Modeling the anisotropic preconditioning behavior of soft biological tissues

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Abstract: In the present contribution, we present a constitutive model to describe the mechanical preconditioning characteristics of soft biological tissues. The model is based on a generalized polyconvex anisotropic strain energy function represented by a series. The preconditioning is considered as an inelastic dissipative process and is taken into account by an evolution of material parameters treated as internal variables. Accordingly, evolution equations are formulated and onset conditions for the softening are defined. A numerical example illustrates the model in application to preconditioning with increasing upper load levels.

Key–Words: Soft Tissues, Constitutive Modeling, Anisotropy, Softening, Preconditioning

1 Introduction

When biological tissues are subjected to consecutive loading and unloading, their mechanical response during the first cycles is usually characterized by remarkable softening, decreasing hysteresis and the tendency to approach a steady state, in which the material response becomes repeatable. This behavior is well-known as preconditioning. The properties in the preconditioned state depend decisively on the deformation history [1], i.e. on the test protocol. In particular, the upper and lower load limits applied have a significant influence [2]. Recently, preconditioning has been modeled in the framework of quasi-linear viscoelasticity theory [1, 3] (QLV) or by combination of QLV with strain softening approaches originating from elastomer modeling [4]. On the other hand, continuum damage mechanics (CDM) [5, 6] has been utilized, in particular to model anisotropic damage in arterial walls [7, 8].

In the present work we propose a CDM based constitutive model to describe the preconditioning behavior of soft tissues including the progressive softening and the dependence on the load limits. This model is anisotropic, three-dimensional and thermodynamically consistent. First, we briefly resume an anisotropic hyperelastic model [9] able to describe the nonlinear elastic behavior of soft tissues. The strain energy function of the model is polyconvex and coercive [10] so that the material stability and the existence of a global minimizer of the total elastic energy are guaranteed in the elastic domain. In the next step, the anisotropic softening is taken into account by an evolution of material parameters associated with the stiffness of the material. These parameters are considered as internal variables. Accordingly, evolution equations and onset conditions for the softening are proposed. Finally, the performance of the preconditioning model is illustrated by a numerical example.

2 Polyconvex hyperelastic model

Soft tissues may be considered as fiber-reinforced materials consisting of an isotropic matrix and a number of reinforcing (collagen) fiber families [11], the orientation of which is given by unit vectors $\boldsymbol{m}_i, i = 1, 2, ..., n$. We define $n + 1$ structural tensors

$$\mathbf{L}_i = \boldsymbol{m}_i \otimes \boldsymbol{m}_i, \quad i = 1, 2, ..., n, \quad \mathbf{L}_0 = \frac{1}{3}\mathbf{I}, \quad (1)$$

where $\mathbf{I}$ denotes the second-order identity tensor and $\mathbf{L}_0$ is assumed to be associated with the isotropic matrix. Forming linear combinations with the aid of non-negative weight factors $v_i^{(r)}, i = 0, 1, ..., n$, one obtains the so-called generalized structural tensors

$$\tilde{\mathbf{L}}_r = \sum_{i=0}^{n} v_i^{(r)} \mathbf{L}_i, \;\; r = 1, 2, ..., \;\; \sum_{i=0}^{n} v_i^{(r)} = 1, \quad (2)$$

characterized by the property $\mathrm{tr}\tilde{\mathbf{L}}_r = 1$. Further, the generalized invariants

$$\tilde{I}_r = \mathrm{tr}\left[\mathbf{C}\tilde{\mathbf{L}}_r\right], \; \tilde{J}_r = \mathrm{tr}\left[(\mathrm{cof}\mathbf{C})\tilde{\mathbf{L}}_r\right], \quad (3)$$

$r = 1, 2, ..., s$, are introduced where $\mathbf{C}$ denotes the right Cauchy-Green tensor and $\mathrm{cof}\mathbf{C} = \mathbf{C}^{-\mathrm{T}}\mathrm{det}\mathbf{C}$ is its cofactor. On the basis of these generalized invariants and $\mathrm{III}_{\mathbf{C}} = \mathrm{det}\mathbf{C}$, a class of polyconvex strain energy functions for fiber-reinforced materials is given by the series representation [9] with an arbitrary number of terms s:

$$W = \frac{1}{4}\sum_{r=1}^{s}\mu_r\left[f_r(\tilde{I}_r) + g_r(\tilde{J}) + h_r(\mathrm{III}_{\mathbf{C}}^{1/2})\right], \quad (4)$$

where $f_r(\tilde{I}_r)$ and $g_r(\tilde{J}_r)$ are convex and monotone increasing and $h_r(\mathrm{III}_{\mathbf{C}}^{1/2})$ are convex functions [9, 12]. The condition of the energy and stress free reference configuration is thus easily satisfied [9]. Taking the constraint $\mathrm{III}_{\mathbf{C}} = 1$ into account, the strain energy (4) can be specified for incompressible materials by setting

$$W = \frac{1}{4}\sum_{r=1}^{s}\mu_r\left[f_r(\tilde{I}_r) + g_r(\tilde{K}_r)\right], \ \tilde{K}_r = \mathrm{tr}(\mathbf{C}^{-1}\tilde{\mathbf{L}}_r). \quad (5)$$

The above model shows good agreement with experimental data on various soft tissues [9, 12].

3 Dissipative model

3.1 Evolution of material parameters

Softening is taken into account by evolution of the material parameters $\mu_r, r = 1, 2, ..., s$, appearing as linear factors in the strain energy function (4). These parameters are considered as internal variables. Accordingly, the elastic potential (4) is extended to the free energy

$$\Psi = \tilde{\Psi}(\tilde{I}_r, \tilde{J}_r, \mathrm{III}_{\mathbf{C}}, \mu_r), \quad r = 1, 2, ..., s. \quad (6)$$

Exploiting the second law of thermodynamics under isothermal conditions one can write the second Piola-Kirchhoff stress tensor $\mathbf{S}$ and the internal dissipation D as

$$\mathbf{S} = 2\frac{\partial\Psi}{\partial\mathbf{C}}, \quad (7)$$

$$D = -\sum_{r=1}^{s}\frac{\partial\Psi}{\partial\mu_r}\dot{\mu}_r = -\sum_{r=1}^{s}\psi_r\dot{\mu}_r \geq 0, \quad (8)$$

where the thermodynamic variables $\psi_r \geq 0$ conjugate to μ_r have been introduced. Similarly to classical continuum damage mechanics, we define yield surfaces in strain space by

$$\varphi_r(\mathbf{C}, \kappa_r) = \psi_r(\mathbf{C}) - \kappa_r = 0, \quad r = 1, 2, ..., s, \quad (9)$$

on which the criterion for the evolution is satisfied. The parameters κ_r define the onset of the softening described by μ_r. The normals to the surfaces $\mathbf{N}_r = \partial\varphi_r/\partial\mathbf{C}$ allow to distinguish between unloading $\dot{\psi} < 0$, neutral loading $\dot{\psi} = 0$ and loading $\dot{\psi} > 0$, where $\dot{\psi}_r = \mathbf{N}_r : \dot{\mathbf{C}}$ results from the chain rule. Finally, the evolution equations for μ_r are written by

$$\dot{\mu}_r = \begin{cases} d_r(\psi_r, \mu_r)\dot{\psi}_r & \text{if } \dot{\psi}_r > 0 \text{ and } \varphi_r = 0 \\ 0 & \text{else,} \end{cases} \quad (10)$$

where $r = 1, 2, ..., s$ and $d_r(\psi_r, \mu_r) \leq 0$ are scalar functions. Accordingly, $\dot{\mu}_r \leq 0, r = 1, 2, ..., s$, so that softening is associated with a decrease of the parameters μ_r. Note that the dissipation inequality $(7)_2$ is thus satisfied.

3.2 Preconditioning

Softening associated with preconditioning is assumed to be a progressive process taking place during loading until a stable state is reached. Thus, we set in (9)

$$\kappa_r = \psi_r, \quad r = 1, 2, ..., s \quad (11)$$

and consider softening functions of the particular form

$$d_r(\mu_r) = -k_r(\mu_r - \bar{\mu}_r), \quad r = 1, 2, ..., s \quad (12)$$

with dimensionless scalar factors $k_r > 0$ and parameters $\bar{\mu}_r$ representing the value of μ_r in the entirely preconditioned state.

3.3 Preconditioning with different load levels

If the upper and lower amplitudes of the cyclic loading are changed after the tissue response has stabilized, the tissue has to be preconditioned anew [2]. This suggests that the attainable degree of softening depends on the loading history and in particular on the maximum load experienced in the past. In the model, this can be regarded by a dependence of the value $\bar{\mu}_r$ on the maximum of the associated thermodynamic variable ψ_r, say $\bar{\psi}_r$. Thus, we have

$$d_r(\bar{\psi}_r, \mu_r) = -k_r(\mu_r - \hat{\bar{\mu}}_r(\bar{\psi}_r)), \quad (13)$$

where

$$\bar{\psi}_r = \max_{\tau\in(-\infty,t]}\left[\psi_r(\mathbf{C}(\tau))\right], \quad r = 1, 2, ..., s. \quad (14)$$

We illustrate the model by simulating an experiment in which a biological tissue is subject to uniaxial cyclic loading with increasing stress levels as depicted in Figure 1. The tissue is modeled as an incompressible, fiber-reinforced material with two mechanically equivalent fiber families. The functions f_r and g_r are given by an exponential representation [12]. Furthermore, we restrict the number of terms in the strain energy to a single term ($s = 1$), and omit the index for demonstration purposes. In view of (5) and (6), the free energy thus takes the form

$$\begin{aligned} \Psi &= \tilde{\Psi}(\tilde{I}, \tilde{K}, \mu) \\ &= \frac{1}{4}\mu \left[\frac{1}{\alpha} \left(e^{\alpha(\tilde{I}-1)} - 1 \right) + \frac{1}{\beta} \left(e^{\beta(\tilde{K}-1)} - 1 \right) \right]. \end{aligned} \tag{15}$$

Generally, the functional dependence of $\hat{\bar{\mu}}_r$ on the maximum $\bar{\psi}_r$ may be studied experimentally. Here, we suppose the following form

$$\hat{\bar{\mu}}(\bar{\psi}) = \mu_{ini} \exp \left[- \left(\frac{\bar{\psi}}{a} \right)^b \right], \tag{16}$$

where $a > 0$ and $b \geq 1$ are scalar constants. The nominal stress in the loading direction simulated on the basis of (15) is plotted vs. stretch in Figure 1.

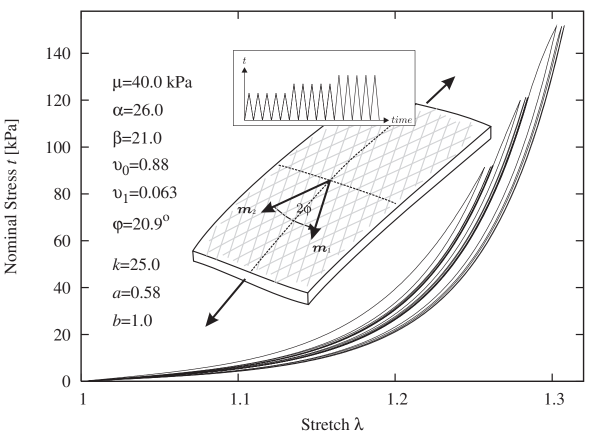

Figure 1: Simulation of the preconditioning of a tissue sample with increasing upper limits.

4 Conclusions

We proposed a dissipative model for the preconditioning behavior of soft tissues based. The model describes anisotropic elastic behavior even if only one term is considered. Already a second term renders the model fully anisotropic both with respect to elastic and inelastic properties. Polyconvexity and coercivity of the underlying strain energy function guarantee material stability and the existence of the solution of a boundary value problem in the elastic domain. The general preconditioning characteristics of soft tissues are well described by the model. Further evaluation in comparison with experimental data on the softening behavior will be necessary.

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