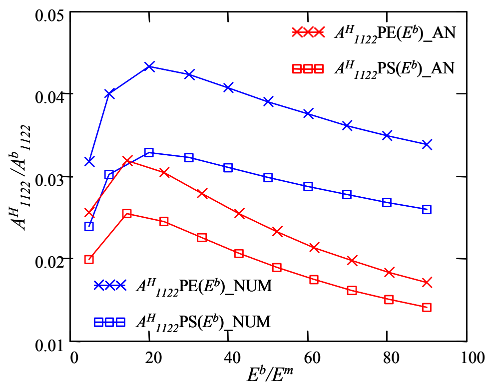

Figure 11 Trend of membrane modulus A^H_{1122}.

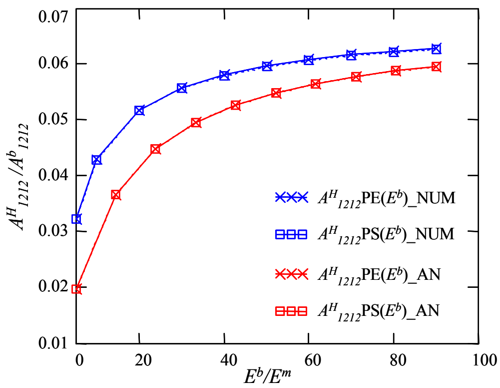

Figure 12 Trend of membrane modulus $A^H{}_{1212}$.

The comparison evinced that:

- the membrane modulus $A^H{}_{1111}$ evaluated analytically (AN) is bigger than this evaluated numerically (NUM) and the difference increase increasing ξ^{-1}.
- the membrane modulus $A^H{}_{2222}$ evaluated analytically is included between those evaluated numerically in plane strain and in plane stress domains. The values are close to plane strain envelope.
- the membrane modulus $A^H{}_{1122}$ evaluated numerically is bigger than that evaluated analytically. The gap between plane strain and plane stress hypothesis is more evident in the numerical model.
- the membrane modulus $A^H{}_{1212}$ evaluated in plane strain and in plane stress hypothesis does not differ both in the analytical and numerical models. Also in this case the numerical model evaluates a value bigger than the analytical one, but the difference decrease increasing ξ^{-1}.

Generally, the difference between the membrane moduli evaluated in plane strain and in plane stress hypotheses are bigger in numerical model than in analytical one. This is due to the constitutive function used for modelling the joint interface in the analytical model: the joint elastic constants are always evaluated in plane strain hypothesis, whereas in the numerical model the constitutive law is the same both for block and for joint.

5.1 Effects of FRP repointing technique

In this paragraph the effects of FRP repointing technique is analysed. The numerical analysis was carried on by the numerical model and the un-strengthened masonry was compared with the strengthened one. Three type of FRP materials were assumed which differ for longitudinal Young modulus: S=145 GPa; M=210 GPa; H=300GPa.

The efficiency of FRP repointing technique is shown in the following diagrams (Fig. 13, 14, 15, 16).

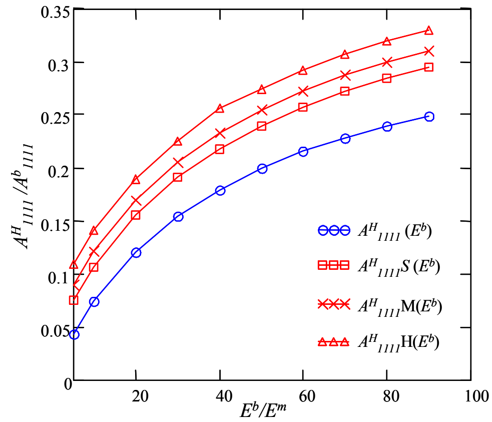

Figure 13 $A^H{}_{1111}$ comparison between un-strengthened and strengthened masonry.

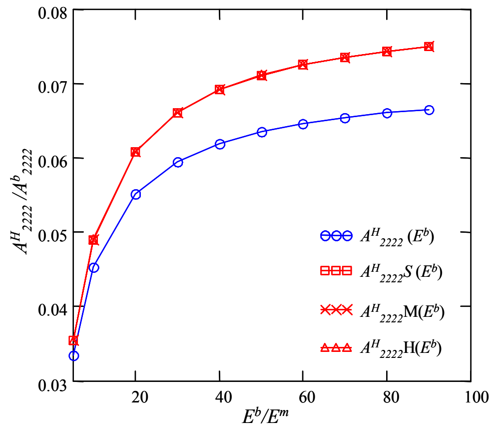

Figure 14 $A^H{}_{2222}$ comparison between un-strengthened and strengthened masonry.

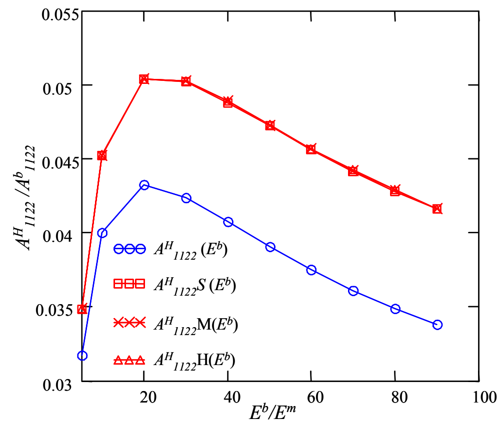

Figure 15 $A^H{}_{1122}$ comparison between un-strengthened and strengthened masonry.

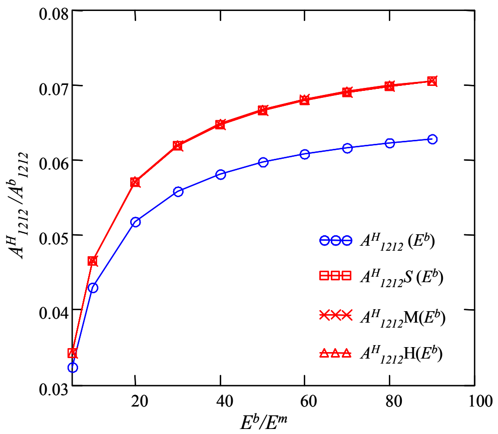

Figure 16 A^H_{1212} comparison between un-strengthened and strengthened masonry.

The increment of membrane stiffness in masonry panel is more evident along y_1 axis direction. The membane modulus A^H_{1111} increases depending on FRP longitudinal stiffness E^{FRP} (Fig. 13), whereas FRP stiffness is not relevant on the others membrane moduli (Fig. 14, 15, 16).
Anyway the positive contribution of FRP repointing is evident because the bed joint stiffness is increased along y_1 axis direction and the transversal elongation is reduced so that less tensile stress are transmitted from mortar bed joint to blocks.
The effect of FRP repointing technique is evaluated as:

$$\Delta A^H_{\alpha\beta\gamma\delta} = \frac{A^H_{\alpha\beta\gamma\delta}FRP - A^H_{\alpha\beta\gamma\delta}}{A^H_{\alpha\beta\gamma\delta}FRP} \cdot 100 \qquad (13)$$

where $A^H_{\alpha\beta\gamma\delta}FRP$ is the membrane moduli evaluate in strengthened masonry and $A^H_{\alpha\beta\gamma\delta}$ is the membrane moduli evaluate in un-strengthened masonry. The membrane moduli $A^H_{\alpha\beta\gamma\delta}$ is plotted versus the ratio ξ^{-1}, considering slow (S), medium (M) and high (H) longitudinal Young modulus of FRP for A^H_{1111}, whereas only the slow modulus is considered for the other moduli.

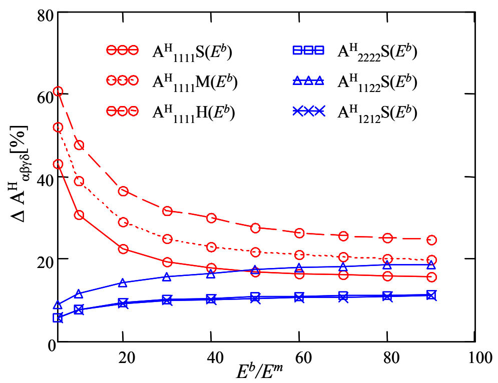

Figure 17 $\Delta A^H_{\alpha\beta\gamma\delta}$ increment of in-plane stiffness for strengthened masonry in comparison with un-strengthened one.

The membrane stiffness A^H_{1111} reduces increasing the ratio ξ^{-1} in function of the parameter E^{FRP}, whereas the membrane stiffness A^H_{2222}, A^H_{1122} and A^H_{1212} increase slowly in the full range, showing more evident increment for small ratio ξ^{-1}.

6 Masonry panel

A 2D full F.E. model as been built to represent masonry panel with single block in the thickness of panel. The aim is to compare the 2D heterogeneous model with the homogenised 2D model. Plate elements (4 node) are used both for blocks, mortar joints and FRP repointing (Fig. 18).
The masonry panel is 1160 mm height and 1550 mm width. The bed and head joints thickness is 10 mm; the block are 250×120×55 mm. In the strengthened panel, the bed joint is composed of: one layer of mortar joint (4.4 mm), one layer of FRP (1.2 mm) and one layer of mortar joint (4.4 mm).

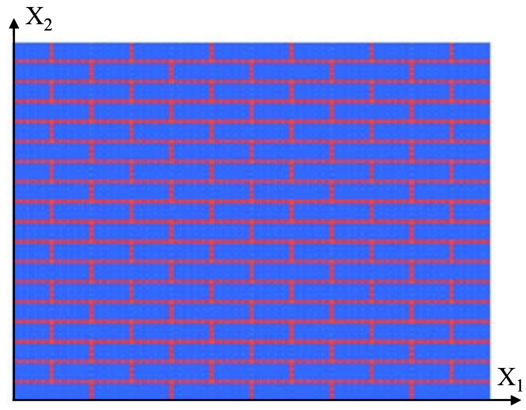

Figure 18 2D heterogeneous numerical model.

The panel is loaded by an unit horizontal displacement at the top (u_{11}=1 mm), whereas the lower end is clamped. The comparison between the 2D heterogeneous F.E. model and 2D homogenised model is carried out for $10 \le \xi^{-1} \le 90$.
The vertical strain ε_{22} of the cross section at 100 mm and 125 mm from the bottom is considered. The first cross section considers a layer of blocks (*B*) and vertical joints; the second layer considers a layer of mortar joint (*J*) or mortar joint/FRP (*J-FRP*). Considering the case of plane strain, the numerical analysis was carried on considering the heterogeneous masonry panel and comparing the results with these carried out considering the corresponding homogenised one. The same analysis was conducted for strengthened masonry panel, evaluating the effect of slow, medium and high FRP longitudinal Young modulus.

The numerical results carried out show that a good agreement between 2D heterogeneous model and 2D homogenised model for both un-strengthened and strengthened cases.

In figure 19, the results carried out by the two procedures of homogenisation, analytical (AN) and numerical (NUM), are compared with the heterogeneous case for ξ^{-1}=50.

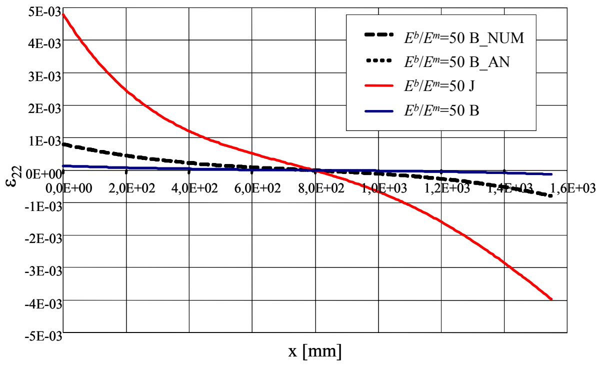

Figure 19 ε_{22} in layers *B* and *J*, in 2D heterogeneous model and 2D homogenised model, evaluated by analytical and numerical homogenisation procedure.

The homogenised model is included between the strain distribution at *B* layer and *J* layer; the homogenised solution is more close to *B* layer distribution because the area fraction of block is bigger than that of joint and this is more evident increasing the ratio ξ^{-1}. The 2D homogenised model has the same strain distribution along *B* layer and *J* layer, so that only *B* layer is considered for this model.

In figure 20 the un-strengthened heterogeneous masonry panel is compared with the strengthened one for ξ^{-1}=50.

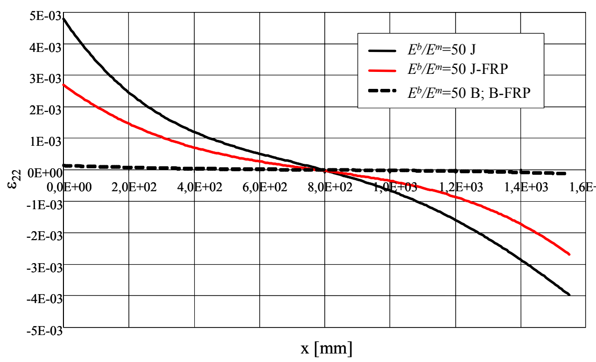

Figure 20 ε_{22} along layers *B* and *J*, for ξ^{-1}=50 ratio, for 2D heterogeneous model and 2D homogenised FRP model evaluated by numerical homogenisation procedure.

The ε_{22} along layers *B* is the same in un-strengthened and strengthened masonry, whereas along the bed joint the strain distribution is modified by the presence of FRP repointing. The strengthened bed joint is more stiff so that the strain distribution is minor than in bed joint made only of mortar. The stiffness of bed joint is function of E^b/E^m ratio and it is not influenced by FRP material longitudinal Young modulus.

7 Conclusion

The proposed 2D homogenised model allows to investigate easily the membrane behaviour of masonry panel, when a wider set of internal parameters varies (i.e. relative size of the joints, relative deformability of the joints).

The FRP repointing technique is a suitable strengthening technique to increase the stiffness of masonry panel for in-plane loading.

The research should be developed evaluating the influence of FRP repointing in the analytical model and considering the FRP anisotropy in the numerical one. The next step would be to implement the analysis by a 3D numerical model that may take into account the behaviour along the masonry wall thickness.

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