Study on the Gyroscope Stability Subjected to Impulse and Step by Step Perturbations

DIANA COTOROS*, MIHAELA BARITZ**
*Department of Mechanics, **Department of Precision Mechanics
University Transilvania of Brasov
Address 29, Eroilor St. Brasov
ROMANIA

Abstract: - The paper aims at proving the stability of a gyroscopic device subjected to different types of impulse and step by step perturbations, by using a MathCAD program. The motion equations solutions are graphically represented in a stationary situation and also after simulating various perturbations, simulating them by help of some mathematical functions.

Key-Words: - gyroscope, perturbation, stability, impulse, step by step, MathCAD

1 Introduction

The main part of a gyroscopic device is represented by the gyroscope itself. The gyroscope is a rigid body, having a revolution symmetry and performing a fixed point motion.

The reason of using gyroscope in all kinds of navigation devices for aircrafts, ships or sofisticated landvehicles is the property of rotation axis stability.

Stability in motion means maintaining a certain trajectory, even if subjected to perturbing factors, in the vicinity of the non-perturbed motion trejectory. If the stability condition is not achieved, the indications of the gyroscopic device may be affected and also the safety of the vehicle and persons using it. This is why the indications stability should be beyond doubt.

It is obvious that during their travel, aircrafts, ships or landvehicles will be inevitably subjected to various types of perturbations due to bad weather (wind blows, rain, turbulence, tide, waves) or external factors (shocks, vibrations) according to the path they are moving onto.

The present paper proposes a possibility of simulating the actions of these perturbations upon a dynamically tuned gyroscope and analyze by graphical representations of motions equations solutions, their effect upon syability.

The dynamically tuned gyroscope consists of a symmetrical or non symmetrical rotor which rotates at a high angular velocity and has such an elastic connection to the its frame, that a gyroscopic torque occurs as a result of its transport motion.

2 Problem Formulation

The matrix form of the generalized motion equations of a dynamically tuned gyroscope is:

$$[J_\Phi][\ddot{\Phi}] + [c_\Phi][\dot{\Phi}] + [k_\Phi][\Phi] = [J_\Psi][\ddot{\Psi}] + [c_\Psi][\dot{\Psi}] \qquad (1)$$

where $[J_\Phi]$ and $[J_\Psi]$ represent the skew matrix of the moments of inertia for the rotors and frame, $[c_\Phi]$ and $[c_\Psi]$ the matrix of damping coefficients, $[k_\Phi]$ the matrix of elastic coefficients, $[\Phi]$ the vector of the rotor angular displacements about the shaft and $[\Psi]$ the angular displacements vector of the gyroscope frame.

As we are dealing with a system of non homogeneous differential equations, we will start by determining the eigenvalues of the characteristic equations.

$$Z(t) = \begin{bmatrix} e^{\lambda_0 t} & e^{\lambda_1 t} & 0 & 0 \\ \lambda_0 e^{\lambda_0 t} & \lambda_1 e^{\lambda_1 t} & 0 & 0 \\ 0 & 0 & e^{\lambda_{10} t} & e^{\lambda_{11} t} \\ 0 & 0 & \lambda_{10} e^{\lambda_{10} t} & \lambda_{11} e^{\lambda_{11} t} \end{bmatrix} \qquad (2)$$

where λ_0, λ_1, λ_{10}, λ_{11} represent the solutions of the characteristic equations.

The solutions of the homogeneous equations become

$$xo(t) = Z(t) \cdot C \qquad (3)$$

where C is obviously a constant matrix, obtained according to the initial conditions.

These are the solutions of the motion equations if there is no perturbations to affect the gyroscope operation.

If a perturbation occurs, we need to give it a mathematical form and use it in order to obtain the general solution. Thus, we are going to add a particular solution to the homogeneous one. The particular solution will be the result of the mathematical model of the considered type of perturbation.

3 Problem Solution

If we consider for example f(t) the mathematical representation of the external perturbation, the general solution will be:

$$xg(t,j) = xo(t)_j + \int_0^t \phi(t-\tau,1)_j \cdot f(t-\tau)d\tau \qquad (4)$$

where $\phi(t,j) = Z(t)\cdot\left[Z(0)^{-1}\right]^{<j>}$.

In order to express the action of shocks, small impacts or even unbalanced rotors we considered some impulse type of functions with variable parameters, whose actions at different moments are represented in Fig.1 and Fig.2.

Thus, for $f1(k,t) = k\,t\exp(-4\,t)$, we get

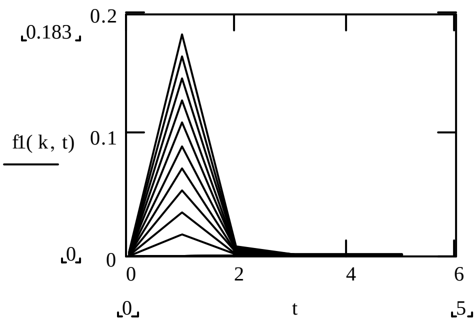

Fig.1

while for a variation given by $f2(k,t) = 4\,t\exp(-k\,t)$, we get the following representation:

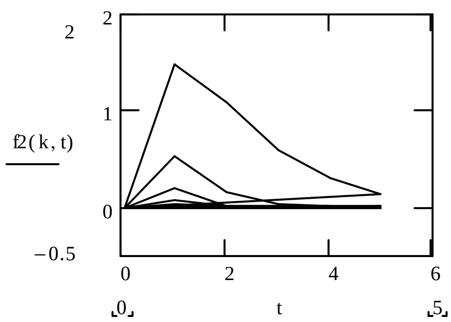

Fig.2

In the following figures (fig.3 and fig.4) we represented by comparison the homogeneous solution (with no perturbation) with a solid line and the various representations for perturbed solutions. In fig.3 the perturbation was modelled using the function f1, with the changing parameter k, while in fig.4 we represented the perturbation according to function f2, again with a variable parameter k, as shown above. The general solutions, affected by perturbations were represented with dotted lines, one for each value of the parameter k.

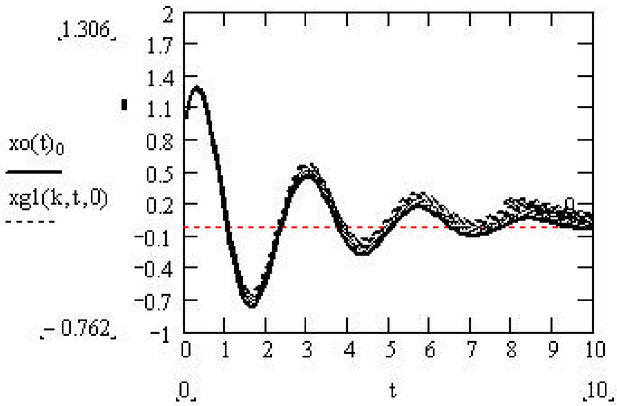

Fig.3

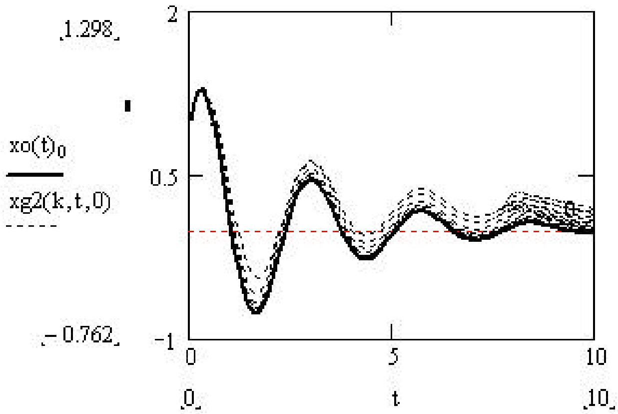

Fig.4

It is obvious from the figures above that the non-homogeneous solution (obtained by introducing the impulse perturbation) tends to remain in the vicinity of the initial one, proving that the stability is ensured, regardless of the various values of the parameters k.

In order to show in a more eloquent way the stability of the motion equations solutions, we also used another representation, namely the representation in the phases plane, involving both position and velocity. Thus, in fig.5, we represented the phases plane for the first perturbation given by f1, while in fig.6, the second pertrubation given by f2 was represented.

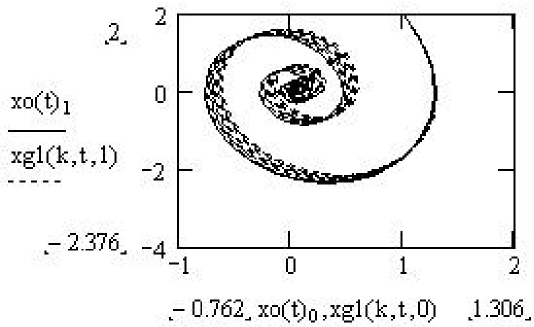

Fig.5

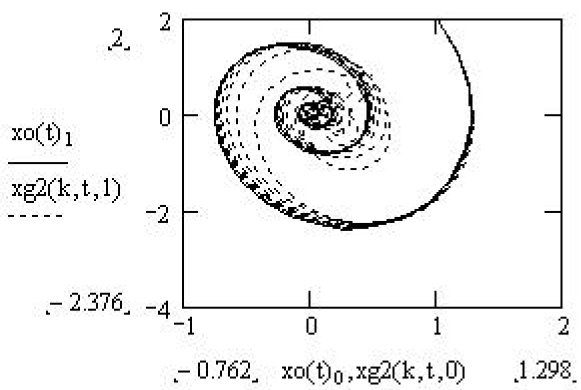

Fig.6

In order to express other types of perturbations, like strong wind blows, tides, waves or some kinds of vibrations, we introduced a step by step function g(t), using again variable parameters. The perturbations we get are represented in fig.7 and fig.8. Thus, for $g1(t) = k(1 - \exp(-4t))$

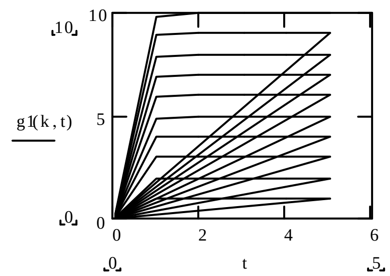

Fig.7

while for $g2(t) = 4(1 - \exp(-kt))$, the function representation becomes

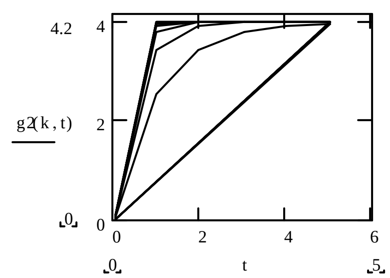

Fig.8

By applying the perturbation given by the variable functions g1(t) and g2(t) shown above, by comparison to the stationary solution given by the homogeneous solution, we get the representations in fig.9 and fig.10.

Analyzing the obtained results we can see that though the effect seems a little more powerful than in the previous case (dotted lines), the stability will not be lost, the solutions remain in the vicinity of the non perturbed one (solid line).

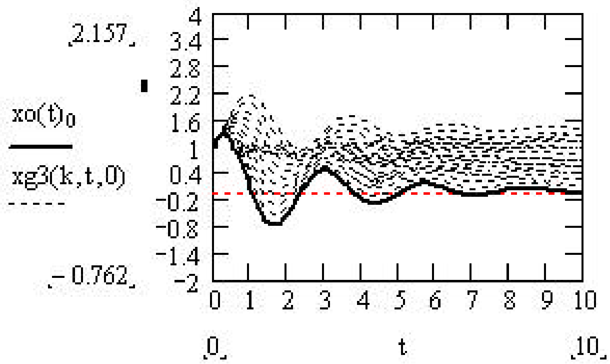

Fig.9

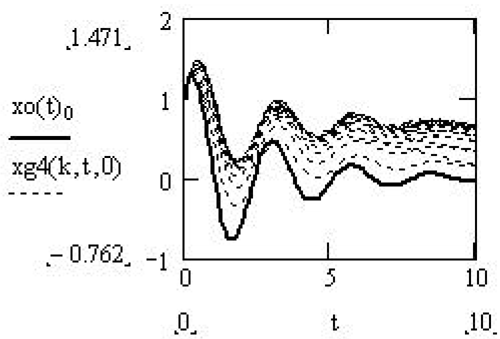

Fig.10

As we did in the previous situation we used the representations in the phases plane to emphasize the solutions stability, by presenting the dependence between the position and velocity. The results are expressed in fig.11 for the perturbation g1 and in fig.12 for the pertrubation expressed by g2. The significance of the lines remains the same.

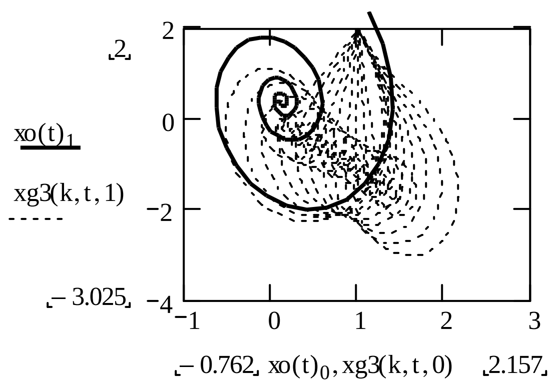

Fig.11

The representations in the phases plane prove again the fact that after the perturbations, the solutions of the motions equations tend to return to the vicinity of the trajectory given by the not perturbed motion. Of course the evolutions are not identical but obviously the systems tends to stability, ensuring thus the correct indications of the gyroscopic devices even subjected to external perturbing factors.

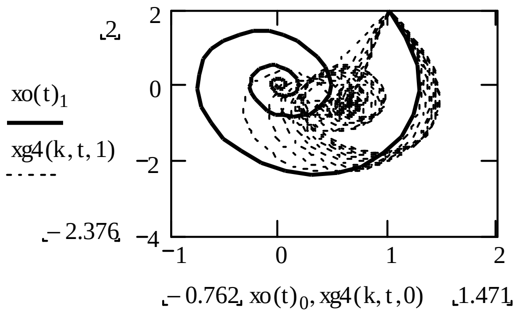

Fig.12

4 Conclusion

The gyroscopic devices used in many applications like aircrafts, ships or some landvehicles should be reliable instruments, they indicating the position, velocity, trajectory and many other parameters, which are vital for the safe operation of these means of transportation. Unfortunately they are subjected to the influence of some external factors that can not be avoided. This is why, the main component part of these devices, the gyroscope should be able to compensate the results of these perturbations.

By representing the solutions of the motion equqtions for a dynamically tuned gyroscope, we were able to prove the following:

- in case of shocks or small impacts, simulated by help of various impulse type functions, the general solutions remain tightly in the vicinity of the homogeneous solutions (meaning the motion without perturbations)
- in case of repeated wind blows, tides, waves or vibrations, simulated by step by step functions, the influence is greater but still the trajectories of the general solutions return fast to the vicinity of the homogeneous one.

So, the dynamically tuned gyroscope ensures the stability by itself, without introducing expensive and bulky correction loops or other correcting devices, as when we are dealing with classical gyroscopes.

The future research will be directed towards some random types of perturbations and checking the results on test stands. Of course, for any type of perturbation that might occur, we may start with the less expensive way of checking stability, using mathematical models and MathCAD representations.

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