Hybrid Model for Structural Reliability With Imprecise Probabilistic Characteristics

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Abstract: -A hybrid model of a probabilistic and non-probabilistic reliability theory is developed to predict the structural reliability when the probabilistic characteristics parameters of structural properties are imprecisely known. In this study, these parameters are described by appropriate ellipsoid. Interval of the structural reliability will be sought, i.e. estimating the worst possible reliability and the best possible reliability by ellipsoid-bound convex model method. A numerical example of a 60-bar power pagoda is used to illustrate the feasibility and validity of the proposed theory.

Key-Words: - Structural reliability; Reliability interval; Hybrid model; Probabilistic; Non-probabilistic reliability; Ellipsoid-bound convex model

1 Introduction

Structural reliability analysis plays an important role in the analysis and design of structures. In the engineering applications, probabilistic reliability theory appears to be presently the most important method[1-3]. The recent researches show that the reliability sensitivity of structure system strongly depends on the parameters of the probability model[4-8]. However, very often, sufficient information on the probabilistic characteristics is absent, and as a result, one may possess only imprecise or limited data on the probabilistic characteristics. The natural question arises as to how to deal with this situation which is almost invariably encountered in an engineering practice. There are several alternative approaches to deal with these problems, including probabilistic (random) description, fuzzy sets description and convex set description. But almost total lack of communication. Stochasticians almost exclusively utilize probabilistic methods and this models are generally informative-intensive[2,3,9], analysts of fuzzy sets employ fuzzy logic[10], whereas investigators dealing with anti-optimization (i.e. convex modeling, interval analysis) utilize models based on unknown-but-bounded quantities[5,8]. Therefore, the development of rigourous mathematical methods of combining the existing information for obtaining general estimates of the reliability of the entire system represents an actual problem[5,6]. Such a combination of the probabilistic and non-probabilistic analysis approach was performed in the shuttle applications by Elishakoff, Lin and Zhu[11] as well as for the analysis of uncertainty in passenger aircraft design for composite materials by Elishakoff, Li and Starnes[12]. This analysis is referred to also as robust uncertainty modeling or info-gap uncertainty[13,14].

In this study, we will combine ellipsoid-bound convex model analysis method with probabilistic methodology to evaluate the lower bounds and upper bounds of the structural reliability index and reliability. These bounds will be very useful in practice and could predict the maximum and minimum reliability when the experimental data are very limited.

2 Probabilistic Theory in Structural Reliability

Reliability analysis is to analytically formulate the failure given a failure criteria or failure mode. As a simplification, it is assumed that all states of the structure are divided into two states: failure state and safe state. Define the limit state function or failure function that represents the working state of structures as

$$M = g(X) \tag{1}$$

where $X = (X_1, X_2, \cdots, X_n)$ is an n-dimension random variable vector. If $M > 0$, the structure is in the safe state, and if $M < 0$, the structure is in the failure state. $M = 0$ defines the limit state surface which separates the failure region from the safe region. Given a limit state function $M = g(X)$ and a joint density function $f_X(x)$ of the random vector

$X=(X_1,X_2,\cdots,X_n)$, the structural reliability or probability of survival R is computed by

$$R=\iint_{g(X)>0}\cdots\int f_X(x)dx \tag{2}$$

The probability of failure P_F is the complement of R and is computed as $(1-R)$.

In general, an analytical evaluation of the integral given by Eq (2) is not possible due to the complexity of both $f_X(x)$ and $g(X)$. Also, if the number of random variables is large, a numerical integration of the problem is not feasible. Therefore, approximate method, first order reliability method (FORM), is used to obtain the failure probability. The description of this method can be divided into three steps. In the first step, the vector of basic variables $X=(X_1,X_2,\cdots,X_n)$ is transformed into an independent standard (zero mean and unit standard deviation) normal vector $Y=(Y_1,Y_2,\cdots,Y_n)$ using a probability preserving transformation. In the second step, the failure surface in the *y*-space is linearly approximated. In the final step, the probability content of the *y*-space can be exactly computed for the linear domain.

The structural reliability is given by

$$R=P\{g(X)>0\} \tag{3}$$

where $X=(X_i,\cdots,X_k)^T$ is the basic variable and $g(X)$ is the nonlinear failure function. The structural reliability R could be estimated by the following presented method. Suppose that there exists a transformation $Y=T(X)$, which can transform the variable $X=(X_i,\cdots,X_k)^T$ to the independent standard normal variable $Y=(Y_i,\cdots,Y_k)^T$. Then Eq.(3) can be transformed into

$$\begin{aligned} R&=P(M>0)=P(g(X)>0)\\ &=P(g(T^{-1}(Y))>0)=P(h(Y)>0) \end{aligned} \tag{4}$$

where $h(Y)=g(T^{-1}(Y))$. If FORM is used, by virtue of the linearization $h(Y)\approx\beta+\alpha Y$ and the transformation $Z=\alpha Y$, then Z becomes a standard normal variable, and R can be approximated as

$$R=P(h(Y)>0)\approx P(Z>-\beta)=\Phi(\beta) \tag{5}$$

where $\Phi(\cdot)$ is the standard normal distribution function $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp(-t^2/2)dt$ and the so-called reliability index β is defined as

$$\beta=\frac{\mu_M}{\sigma_M} \tag{6}$$

The one-one relation between β and R is given by the following equation

$$R=\Phi(\beta) \quad \text{or} \quad \beta=\Phi^{-1}(R) \tag{7}$$

Structural reliability relies on two kinds of design parameters: one is the stress or distortion of the structures or component parts caused by the various external loads, which is called stress resultant, denoted as s; the other is the capacity of enduring the loads for structures, component parts or their materials, which is called resistance or intensity, denoted as r. In the construction time and service time of structures, they exist in the manners of either *Safe* or *Failure*. In this paper, the limit state function $M=g(X)$ is taken as the linear function of s and r.

$$M=g(X)=r-s \tag{8}$$

In the analysis of probabilistic reliability, the stress resultant s and resistance r are supposed to be random variables, hence M is also a random variable.

In the limit state function (8), it is assumed that r and s obey the same kind of probabilistic distribution , and their mean values and standard variances are, respectively, μ_r, μ_s and σ_r, σ_s. Hence, the distribution of the limit state function $M=r-s$ is the same with r and s, and its mean value and standard variance are, respectively,

$$\mu_M=\mu_r-\mu_s \quad , \quad \sigma_M=\sqrt{\sigma_r^2+\sigma_s^2} \tag{9}$$

The structural reliability index β and the structural reliability R can be obtained by use of Eqs.(5), (6) and (9)

$$\beta=\frac{\mu_M}{\sigma_M}=\frac{\mu_r-\mu_s}{\sqrt{\sigma_r^2+\sigma_s^2}} \tag{10}$$

and

$$R=\Phi(\beta)=\Phi\left(\frac{\mu_r-\mu_s}{\sqrt{\sigma_r^2+\sigma_s^2}}\right) \tag{11}$$

Eqs.(10) and (11) are the two basic formulas for calculating the structural reliability index and reliability. From the above analysis, we can conclude that the evaluation of the reliability index hinges on the calculations of the mean values and standard variances of the structural stress and intensity.

3 Limitations of Pure Probability

The mathematical theory of probability has proven useful in many technological applications. However, it has limitations which, when clearly identified, facilitate our understanding of the non-probabilistic alternatives.

One main criticism of probabilistic concepts of uncertainty arises in discussion of prior probability and Bayesian inference and decision theory. A classical objection to Bayesian statistics hits at the source of the prior distribution and utility functions. Uniqueness is formulating prior distributions is illusive: a given quantity of prior information is often not represented by a unique prior probability distribution. The difficulty of quantifying prior knowledge is seen quite clearly in such quandaries as the 3-box riddle[14], the prisoner's dilemma and similar problems where alternative decisions each seem fully consistent with the initial information.

Probabilistic models have been used in recent decades to represent the uncertainty in engineering. The concern about these models arises from the fact that a stochastic model represents typical events much more reliably than rare events, especially when the model is based on limited information. Rare events in probabilistic models are described by the tails of the distribution, while probability distributions are usually specified in terms of mean and mean-variation parameters. This makes probabilistic models risky design tools, since it is rare events, the catastrophic ones, which must underlie the reliable design.

When a probabilistic description of the unknown elements is at hand, one is naturally led to consider stochastic models. When only partial information, or no information at all, is available, however, there is understandably a reluctance to rely on such models. In presuming that probability distributions exist they seem inherently misdirected. It can be seen that one's thinking about uncertainty can, and sometimes should be non-probabilistic.

Furthermore, according to the law of large numbers[15], only if the number of observations in the sample becomes large — that is, n approaches infinity, the sample mean and variance converge to the real mean value and variance. However, in practice, n may be impossible to be large enough to obtain the exact mean and variance value so that there often exists uncertainties in the parameters of probabilistic characteristics.

4 Hybrid Model for Structural Reliability

Indeed, the indeterminacy about the uncertain variables involved could be stated in terms of these variables belonging to certain sets, such as

- The uncertain parameter x is bounded,

$$|x| \le a \tag{12}$$

- The uncertain function has envelope bounds,

$$x_{lower}(t) \le x(t) \le x_{upper}(t) \tag{13}$$

where $x_{lower}(t)$ and $x_{upper}(t)$ are deterministic functions which delimit the range of variation of $x(t)$.

- The uncertain function has an integral square bound,

$$\int_{-\infty}^{+\infty} x^2(t)dt \le a \tag{14}$$

- The uncertain parameters vector has an instantaneous ellipsoid-bound,

$$(x - x_0)^T W (x - x_0) \le \alpha^2 \tag{15}$$

where W is a positive definite matrix, $x_0 = (x_{0i})$ is the nominal value of the vector $x = (x_i)$, and α is the radius of the convex set.

Consider a realistic situation when on one hand the mean values μ_r and μ_s and the standard deviations σ_r and σ_s of the resistance r and the stress resultant s are uncertain, but on the other hand, insufficient information is available on μ_r, μ_s and σ_r, σ_s to justify the above probabilistic framework. It is assumed that we possess only scarce information on the probabilistic characters μ_r, μ_s and σ_r, σ_s, namely, the uncertainties in the mean values μ_r, μ_s and the standard deviations σ_r, σ_s are bounded sets as the fourth kind of convex modelling mentioned above

$$\begin{aligned} &\frac{(\mu_r - \mu_{r0})^2}{e_r^2} + \frac{(\mu_s - \mu_{s0})^2}{e_s^2} \le \theta_1^2 \\ &\frac{(\sigma_r - \sigma_{r0})^2}{g_r^2} + \frac{(\sigma_s - \sigma_{s0})^2}{g_s^2} \le \theta_2^2 \end{aligned} \tag{16}$$

where μ_{r0}, μ_{s0} and σ_{r0}, σ_{s0} are the center points of the ellipsoids, and are chosen as the nominal or typical input; e_r^0, e_s^0 and g_r^0, g_s^0 are the radii of the ellipsoids; θ_1 and θ_2 are positive constants and determine the sizes of the ellipsoids. These values are all based on available limited information on the resistance r and the stress resultant s.

Let us consider the structural reliability (11) subject to the constraint condition (16). For convenience, the constraint conditions (16) may be written as follows

$$Z_1(\mu_r,\mu_s,e_r,e_s,\theta_1) = \left\{(\mu_r,\mu_s): \frac{(\mu_r-\mu_{r0})^2}{e_r^2}+\frac{(\mu_s-\mu_{s0})^2}{e_s^2}\le\theta_1^2\right\} \quad (17)$$

and

$$Z_2(\sigma_r,\sigma_s,g_r,g_s,\theta_2) = \left\{(\sigma_r,\sigma_s): \frac{(\sigma_r-\sigma_{r0})^2}{g_r^2}+\frac{(\sigma_s-\sigma_{s0})^2}{g_s^2}\le\theta_2^2\right\} \quad (18)$$

There are many applications in Eq.(11) with the constraint conditions (16) or (17) and (18), where the mean values μ_r and μ_s and the standard deviations σ_r and σ_s are not precisely known. Because the mean values μ_r and μ_s and the standard deviations σ_r and σ_s are uncertain but bounded, the associated probabilistic reliability of the structure similarly constitutes the bounded variable. That is to say, the probabilistic reliability of the structure with bounded probabilistic characteristics will become a set as follows

$$\Gamma = \left\{ R=\Phi(\beta):\quad \beta=\frac{\mu_r-\mu_s}{\sqrt{\sigma_r^2+\sigma_s^2}}, \right.$$
$$\frac{(\mu_r-\mu_{r0})^2}{e_r^2}+\frac{(\mu_s-\mu_{s0})^2}{e_s^2}\le\theta_1^2, \quad (19)$$
$$\left. \frac{(\sigma_r-\sigma_{r0})^2}{g_r^2}+\frac{(\sigma_s-\sigma_{s0})^2}{g_s^2}\le\theta_2^2 \right\}$$

We should stress that Γ may be generally of complicated geometric shape so that it may be usually impractical to try to solve them. Instead, in this study, we are interested in the set or the interval containing the structural probability reliability with uncertain but bounded probabilistic parameters. Therefore, it is a common practice to seek the interval of the probabilistic reliability

$$R^I=[\underline{R},\overline{R}]=[R_{\min},R_{\max}] \quad (20)$$

where

$$\underline{R}=R_{\min},\ \overline{R}=R_{\max} \quad (21)$$

which is the smallest width interval enclosing all possible probabilistic reliability values. $R_{\min}$ is the worst possible reliability or minimum value and $R_{\max}$ is the best possible reliablity or maximum. Obviously, the maximum value problem and the minimum value problem in Eq.(20) are global optimization problems.

5 Determination for Intervals of Structural Reliability Index and Reliability

In this section, the intervals of the structural reliability index and reliability will be computed.

Based on Eq.(7) and the monotonicity of function (11), the extreme value problem of the structural reliability (20) can be transformed into the following extreme value problem of the structural reliability index

$$\mathrm{H} = \left\{ \beta:\beta=\frac{\mu_r-\mu_s}{\sqrt{\sigma_r^2+\sigma_s^2}}, \right.$$
$$\frac{(\mu_r-\mu_{r0})^2}{e_r^2}+\frac{(\mu_s-\mu_{s0})^2}{e_s^2}\le\theta_1^2, \quad (22)$$
$$\left. \frac{(\sigma_r-\sigma_{r0})^2}{g_r^2}+\frac{(\sigma_s-\sigma_{s0})^2}{g_s^2}\le\theta_2^2 \right\}$$

Thus, the set or interval of the structural reliability index can be expressed as

$$\beta^I=[\underline{\beta},\overline{\beta}]=[\beta_{\min},\beta_{\max}] \quad (23)$$

where

$$\underline{\beta}=\beta_{\min}\ ,\ \overline{\beta}=\beta_{\max} \quad (24)$$

Under the condition that μ_Y and σ_Y are statistically independent, let us consider the extreme value problem of the structural reliability index β. Clearly, based on the multi-objective optimization theory, the maximum value and the minimum value can be, respectively, expressed as

$$\beta_{\max}=\frac{(\mu_Y)_{\max}}{(\sigma_Y)_{\min}}\ ,\ \beta_{\min}=\frac{(\mu_Y)_{\min}}{(\sigma_Y)_{\max}} \quad (25)$$

where $\mu_Y=\mu_r-\mu_s$ and $\sigma_Y=\sqrt{\sigma_r^2+\sigma_s^2}$. Thus, the extreme value problem of the structural reliability index is transformed into the extreme values problems of μ_Y and σ_Y.

For the extreme value problem

$$\textit{extreme}\quad \mu_Y=\mu_r-\mu_s \quad (26)$$

subject to

$$\frac{(\mu_r-\mu_{r0})^2}{e_r^2}+\frac{(\mu_s-\mu_{s0})^2}{e_s^2}\le\theta_1^2 \quad (27)$$

According to the optimization theory, since μ_Y is a linear function of uncertain parameters μ_r and μ_s, and $Z_1(\mu_r,\mu_s,e_r,e_s,\theta_1)$ is a convex set, the extreme values of μ_Y will occur on the boundary of the set $Z_1(\mu_r,\mu_s,e_r,e_s,\theta_1)$. The boundary of the set

$Z_1(\mu_r, \mu_s, e_r, e_s, \theta_1)$ represents an ellipsoidal shell, i.e.,

$$S_1(\mu_r, \mu_s, e_r, e_s, \theta_1) = \left\{ (\mu_r, \mu_s) : \frac{(\mu_r - \mu_{r0})^2}{e_r^2} + \frac{(\mu_s - \mu_{s0})^2}{e_s^2} = \theta_1^2 \right\} \quad (28)$$

By the method of Lagrange multipliers, the Lagrangian function can be written as

$$L_1 = \mu_r - \mu_s + \lambda_1 \left(\frac{(\mu_r - \mu_{r0})^2}{e_r^2} + \frac{(\mu_s - \mu_{s0})^2}{e_s^2} - \theta_1^2 \right) \quad (29)$$

where λ_1 is the Lagrange multiplier.

Necessary conditions for taking the extreme values are

$$\frac{\partial L_1}{\partial \mu_r} = 1 + 2\lambda_1 \frac{(\mu_r - \mu_{r0})}{e_r^2} = 0 \quad (30)$$

and

$$\frac{\partial L_1}{\partial \mu_s} = -1 + 2\lambda_1 \frac{(\mu_s - \mu_{s0})}{e_s^2} = 0 \quad (31)$$

resulting in

$$\mu_r - \mu_{r0} = -\frac{e_r^2}{2\lambda_1} \quad (32)$$

and

$$\mu_s - \mu_{s0} = \frac{e_s^2}{2\lambda_1} \quad (33)$$

Substitution of Eqs.(32) and (33) into the following constraint condition

$$\frac{(\mu_r - \mu_{r0})^2}{e_r^2} + \frac{(\mu_s - \mu_{s0})^2}{e_s^2} = \theta_1^2 \quad (34)$$

The Lagrange multiplier is obtained as

$$\lambda_1 = \pm \frac{\sqrt{e_r^2 + e_s^2}}{2\theta_1} \quad (35)$$

Substituting Eq.(35) into Eqs.(32) and (33) yields the extreme points of the mean values μ_r and μ_s

$$\mu_r = \mu_{r0} \mp \frac{e_r^2 \theta_1}{\sqrt{e_r^2 + e_s^2}} \quad (36)$$

and

$$\mu_s = \mu_{s0} \pm \frac{e_s^2 \theta_1}{\sqrt{e_r^2 + e_s^2}} \quad (37)$$

So the maximum values and the minimum values of the mean values μ_r and μ_s can be expressed as

$$(\mu_r)_{\max} = \mu_{r0} + \frac{e_r^2 \theta_1}{\sqrt{e_r^2 + e_s^2}}, \quad (\mu_r)_{\min} = \mu_{r0} - \frac{e_r^2 \theta_1}{\sqrt{e_r^2 + e_s^2}} \quad (38)$$

and

$$(\mu_s)_{\max} = \mu_{s0} + \frac{e_s^2 \theta_1}{\sqrt{e_r^2 + e_s^2}}, \quad (\mu_s)_{\min} = \mu_{s0} - \frac{e_s^2 \theta_1}{\sqrt{e_r^2 + e_s^2}} \quad (39)$$

Thus, the maximum values and the minimum values of μ_Y can be determined by

$$(\mu_Y)_{\max} = (\mu_r)_{\max} - (\mu_s)_{\min} = \mu_{r0} - \mu_{s0} + \theta_1 \sqrt{e_r^2 + e_s^2} \quad (40)$$

and

$$(\mu_Y)_{\min} = (\mu_r)_{\min} - (\mu_s)_{\max} = \mu_{r0} - \mu_{s0} - \theta_1 \sqrt{e_r^2 + e_s^2} \quad (41)$$

For the extreme value problem

$$extreme \quad \sigma_Y = \sqrt{\sigma_r^2 + \sigma_s^2} \quad (42)$$

subject to

$$\frac{(\sigma_r - \sigma_{r0})^2}{g_r^2} + \frac{(\sigma_s - \sigma_{s0})^2}{g_s^2} \le \theta_2^2 \quad (43)$$

In virtue of the following variable transformations

$$u_r = \sigma_r - \sigma_{r0} \quad , \quad u_s = \sigma_s - \sigma_{s0} \quad (44)$$

The optimization problem (42) can be converted into the following form

$$extreme \quad \sigma_Y = \sqrt{(u_r + \sigma_{r0})^2 + (u_s + \sigma_{s0})^2} = \sqrt{\varphi_0 + 2g^T \delta + \delta^T \delta} \quad (45)$$

subject to the constraint condition

$$\frac{u_r^2}{g_r^2} + \frac{u_s^2}{g_s^2} \le \theta_2^2 \quad (46)$$

where $\varphi_0 = (\sigma_{r0}^2, \sigma_{s0}^2)^T$, $g = (\sigma_{r0}, \sigma_{s0})^T$, $\delta = (u_r, u_s)^T$.

For convenience, the constraint condition (46) can be written

$$\delta^T \Omega \delta \le \theta_2^2 \quad (47)$$

where $\Omega = diag(\frac{1}{g_r^2}, \frac{1}{g_s^2})$.

Obviously, the extreme value problem $(\sigma_Y)_{extreme}$ is equivalent to the extreme value problem $(\varphi_0 + 2g^T\delta + \delta^T\delta)_{extreme}$, i.e.,

$$(\sigma_Y)_{extreme} = \sqrt{(\varphi_0 + 2g^T\delta + \delta^T\delta)_{extreme}} \tag{48}$$

Namely, the following extreme value problem will be solved

$$w = \varphi_0 + 2g^T\delta + \delta^T\delta \tag{49}$$

subject to the constraint condition (47).

By virtue of the method of Lagrange multipliers, and define the Lagrangian function as

$$L(\delta,\eta) = \varphi_0 + 2g^T\delta + \delta^T\delta + \eta(\delta^T\Omega\delta - \theta_2^2) \tag{50}$$

where η is the Lagrange multiplier. According to the extreme condition, we can obtain

$$\frac{\partial L}{\partial \delta} = 2g + 2\delta + 2\eta\Omega\delta = 0 \tag{51}$$

Moreover, the constraint condition must be satisfied

$$\delta^T\Omega\delta \le \theta_2^2 \tag{52}$$

Since Eq.(52) is an inequality, the Lagrange multiplier must satisfy one of the following relations[15]

$$\eta = 0 \quad \text{if} \quad \delta^T\Omega\delta < \theta_2^2 \tag{53}$$

and

$$\eta \ge 0 \quad \text{if} \quad \delta^T\Omega\delta = \theta_2^2 \tag{54}$$

From Eq.(51) and (53), we can obtain

$$\delta = -g \tag{55}$$

Substitution of Eq.(55) into Eq.(49) yields

$$w_{ext}^{(r)} = \varphi_0 - g^T g = 0 \tag{56}$$

By solving Eq.(51) and Eq.(54), we can obtain two Lagrange multiplier η_1 and η_2. By substituting them into Eq.(49), two extremum values can be obtained as follows, respectively

$$\begin{aligned} w_{ext}^{(s)} &= \varphi_0 - 2g^T(I+\eta_1\Omega)^{-1}g \\ &\quad + 2g^T(I+\eta_1\Omega)^{-1}(I+\eta_1\Omega)^{-1}g \end{aligned} \tag{57}$$

and

$$\begin{aligned} w_{ext}^{(t)} &= \varphi_0 - 2g^T(I+\eta_2\Omega)^{-1}g \\ &\quad + 2g^T(I+\eta_2\Omega)^{-1}(I+\eta_2\Omega)^{-1}g \end{aligned} \tag{58}$$

From the above equations (56), (57)and (58), three possible extremum values are obtained. By choosing the maximum value from the three possible extremum values as the upper bound of the objective function, and the minimum value as the lower bound, so we have

$$w_{\max} = \max(w_{ext}^{(r)}, w_{ext}^{(s)}, w_{ext}^{(t)}) \tag{59}$$

and

$$w_{\min} = \min(w_{ext}^{(r)}, w_{ext}^{(s)}, w_{ext}^{(t)}) \tag{60}$$

Then

$$(\sigma_Y)_{\max} = \sqrt{w_{\max}} \tag{61}$$

and

$$(\sigma_Y)_{\min} = \sqrt{w_{\min}} \tag{62}$$

In virtue of Eq.(25), from Eqs.(40), (41) and (61), (62), the maximum value or upper bound and the minimum value or lower bound of the structural reliability index can be obtained as follows

$$\overline{\beta} = \beta_{\max} = \frac{\mu_{r0} - \mu_{s0} + \theta_1\sqrt{e_r^2 + e_s^2}}{\sqrt{w_{\min}}} \tag{63}$$

and

$$\underline{\beta} = \beta_{\min} = \frac{\mu_{r0} - \mu_{s0} - \theta_1\sqrt{e_r^2 + e_s^2}}{\sqrt{w_{\max}}} \tag{64}$$

Consequently, according to the one-one relation (7), the best possible value or the upper bound and the worst possible value or the lower bound of the structural reliability can be calculated, respectively

$$\overline{R} = \Phi(\overline{\beta}) \tag{65}$$

and

$$\underline{R} = \Phi(\underline{\beta}) \tag{66}$$

6 Numerical Example and Discussions

Consider a 60-bar power pagoda as shown in Fig.1. The length of rods can be obtained from Fig.1. Two horizontal loads $P = 40000N$ are applied at node 21 and 22 respectively. The mean value of Young's modulus and cross-sectional area of the bars are, respectively, $\mu_E = 2.1\times10^{11} N/m^2$ and $\mu_A = 1.0\times10^{-3} m^2$. The uncertain parameter vector $R = (E, A)^T$ is normally distributed with a coefficient of variation 0.05. The allowable stress is normal distributed, and the distribution form is $r \sim N(\mu_{r0}, \sigma_{r0}) = N(1.5\times10^8, 6.42\times10^6)$. The mean value and standard variance of the applied stress are, respectively, $\mu_{s0} = 1.35\times10^8$, $\sigma_{s0} = 6.21\times10^6$.

The results of Eqs.(10) and (11) calculated by deterministic probabilistic characteristics parameters are: structural reliability index, $\beta = 1.6794$, and reliability, $R = 95.3\%$.

Take into account the uncertainty in the probabilistic characteristics μ_r , μ_s and σ_r , σ_s of the applied stress and allowable stress. Suppose that their mean values and the standard variances change in the following ellipsoids, respectively,

$$\frac{(\mu_r-\mu_{r0})^2}{(\alpha_1\mu_{r0})^2}+\frac{(\mu_s-\mu_{s0})^2}{(\alpha_2\mu_{s0})^2}\le 1,$$

$$\frac{\left(\sigma_r-\sigma_{r0}\right)^2}{(\alpha_3\sigma_{r0})^2}+\frac{\left(\sigma_s-\sigma_{s0}\right)^2}{(\alpha_4\sigma_{s0})^2}\le 1$$

where α_1, α_2, α_3 and α_4 are uncertain coefficients of the ellipsoids semi-radius. In this numerical example, they are chosen in such a manner that the variations at most will correspond to $\pm 5\%$ variation of each parameter. In practice, the bounds of these uncertainties will lie on the measured experiments data or experience.

Results of the calculation of the structural reliability index and reliability versus parameter α based on Eqs.(63), (64) and (65), (66) are shown in Table 1, Table 2 and Fig.2-Fig.5, where $\overline{\beta}$ and $\underline{\beta}$ are the upper bound and lower bound of the structural reliability index, and $\overline{R}$ and $\underline{R}$ are the upper bound and lower bound of the structural reliability corresponding to different cases of coefficient α_i, $i=1, 2, 3, 4$, respectively. Table 1 shows the upper bound $\overline{\beta}$ and lower bound $\underline{\beta}$ of the structural reliability index versus α_1 and α_2, for $\alpha_3=\alpha_4=0.0$, namely, the mean values are only considered to be uncertain and the standard variances are deterministic. Whereas in Table 2, the standard variances are uncertain, the mean values are deterministic, namely $\alpha_1=\alpha_2=0.0$ and $\alpha_3=\alpha_4\ne 0.0$. In Table 3, not only the mean value but also the standard variable are uncertain. Fig.2-Fig.5 depict the varying curves of $\overline{R}$ and $\underline{R}$ with the variations of uncertain factors α_i $(i=1,2,3,4)$.

The numerical results indicate that uncertainties in probabilistic characteristics properties have significant effects on the structural reliability. From these tables and figures, we can found that interval [$\underline{\beta}$, $\overline{\beta}$] of the structural reliability index and interval [$\underline{R}$, $\overline{R}$] of the structural reliability become wider with the increasing of uncertainties of probabilistic characteristics parameters.

7 Conclusions

Imprecise or uncertain probabilistic properties which are results from small sample random tests or incomplete information on uncertain variables are considered. A hybrid model of a probabilistic and non-probabilistic structural reliability theory is presented in this study to predict the variation of the structural reliability with uncertain probabilistic characteristics parameters. These probabilistic characteristics parameters are assumed to be described by appropriate ellipsoid.

Uncertainties in probabilistic characteristics properties have significant effects on the structural reliability. It is remarkable that this model is able to predict the worst possible value and the best possible value of the structural reliability index and reliability due to uncertainty. The reliability interval will be very useful in practice and could be directly incorporated into design when experimental data are very limited and the conventional probabilistic reliability approach cannot be utilized.

The hybrid model bridge the communication between stochasticians and analysts of sets. Consequently, the situations that they almost exclusively utilize probabilistic methods or anti-optimization methods will be broken.

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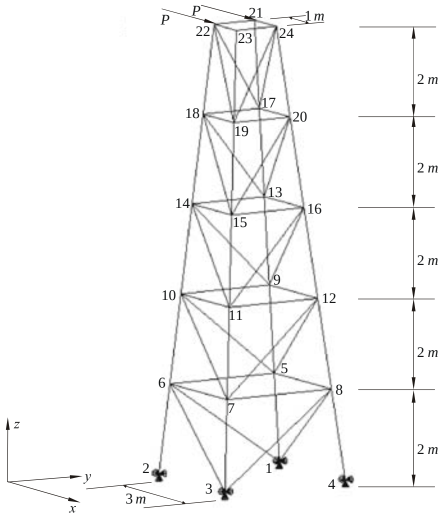

Fig.1 A 60-bar Power pagoda

Table 1 The structural reliability index β versus α_1 and α_2 when ($\alpha_3 = \alpha_4 = 0.0$)

α_1 \ α_2, β	0.00		0.01		0.03		0.05	
	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$
0.00	1.67936	1.67936	1.83050	1.52821	2.13278	1.22593	2.43506	0.92365
0.01	1.84729	1.51142	1.90529	1.45342	2.16288	1.19583	2.45350	0.90521
0.02	2.01523	1.34348	2.04767	1.31104	2.24363	1.11508	2.50634	0.85237
0.03	2.18316	1.17555	2.20534	1.15337	2.35716	1.00155	2.58761	0.77110
0.04	2.35110	1.00761	2.36789	0.99082	2.48981	0.86890	2.69046	0.66825
0.05	2.51903	0.83968	2.53253	0.82618	2.63364	0.72507	2.80903	0.54968

Table 2 The structural reliability index β versus α_3 and α_4 when ($\alpha_1 = \alpha_2 = 0.0$)

α_3 \ α_4, β	0.00	0.01		0.03		0.05	
	β	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$
0.00	1.67936	1.67936	1.67936	1.67936	1.67936	1.67936	1.67936
0.01	1.67936	1.69131	1.66757	1.69565	1.66337	1.69624	1.66281
0.02	1.67936	1.69435	1.66462	1.70794	1.65171	1.71151	1.64839
0.03	1.67936	1.69522	1.66379	1.71573	1.64450	1.72403	1.63694
0.04	1.67936	1.69556	1.66346	1.72043	1.64020	1.73367	1.62834
0.05	1.67936	1.69572	1.66330	1.72332	1.63758	1.74086	1.62205

Table 3 The structural reliability index β versus α $(\alpha=\alpha_1=\alpha_2=\alpha_3=\alpha_4)$

β \ α	0.01	0.02	0.03	0.04	0.05
$\bar{\beta}$	1.67936	1.91885	2.16178	2.40821	2.65823
$\underline{\beta}$	1.67936	1.44322	1.21038	0.98076	0.75430

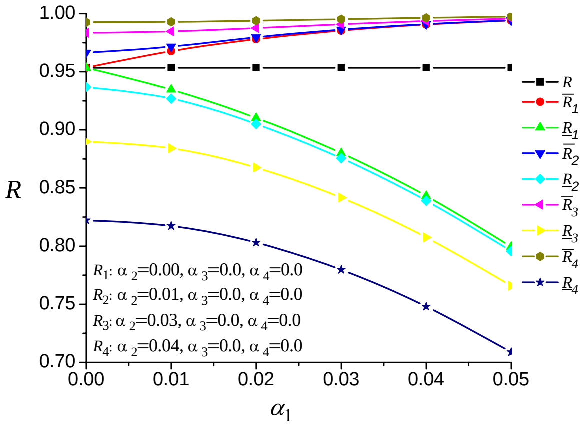

Fig.2 The structural reliability R versus $\alpha_1\,(\alpha_3=\alpha_4=0.0)$

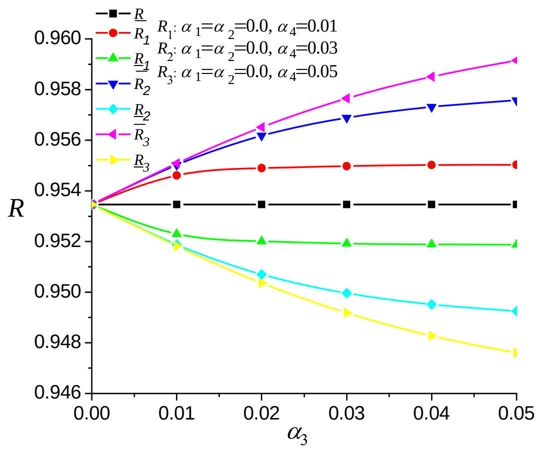

Fig.4 The structural reliability R versus $\alpha_3\,(\alpha_1=\alpha_2=0.0)$

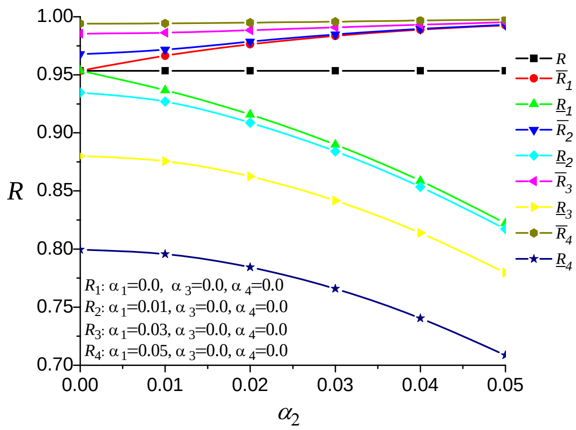

Fig.3 The structural reliability R versus $\alpha_2\,(\alpha_3=\alpha_4=0.0)$

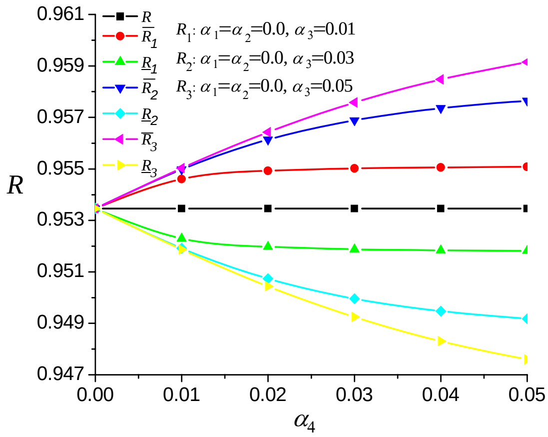

Fig.5 The structural reliability R versus $\alpha_4\,(\alpha_1=\alpha_2=0.0)$