Effective Coefficients in Anisotropic Porous Medium with Multiscale Log-Normal Conductivity

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Abstract: Effective coefficients problem is solved for the anisotropic medium. The conductivity is assumed to be a random multifractal of lognormal distribution. Dimensions of solution domain of the problem are considered to be large compared to the sizes of heterogeneities of the medium. Subgrid modeling approach, associated with problems of subsurface hydrodynamics is presented. Theoretical result is compared to the results of direct 3D numerical modeling and results of conventional perturbation theory.

Key-Words: - Random fields, effective parameters, subgrid modeling, scaling, multifractal, anisotropy

1 Introduction

In studying an anisotropic heterogeneous medium small-scale details of conductivity function are considered to be within the statistical approach. One can thus introduce effective parameters [1]. If dimensions of solution domain of the problem are a large compared to the sizes of heterogeneities of the medium, the boundary conditions influence on the effective coefficients is weak. However in this case the effective coefficients should be partially taken into account in high orders of perturbation theory in order to improve the accuracy of the derived relationships. To solve such a problem one uses renormalization group (RG) methods and subgrid modeling method. According to [2], the renormalization group methods partially take into account high orders of perturbation theory. Same arguments are also applicable to the subgrid modeling. RG methods for the filtration theory in isotropic medium are developed by many authors [3], [4], [5]. In paper [6] authors deduced the subgrid formulas for the effective permeability using the ideas of Wilson renormalization group (RG) [7] for isotropic medium. In the present study we derive the subgrid modeling formulas for solving a problem of filtration in anisotropic fractal porous medium. If a medium is assumed to satisfy the refined Kolmogorov's scaling hypothesis [8] equations of subgrid model take an especially simple form. Refined formulas of perturbation theory are in better agreement with the results of direct numerical modeling than the formulas of conventional perturbation theory.

2 Statement of the problem

Let an incompressible fluid flow through a heterogeneous medium with a conductivity coefficient $\varepsilon(\mathbf{x})$. At low Reynolds numbers filtration velocity $\mathbf{v}$ and pressure p are related by Darcy's law $\mathbf{v} = -\varepsilon(\mathbf{x})\nabla p$. Incompressibility condition $div\,\mathbf{v} = 0$ yields the equation

$$\nabla_i \varepsilon(\mathbf{x})\nabla_i p(\mathbf{x}) = 0, \; p(\mathbf{x})\big|_S = p_0(\mathbf{x}), \qquad (1)$$

where S is a boundary of the domain V. Suppose the field of conductivity is known. This assumes that it is measured at each point $\mathbf{x}$ as the fluid is pumped through a sample of small size l_0. A random function of spatial coordinates $\varepsilon(\mathbf{x})$ is considered as a limit of conductivity $\varepsilon(\mathbf{x})_{l_0}$. As $l_0 \to 0$, we have $\varepsilon(\mathbf{x})_{l_0} \to \varepsilon(\mathbf{x})$. To pass to a coarser grid l_1, one can smooth the resultant field $\varepsilon(\mathbf{x})_{l_0}$ using the scale $l_1 > l_0$. However obtained field is not the true conductivity that describes filtration in the interval of scales (l_1, L). Here L is a maximum scale of heterogeneities. To find conductivity on a coarser grid, one has to repeat the measurements, pumping the fluid through a larger sample of size l_1. This procedure is necessary since the fluctuations of conductivity within the scale interval (l_0, l_1) have are correlated to pressure fluctuations induced by them. Similar to [8], we consider a dimensionless field ψ equal to the ratio of conductivities smoothed using two different scales $\psi(\mathbf{x},l,l_1)=\varepsilon(\mathbf{x})_{l_1}/\varepsilon(\mathbf{x})_l$, where $\varepsilon(\mathbf{x})_l$ is the conductivity $\varepsilon(\mathbf{x})_{l_0}$ smoothed over scale l, $l_1<l$. The field function $\psi(\mathbf{x},l,l_1)$ has too many arguments. We define a simpler field $\varphi(\mathbf{x},l)=\partial\psi(\mathbf{x},l,l\lambda)/\partial\lambda\,|_{\lambda=1}$, $\lambda=l_1/l$, that contains the same information. Therefore we have the relation

$$\frac{\partial\ln\varepsilon(\mathbf{x})_l}{\partial\ln l}=\varphi(\mathbf{x},l). \qquad (2)$$

The solution of Eq. (2) has the form

$$\varepsilon(\mathbf{x})_{l0}=\varepsilon_0\exp\left[-\int_{l_0}^{L}\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]. \qquad (3)$$

We suppose that the conductivity has heterogeneities of the scale l_1 from the interval (l_0,L), where l_0 is minimum and L is maximum scale of dimensions $L^3\ll V$, $\varepsilon(\mathbf{x})=\varepsilon(\mathbf{x})_{l0}$. The field $\varphi(\mathbf{x},l)$ is assumed to be statistically homogeneous so a correlation function is

$$\langle\varphi(\mathbf{x},l)\varphi(\mathbf{y},l_1)\rangle-\langle\varphi(\mathbf{x},l)\rangle\langle\varphi(\mathbf{y},l_1)\rangle$$
$$=\Phi(\mathbf{x}-\mathbf{y},l,l_1)$$

Here $\langle\ \rangle$ is probability averaging. For simplicity we use the same notation Φ in the right-hand side. For example if function φ is statistically invariant to the scale transform, its correlation function is equal to $\Phi\left(\sum_i\alpha_i(x_i-y_i)^2/l^2\right)$, where α_i are constants. In this approximation fields $\varphi(\mathbf{x},l),\varphi(\mathbf{y},l_1)$ at any $\mathbf{x},\mathbf{y}$ are considered to be statistically independent. This assumption is common in the scaling models and reflects the decay of statistical dependence when the scales of fluctuations become different in magnitude. It means that $\varphi(\mathbf{x},l),\varphi(\mathbf{y},l_1)$ are delta correlated in the logarithmic scale. The latter was proposed in [8]. To describe the probability distribution for the integral from (7) for large L/l_0, we use the theorem about sums of independent variables. If the variance of $\varphi(\mathbf{x},l)$ at a given point exists, then the theorem says that the integral from (7) for very large L/l_0 tends to a normal field. In the opposite case (the second correlation function does not exist), the integral tends to a field described by a stable distribution [9]. For simplicity, it is assumed that $\varphi(\mathbf{x},l)$ has normal distribution.

3 Subgrid model

Conductivity function $\varepsilon(\mathbf{x})$ is divided into two components with respect to the scale l. The large-scale component (ongrid) $\varepsilon(\mathbf{x},l)$ is obtained by statistical averaging over all $\varphi(\mathbf{x},l_1)$ with $l_0<l_1<l$, where $dl=l-l_0$ is small. A small-scale (subgrid) component is equal to $\varepsilon'(\mathbf{x})=\varepsilon(\mathbf{x})-\varepsilon(\mathbf{x},l)$:

$$\varepsilon(\mathbf{x},l)=\varepsilon_0\exp\left[-\int_l^L\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]\left\langle\exp\left[-\int_{l_0}^{l}\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right]\right\rangle,$$

$$\varepsilon'(\mathbf{x})=\varepsilon(\mathbf{x},l)\left[\frac{\exp\left(-\int_{l_0}^{l}\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right)}{\left\langle\exp\left(-\int_{l_0}^{l}\varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right)\right\rangle}-1\right], \qquad (4)$$

A large-scale (ongrid) component of the pressure $p(\mathbf{x},l)$ is obtained as averaging solutions of Eq. (1), in which a large-scale component of conductivity is fixed and a small component $\varepsilon'(\mathbf{x})$ is a random variable. A subgrid component of the pressure is $p'(\mathbf{x})=p(\mathbf{x})-p(\mathbf{x},l)$. Substituting the expression for $\varepsilon(\mathbf{x}), p(\mathbf{x})$ in Eq.(1) and averaging over the small-scale component, we obtain:

$$\nabla_i\left[\varepsilon(\mathbf{x},l)\nabla_i p(\mathbf{x},l)+<\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})>_{\varepsilon(\mathbf{x},l)}\right]=0, \qquad (5)$$

where $\langle\ \rangle_{\varepsilon(\mathbf{x},l)}$ is averaging over $l_1<l$ when $\varepsilon(\mathbf{x},l)$ is fixed. Second term in Eq.(5) is unknown. It cannot be dropped without preliminary estimation, since the correlation between the conductivity and the pressure gradient may be substantial. The choice of the form of the second term in (5) determines the subgrid model. This expression is estimated using perturbation theory. Subtracting (5) from (1) and ignoring the terms of second order of smallness including $\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})-<\varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})>_{\varepsilon(\mathbf{x},l)}$, we obtain the subgrid equation for the pressure:

$$\Delta p'(\mathbf{x})=-\frac{1}{\varepsilon(\mathbf{x},l)}\nabla_i\varepsilon'(\mathbf{x})\nabla_i p(\mathbf{x},l). \qquad (6)$$

The variables $\varepsilon(\mathbf{x},l)$, $p(\mathbf{x},l)$ in the right-hand side of Eq.(6) are considered to be known, these variables and their derivatives varying slower than, $\varepsilon'(\mathbf{x})$ and their derivatives. Therefore,

$$p'(\mathbf{x})\approx\frac{1}{4\pi\varepsilon(\mathbf{x},l)}\int_V\frac{1}{r}\nabla_j\varepsilon'(\mathbf{x}')d\mathbf{x}\nabla_j p(\mathbf{x},l), \qquad (7)$$

where $r=|\mathbf{x}-\mathbf{x}'|$. From (7), we obtain

$$\left\langle \varepsilon'(\mathbf{x})\nabla_i p'(\mathbf{x})\right\rangle_{\sigma(\mathbf{x},l)} = -\int_V \nabla'_i \frac{1}{4\pi r}\nabla'_j \Phi(\mathbf{x}-\mathbf{x}',l)d\mathbf{x}'\varepsilon(\mathbf{x},l)\nabla_j p(\mathbf{x},l) \tag{8}$$

In a quiet deposition environment, natural sedimentary rock has a stratified structure. Here we assume that the conductivity is isotropic at any point of space, but correlation function of the conductivity fields is anisotropic. Usually natural stratums have this kind of anisotropy. The medium is assumed to be to consist of homogeneous isotropic blocks close to parallelepiped shape. The conductivity of blocks is random. If blocks are ordered enough in space then the medium is macro anisotropic. One can see as an example of stratified medium a medium with layers of the different conductivity. The scales of the conductivity are assumed to be different along the different axes. One needs to know the correlation function $\Phi(\mathbf{x}-\mathbf{x}',l)$. It is admittedly difficult to perform such measurements in unconsolidated formation since mapping $\sigma(\mathbf{x})$ on a dense grid requires drilling a large number of wells. Nevertheless, some field measurements of the conductivity reported in the literature [10], [11]. We evaluate integral (8) for the correlation function

$$\Phi_1(\mathbf{x}-\mathbf{x}',l) = \exp(-u), \tag{9}$$

where

$$u = \left(\alpha_1^2\left((x'_1-x_1)^2+(x'_2-x_2)^2\right)+\alpha_2^2(x'_3-x_3)^2\right)/l^2$$

From (4), as $\varepsilon'(\mathbf{x})$ has log-normal distribution, we have

$$\left\langle \varepsilon'(\mathbf{x})\varepsilon'(\mathbf{x}')\right\rangle \approx \varepsilon(\mathbf{x},l)^2\Phi(\mathbf{x},l)dl/l. \tag{10}$$

Using (10), obtain

$$<\varepsilon'\nabla_i p'>_{\varepsilon(\mathbf{x},l)} \approx -\Phi_0(l)\eta_1\,\varepsilon(\mathbf{x},l)\nabla_i p(\mathbf{x},l)dl/l, \tag{11}$$

where $\Phi_0(l)=\Phi(0,l)$, for $a_1<\alpha_2$, $i=1,2$

$\eta_1 = (c^2+1)/(2c^2)\left[\arctan(c)/c-1/(c^2+1)\right]$,

$c^2 = (\alpha_3^2-\alpha_1^2)/\alpha_1^2$ and for $a_1>\alpha_2$,

$$\eta_1 = \frac{c^2-1}{2c^2}\left(\frac{1}{2c}\ln\frac{1+c}{1-c}-\frac{1}{1-c^2}\right),\ c^2 = \frac{(\alpha_1^2-\alpha_2^2)}{\alpha_1^2}.$$

For $i=3$

$$<\varepsilon'\nabla_3 p'>_{\varepsilon(\mathbf{x},l)} \approx -\Phi_0(l)\eta_2\,\varepsilon(\mathbf{x},l)\nabla_3 p(\mathbf{x},l)dl/l, \tag{12}$$

where $\eta_2 = (c^2+1)\left[1-\arctan(c)/c\right]/c^2$ and for $a_1>\alpha_2$, $\eta_2 = \frac{1-c^2}{c^2}\left(\frac{1}{2c}\ln\frac{1+c}{1-c}-1\right)$. Here, the integration over the finite volume V in (8) is replaced by the integration with infinite limits, because the correlation function Φ is small outside the domain of scale L. Such a substitution gives a coarse estimation near to the boundary, but this does not affect the determined mean values, because $L^3 \ll V$. Substituting (11) in Eq. (5) in the limit $l\to l_0$, we come to the expression for the effective coefficients, which correctly describes a mean value of the filtration velocity:

$$\varepsilon(\mathbf{x})_{ef} = \varepsilon^1{}_{0l}\exp\left[-\int_l^L \varphi(\mathbf{x},l_1)\frac{dl_1}{l_1}\right],$$

where

$$\frac{d\ln\varepsilon_{0l}^i}{d\ln l} = \left(\frac{1}{2}-\eta_i\right)\Phi_0(l)-\langle\varphi\rangle, i=1,2. \tag{13}$$

If a function φ is statistically invariant to the scale transform, the solution to Eq.(13) has especially a simple form:

$$\varepsilon_{0l}^i = \varepsilon_{0L}(l/L)^{(\eta_i-1/2)\Phi_0(l)+\langle\varphi\rangle} \tag{14}$$

, where the constant ε_{0L} describes the filtration velocity for the largest scale $\langle\mathbf{v}\rangle = -\varepsilon_{0L}\nabla\langle p\rangle$. If $\alpha_2\to\alpha_1$ this result corresponds to isotropic case $\eta_1=\eta_2=1/3$ [6]. The same scales along axes x_1, x_2 in correlation function (9) are considered only to avoid evaluating the elliptic integrals, which arise in the process of integration for three different scales. For the sake of getting notions about the effect of the form of correlation function on the effective coefficients let us consider approximation of correlation function

$$\Phi_2(\mathbf{x}) = \begin{cases} 1, & |x_i|\le l/\alpha_i,\ i=1,\ldots,3 \\ 0, & |x_i|>l/\alpha_i. \end{cases} \tag{15}$$

One should apply such function to approximate the correlation function with great caution, because Fourier transform of this function takes a negative value for the some frequencies. If α_i are all equal in value then function $\Phi_2(\mathbf{x})$ is still anisotropic since a parallelepiped transforms into a cube rather than a sphere. Substituting (14) in (8), obtain for $i=1,2$

$$\eta_3 = 2\arctan\left(\alpha_1/\sqrt{2\alpha_2^2+\alpha_1^2}\right)/\pi. \tag{16}$$

For $i=3$

$$\eta_4 = 2\arctan\left(\alpha_3{}^2/\sqrt{2\alpha_2^2+\alpha_1^2}\right)/(\alpha_1\pi) \tag{17}$$

If $\alpha_1=\alpha_2$ and parameters $\eta_3=\eta_4=1/3$, which correspond to isotropic case, then $\Phi_2(\mathbf{x})$ is anisotropic function. Parameters η_i as function of α_1/α_2 are in the table 1.

Table 1

α_1/α_2	η_1/η_2	η_3/η_4
0.01	$\frac{-7.76\times10^{-3}}{-0.985}$	$\frac{-4.50\times10^{-3}}{-0.991}$
0.05	$\frac{-3.69\times10^{-2}}{-0.926}$	$\frac{-2.248\times10^{-2}}{-0.955}$
0.25	$\frac{-0.148}{-0.704}$	$\frac{-0.110}{-0.780}$
0.5	$\frac{-0.236}{-0.527}$	$\frac{-0.205}{-0.590}$
1	$\frac{-0.333}{-0.333}$	$\frac{-0.333}{-0.333}$
5	$\frac{-0.472}{-5.58\times10^{-2}}$	$\frac{-0.488}{-2.45\times10^{-2}}$
10	$\frac{-0.490}{-2.029\times10^{-2}}$	$\frac{-0.497}{-6.30\times10^{-3}}$
20	$\frac{-0.497}{-6.75\times10^{-3}}$	$\frac{-0.499}{-1.59\times10^{-3}}$

It follows from the table that the results of calculation of coefficients η_i by function $\Phi_1(\mathbf{x}-\mathbf{x}',l)$ are close to the results with approximation $\Phi_2(\mathbf{x}-\mathbf{x}',l)$ except the case when η_i are small. Therefore such approximation of the correlation function can be handled by computation of the effective coefficients.

3 Numerical modeling

For the numerical calculation we use dimensionless variables**.** The problem is solved for $\varepsilon_0=1$ in a unit cube. In a first case the pressure is set constant $p|_{y=0}=p_1,\ p|_{y=1}=p_2,\ p_1-p_2=1$ on the edges of the cube $y=0$ and $y=1$. On the opposite edges of the cube, the pressure is specified by the linear relation for y: $p=p_1+(p_2-p_1)y$. The main filtration flow is directed along Y-axis. In second case the pressure is set to a constant $p|_{z=0}=p_1,\ p|_{z=1}=p_2$ on the edges of the cube $z=0$ and $z=1$. The main filtration flow is directed along the Z-axis. The integral in (3), is replaced by a finite difference formula, in which it is convenient to pass to the logarithm with base 2:

$$\varepsilon(\mathbf{x})_{l_0}=\exp\left[-\ln 2\sum_{i=-4}^{-2}\varphi(\mathbf{x},\tau_i)\Delta\tau\right]. \quad (18)$$

For the spatial variables, we use $256\times256\times256$ grid, the scale step $\Delta\tau=1$, $\tau_i=i\Delta\tau$, $i=-4,...-2$, $l_i=2^{\tau_i}$. The delta correlation in the scale logarithm implies that the fields are generated independently at each scale τ_i. We use correlation functions:

$$\Phi_1(\mathbf{x}-\mathbf{x}',l)=\Phi_0\exp(-u)/\ln 2, \quad (19)$$

$$u=\left(\alpha_1^2\left((x'_1-x_1)^2+(x'_2-x_2)^2\right)+\alpha_2^2(x'_3-x_3)^2\right)/l^2,$$

$$\Phi_2(\mathbf{x},l)=\Phi_0\exp(-u_1)/\ln 2, \quad (20)$$

$$u_1=\left(\alpha_1^2(x'_1-x_1)^2+\alpha_2^2(x'_2-x_2)^2+\alpha_1^2(x'_3-x_3)^2\right)/l^2$$

The structure of the correlation matrix allows us to represent it in the form of a direct product of four matrices of lower dimensionality and apply the algorithm “along rows and columns” for numerical simulation [12].

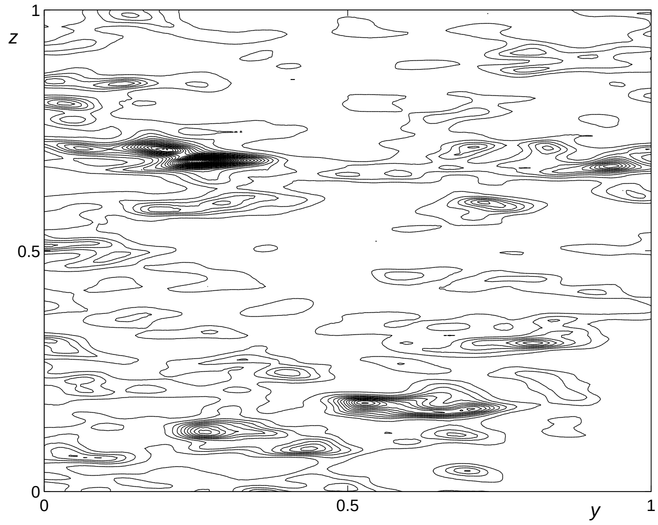

Fig.1 Conductivity isolines for three scales in the mid-span section: $\langle\varphi\rangle=0.15$, $\Phi_0=0.3$, correlation function (18), $\alpha_1=0.25$ $\alpha_2=1$.

Constants $\langle\varphi\rangle,\Phi_0$ should be chosen from experimental data for natural media [10], [11]. In Fig.1, we have self-similar conductivity in the mid-span section for formula (18). The scale of the extreme fluctuations is $L=1/4$. It is not sufficient to replace statistical averaging by only a spatial averaging. We have to add to an ensemble averaging. The filtration velocity is averaged over eighty realizations with space after-averaging. The minimum scale of fluctuations is $1/64$, which is conditioned by the requirement that considered difference problem should provide a good approximation to Eq.(1). To solve Eq.(1), an iterative method is combined with the Fourier transform and the sweep method is used [13]. According to the procedure of the derivation of the subgrid formulas, we have to solve numerically the complete problem. After that one has to perform probability averaging over small-scale fluctuations to verify the formulas. As a result, we obtain a subgrid term, which can be compared to the theoretical expression. The probability averaging requires multiple solutions of the complete problem. We performed a more efficient verification, based on the power dependence of the filtration velocity in a self-similar medium. We determine corresponding mean values using spatial averaging and statistical after-averaging to calculate same mean values using theoretical formulas. We also compare the results obtained with our theoretical formulas to the results obtained with "ordinary" perturbations theory. Effective conductivity should yield the true filtration velocity in the scale interval (l, L). Figures 2-5 show dependences of logarithm of a filtration velocity versus the number of scales k for $\Phi_0 = 0.3$, $\langle \varphi \rangle = 0.15$. Lines 1, 2 show theoretical result for isotropic and anisotropic cases; line 3 shows the results of "conventional" perturbation theory; the results of numerical modeling are marked with circle and asterisk for isotropic and anisotropic cases correspondingly.

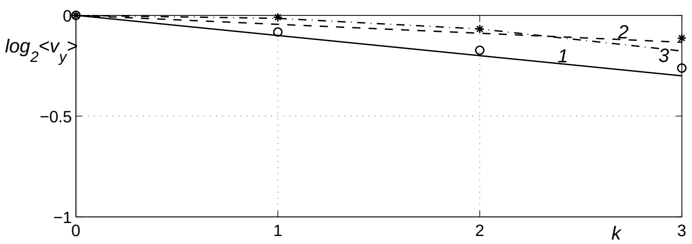

Fig.2 Main filtration flow is directed along Y-axis, correlation function (19); $\alpha_1 = 0.25$ $\alpha_2 = 1$.

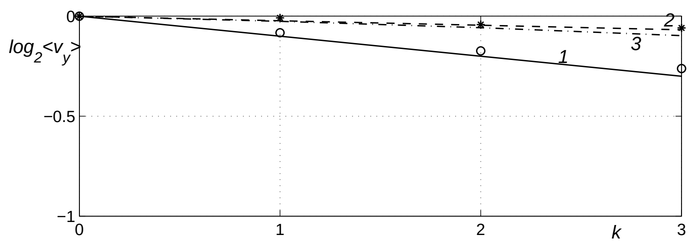

Fig.3 Main filtration flow is directed along Y-axis, correlation function (20), $\alpha_1 = 1$ $\alpha_2 = 0.25$.

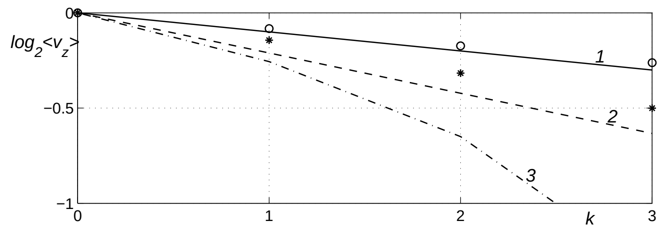

Fig.4 Main filtration flow is directed along Z-axis, correlation function (19), $\alpha_1 = 0.25$ $\alpha_2 = 1$;

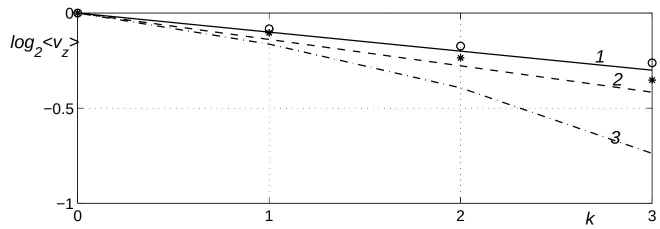

Fig.5 The main filtration flow is directed along Z-axis, correlation function (20), $\alpha_1 = 1$ $\alpha_2 = 0.25$.

In figure 1 the flow is directed along Y-axis, the scales of the fluctuations in the X, Y- axes are greater then scales in Z-axis. Coefficient η_1 is small. Mean value of filtration velocity is obtained by ordinary perturbation theory (line 3) is almost equal to the mean value of velocity obtained by formulas (14) and results of the numerical modeling. In figure 2 the scales of the fluctuations in the Y-axis bigger then scales in X, Z- axes, $a_1 > \alpha_2$.The mean value of filtration velocity is approximately equal to the of velocity in homogeneous medium with unit conductivity. In figures 4-5 the flow is directed along Z-axis, coefficient η_2 is large. The mean value of filtration velocity obtained by our approach agrees better with the results of numerical modeling than the mean value of velocity obtained by the conventional perturbation theory. The mean value of filtration velocity is less than mean value of velocity in isotropic medium. If the scale of the fluctuations in the Y-axis is bigger then scales in X, Z- axes, $a_1 > \alpha_2$ (figure 5) the value of the filtration velocity is close to the mean value of velocity in isotropic medium (line 1).

5 Conclusions

We have obtained the formulas taking into account the contribution of small-scale components to the calculation of mean characteristics of the filtration velocity in anisotropic media. The conductivity was simulated as an extremely heterogeneous field close to multifratal distribution. The later attained if the scale l_0 to in formula (3) tends to zero. Numerical verification is carried out for a medium, in which conductivity possesses the self-similarity property. The power dependences on the scale for the effective conductivity have been calculated. The effective coefficients for anisotropic case, distinct from coefficients for isotropic case, depend on the shape of correlation function Φ. However it was shown that such relationship is weak (see table 1). We can replace the correlation function with approximation (15). It is shown that the subgrid modeling makes possible even with large variance of conductivity to obtain good results. In the approach used, analysis is not beyond the scope of the differential equations apparatus and the theory of random fields. The subjects of investigations are parameter mean values and correlation functions which can be measured.

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