The Interaction of the Semi-Cylindrical Canyon and the Crack for incident SH Wave

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Abstract: - In this paper, the method of Green's function is used to investigate the problem of dynamic stress concentration factor (DSCF) of the semi-cylindrical canyon, the ground motion of the horizontal surface for incident SH wave. The train of thought for this problem is that: Firstly, a Green's function is constructed for the problem, which is a basic solution of displacement field for an elastic half space containing a semi-cylindrical canyon impacted by anti-plane harmonic linear source force at any point of the elastic-space. In terms of the solution of SH-wave's scattering by an elastic half-space with a semi-cylindrical canyon, anti-plane stresses which are the same in quantity but opposite in direction to those mentioned before, are loaded at the region where the crack existent actually, we called this process "crack-division". Finally, the expressions of the displacement and stresses are given when the semi-cylindrical canyon and the crack exist at the same time. Then, using the expressions, we discuss the influence of the ground motion, the DSCF of the semi-cylindrical canyon according to the existent crack and semi-cylindrical canyon .

Key-Words: - Semi-cylindrical canyon; Crack; Green's function; DSCF; Ground motion ; SH-wave ;

1 Introduction

The problem of scattering of SH waves by a semi-cylindrical canyon and other irregular topography is one of the important and interesting questions in earthquake engineering for the latest decades. There are lots of materials obtained by theoretical research and earthquake damage investigation. This problems are complicated, because there are many factors influenced. Researchers solved these problems by analysis and numerical methods. It is hard to obtain analytic solutions under the conditions of the common topography, except for several simple conditions.

Trifunac(1973)[1],Wong and Trifunac(1974) [2] solved the problem of SH wave scattering by a semi-cylindrical and a elliptical canyon using wave function method. Liu Diankui and Han Feng[3-4] solved the problem of scattering of SH waves by canyon topography of arbitrary shape in isotropic and anisotropic media using complex function method. Near the semi-cylindrical canyon, there could be cracks exist. This problem hasn't been studied so far. In the Structural engineering, when we extract oil and coal from a basin, below it there may be a fault. Then the problem of the interaction of the semi-cylindrical canyon and the crack is unavoidable. So we must study this problem conscientiously.

2 Model and Governing equation

The model is shown as Fig.1, an elastic half-space containing a semi-cylindrical canyon and a linear crack.

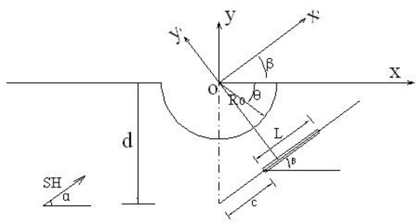


Fig.1 The Model of this Problem

In this paper, the easiest model for the scattering of SH wave: the anti-plane shear SH wave model is studied. The displacement is expressed as $W(x,y,t)$ and the displacement function W satisfies the following governing equation:

$$\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+k^2 W=0 \tag{1}$$

where $k=\frac{\omega}{C_S}$, $C_S=\sqrt{\mu/\rho}$, ω is the circular frequency of the displacement $W(x,y,t)$, C_s stands for the shear wave velocity, ρ and μ are the mass density and the shear modulus of elasticity respectively.

The governing equation of W can also be written in the polar coordinate system as

$$\frac{\partial^2 W}{\partial r^2}+\frac{1}{r}\frac{\partial W}{\partial r}+\frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}+k^2 W=0 \tag{2}$$

The corresponding stresses are given by:

$$\tau_{rz}=\mu\frac{\partial W}{\partial r} \qquad \tau_{\theta z}=\frac{\mu}{r}\frac{\partial W}{\partial \theta} \tag{3}$$

3 Green's function

3.1 The Governing Equation and the Boundary Condition

The Green's function used in this paper is regarded as the displacement response to the elastic half space containing a semi-cylindrical canyon impacted by anti-plane harmonic linear source force at any point of the elastic plane (Fig.2).

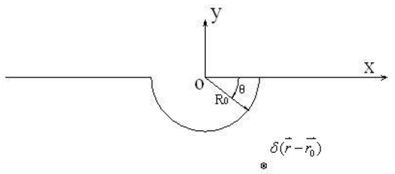


Fig.2 Model to derivate Green Function

In a polar coordinate system, the governing equation of G is following:

$$\frac{\partial^2 G}{\partial r^2}+\frac{1}{r}\frac{\partial G}{\partial r}+\frac{1}{r^2}\frac{\partial^2 G}{\partial \theta^2}+k^2 G=\delta(\vec{r}-\vec{r_0}) \tag{4}$$

$\vec{r_0}$: stands for the position of the linear source force in polar coordinates, $\vec{r_0}=r_0\cdot e^{i\theta_0}$

$\vec{r}$: stands for the position of the arbitrary image in polar coordinates, $\vec{r}=r\cdot e^{i\theta}$

The boundary conditions can be expressed as below:

$$\begin{aligned} \tau_{rz}&=0 \qquad at \quad r=R_0 \\ \tau_{\theta z}&=0 \qquad at \quad \theta=0,\pi \end{aligned} \tag{5}$$

3.2 Derivation of Green's function

The basic solution which satisfies the control equation (4) and the boundary conditions (5) should include two parts of motion: the disturbance of anti-plane linear source force and the scattering wave incited by the semi-cylindrical canyon. The wave displacement of the complete elastic space due to the line source load $\delta(\vec{r}-\vec{r}_0)$ on the arbitrary position of the plane can be given:

$$G^{(i)}=\frac{i}{4\mu}H_0^{(1)}(k\,|\,\vec{r}-\vec{r_0}\,|) \tag{6}$$

Where $H_0^{(1)}(*)$ is the first kind of Hankel function and zero-order. According to addition theorem of Bessel function, Eq. (6)can be written as[5]:

$$G^{(i)}=\frac{i}{4\mu}\sum_{m=0}^{\infty}\varepsilon_m\cos[m(\theta-\theta_0)]\begin{cases}J_m(kr_0)H_m^{(1)}(kr), r>r_0\\ J_m(kr)H_m^{(1)}(kr_0), r<r_0\end{cases} \tag{7}$$

when $m=0$, $\varepsilon_m=1$; $m\geq 1$, $\varepsilon_m=2$.

According to the "symmetry theory"[6], the reflected wave by horizontal interface can be written as:

$$G^{(r)}=\frac{i}{4\mu}H_0^{(1)}(k\,|\,\vec{r}-\overline{\overline{r_0}}\,|) \tag{8}$$

where $\overline{\overline{r_0}}$ stands for the conjugate of the linear source force $\vec{r_0}$ in polar coordinates, and $\overline{\overline{r_0}}=r_0\cdot e^{-i\theta_0}$.

According to addition theorem of Bessel function, Eq. (8) can be written as:

$$G^{(r)}=\frac{i}{4\mu}\sum_{m=0}^{\infty}\varepsilon_m\cos[m(\theta+\theta_0)]\begin{cases}J_m(kr_0)H_m^{(1)}(kr), r>r_0\\ J_m(kr)H_m^{(1)}(kr_0), r<r_0\end{cases} \tag{9}$$

The scattering wave incited by the semi-cylindrical canyon can be written as:

$$G^{(is)}=\sum_{m=0}^{\infty}A_m\cos[m(\theta-\theta_0)]\cdot H_m^{(1)}(kr) \tag{10}$$

$$G^{(rs)}=\sum_{m=0}^{\infty}B_m\cos[m(\theta+\theta_0)]\cdot H_m^{(1)}(kr) \tag{11}$$

where A_m, B_m are unknown coefficients.

Therefore, the total wave field can be written as:

$$G=G^{i}+G^{r}+G^{is}+G^{rs} \tag{12}$$

In order to satisfy the stress free condition on the surface semi-canyon, the total wave field G must satisfy following equation:

$$\mu\cdot\left.\frac{\partial G}{\partial r}\right|_{r=R_0}=0 \tag{13}$$

Because of the $r_0>r=R_0$ in the model of this paper ,That is:

$$\begin{aligned}&\frac{i}{4\mu}\sum_{m=0}^{\infty}\varepsilon_m\cos[m(\theta-\theta_0)]J_m'(kR_0)H_m^{(1)}(kr_0)\\&+\frac{i}{4\mu}\sum_{m=0}^{\infty}\varepsilon_m\cos[m(\theta+\theta_0)]J_m'(kR_0)H_m^{(1)}(kr_0)\\&+\sum_{m=0}^{\infty}A_m\cos[m(\theta-\theta_0)]H_m^{(1)'}(kR_0)\\&+\sum_{m=0}^{\infty}B_m\cos[m(\theta+\theta_0)]H_m^{(1)'}(kR_0)=0\end{aligned} \tag{14}$$

According to Eq. (14), we can obtain:

$$\begin{aligned}A_m&=-\frac{i}{4\mu}\varepsilon_m\frac{J_m'(kR_0)H_m^{(1)}(kr_0)}{H_m^{(1)'}(kR_0)}\\ B_m&=-\frac{i}{4\mu}\varepsilon_m\frac{J_m'(kR_0)H_m^{(1)}(kr_0)}{H_m^{(1)'}(kR_0)}\end{aligned} \tag{15}$$

Substituting Eq. (15) to Eqs. (10) and (11), and substituting Eqs. (7)□(9)□(10) and (11) to Eq. (12), the total wave field G of this problem can be obtained:

$$G(r,r_0,\theta,\theta_0)=\frac{i}{4\mu}\sum_{m=0}^{\infty}\varepsilon_m\{\cos[m(\theta-\theta_0)]$$

$$+\cos[m(\theta+\theta_0)]\}\begin{cases}J_m(kr_0)H_m^{(1)}(kr), r>r_0\\ J_m(kr)H_m^{(1)}(kr_0), r<r_0\end{cases}$$

$$-\sum_{m=0}^{\infty}\frac{i}{2\mu}\varepsilon_m\cos m\theta\cos m\theta_0\frac{J'_m(kR_0)H_m^{(1)}(kr_0)}{H_m^{(1)'}(kR_0)}H_m^{(1)}(kr) \quad (16)$$

Using the addition formula oppositely, we can obtain:

$$G(r,r_0,\theta,\theta_0)=\frac{i}{4\mu}[H_0^{(1)}(k|\bar{r}-\bar{r}_0|)+H_0^{(1)}(k|\bar{r}-\bar{\bar{r}}_0|)]$$

$$-\sum_{m=0}^{\infty}\frac{i}{2\mu}\varepsilon_m\cos m\theta\cos m\theta_0\frac{J'_m(kR_0)H_m^{(1)}(kr_0)}{H_m^{(1)'}(kR_0)}H_m^{(1)}(kr) \quad (17)$$

4 Expression of displacement and stress for the model

The stress on the crack produced by incident SH-wave and the scattering wave incited by the semi-cylindrical canyon can be obtained. A pair of opposite forces is applied to the crack; therefore the resultant force on the crack is zero, which can be thought as crack (showed as Fig.3).

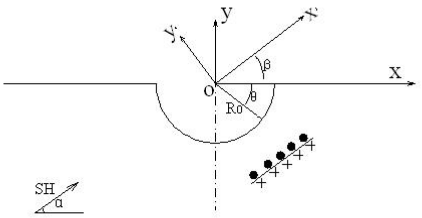


Fig.3 Model of Crack Division

The above constructing process called crack-division technique [7] which can be used to obtain the expression of displacement and stress for the model. The details will be discussed as follows.

4.1 The displacement and stress produced by the semi-cylindrical canyon's scattering

Firstly, we consider the incidence of SH-wave on the infinite linear-elastic half-space containing a semi-cylindrical canyon[8] , the model is showed as Fig 4.

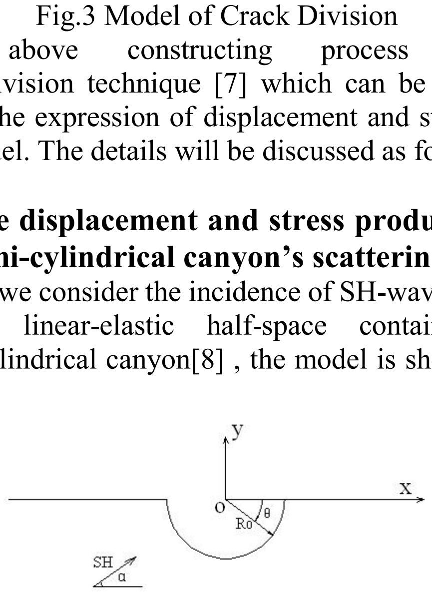


Fig.4 The Scattering of a half-space with semi-cylindrical canyon

The incident displacement field $W^{(i)}$ harmonic to time can be written as follows:

$$W^{(i)}=W_0\sum_{n=0}^{\infty}\varepsilon_n i^n\cos[n(\theta-\alpha)]\cdot J_n(kr) \quad (18)$$

where α is the incident angle, $k=\frac{\omega}{C_S}$ is the shear wave number of the media, $C_S=\sqrt{\mu/\rho}$ is the shear wave velocity of the media.

The reflected wave by horizontal interface can be written as:

$$W^{(r)}=W_0\sum_{n=0}^{\infty}\varepsilon_n i^n\cos[n(\theta+\alpha)]\cdot J_n(kr) \quad (19)$$

The scattering wave incited by the semi-cylindrical canyon can be written as:

$$W^{(is)}=W_0\sum_{n=0}^{\infty}A_n\cdot H_n^{(1)}(kr)\cdot\cos[n(\theta-\alpha)] \quad (20)$$

$$W^{(rs)}=W_0\sum_{n=0}^{\infty}B_n\cdot H_n^{(1)}(kr)\cdot\cos[n(\theta+\alpha)] \quad (21)$$

where A_n , B_n are unknown coefficients.

Using the stress free condition on the surface semi-canyon , we can obtain:

$$\mu\cdot\left.\frac{\partial(W^{(i)}+W^{(r)}+W^{(is)}+W^{(rs)})}{\partial r}\right|_{r=R_0}=0 \quad (22)$$

According to Eq. (22), we can get the solution of the coefficients A_n , B_n :

$$A_n=-\varepsilon_n i^n\frac{J'_n(kR_0)}{H_n^{(1)'}(kR_0)}, B_n=-\varepsilon_n i^n\frac{J'_n(kR_0)}{H_n^{(1)'}(kR_0)} \quad (23)$$

Therefore, the total wave field can be written as:

$$W^{(t)}=W^{(i)}+W^{(r)}+W^{(is)}+W^{(rs)}$$

$$=2W_0\sum_{n=0}^{\infty}\varepsilon_n i^n\left[J_n(kr)-\frac{J'_n(kR_0)}{H_n^{(1)'}(kR_0)}H_n^{(1)}(kr)\right]\cos n\theta\cos n\alpha \quad (24)$$

The corresponding stresses are given by:

$$\tau_{rz}^{(t)}=2\mu W_0k\sum_{n=0}^{\infty}\varepsilon_n i^n[J_n{}'(kr)-\frac{J'_n(kR_0)}{H_n^{(1)'}(kR_0)}H_n^{(1)'}(kr)]\cos n\theta\cos n\alpha \quad (25)$$

$$\tau_{\theta z}^{(t)}=\frac{-2\mu W_0}{r}\sum_{n=0}^{\infty}n\cdot\varepsilon_n\cdot i^n[J_n(kr)-\frac{J'_n(kR_0)}{H_n^{(1)'}(kR_0)}H_n^{(1)}(kr)]\sin n\theta\cos n\alpha \quad (26)$$

When we research the scattering of the semi-cylindrical canyon, the stress field near the surface of the canyon was:

$$\tau_{\theta z}|_{\bar{r}=R_0}=\frac{\mu}{r}\frac{\partial W}{\partial\theta} \quad (27)$$

The maximum stress amplitude in the incident direction generated by the time-harmonic incident wave was at an angle of α is:

$$\tau_0 = \left|\tau_{rz}^{(i)}\right| = \mu W_0 k \tag{28}$$

So we make a definition and conclude the dynamic stress concentration factors (DSCF) around the semi-canyon was that:

$$\tau_1 = \left|\frac{\tau_{\theta z}}{\tau_0}\right| \tag{29}$$

4.2 The displacement and stress produced by the crack and semi–-cylindrical canyon

Now, we consider the scattering problem of incident SH-wave when the semi-cylindrical canyon and crack exist at the same time. According to above incident field and scattering field in the elastic half-space containing only a semi-cylindrical canyon, the crack-division technique is used to construct the model of SH-wave scattering by an elastic half-space containing a semi-cylindrical canyon and a linear crack..

For the example of vertical crack condition (showed as Fig.5 (a)). The constructing process is that: The space is separated along the crack and a pair of anti-plane opposite forces with the multitude $[-\tau_{\theta z}^{(t)}\big|_{\theta=-\frac{\pi}{2}}]$(showed as Eq. (26)) are applied to up and down section of the region where cracks will appear, therefore the resultant force on up(or down) section of those regions were zero which can be thought as crack.

The force $-\tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0}$ is applied on the crack and the addition displacement field can be obtained:

$$-\tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0} \times G(r, r_0, \theta, \theta_0) \tag{30}$$

Integrating along the line of crack, we can obtain:

$$-\int_{\bar{r}_1}^{\bar{r}_2} \tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0} \times G(r, r_0, \theta, \theta_0) d\bar{r}_0 \tag{31}$$

Hence, the total displacement field can be written as follows:

$$W = W^{(t)} - \int_{\bar{r}_1}^{\bar{r}_2} \tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0} \times G(r, r_0, \theta, \theta_0) d\bar{r}_0 \tag{32}$$

And the total stress field at the tip of the crack was that:

$$\tau_{rz}\big|_{\bar{r}=\bar{r}_1} = \tau_{rz}^{(t)}\Big|_{\bar{r}=\bar{r}_1} - \int_{\bar{r}_1}^{\bar{r}_2} \tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0} \times \mu \frac{\partial G}{\partial r}\bigg|_{\bar{r}=\bar{r}_1} d\bar{r}_0 \tag{33}$$

The stress field around the canyon was:

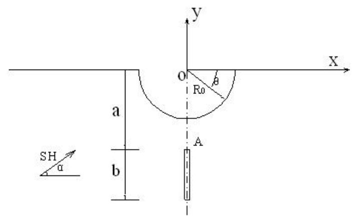
$$\tau_{\theta z}\big|_{\bar{r}=R_0} = \tau_{\theta z}^{(t)}\Big|_{\bar{r}=R_0} - \int_{\bar{r}_1}^{\bar{r}_2} \tau_{\theta z}^{(t)}|_{\bar{r}=\bar{r}_0} \times \frac{\mu}{r}\frac{\partial G}{\partial \theta}\bigg|_{\bar{r}=R_0} d\bar{r}_0 \tag{34}$$

It's the same with the condition of no-crack, we define the DSCF for this model was:

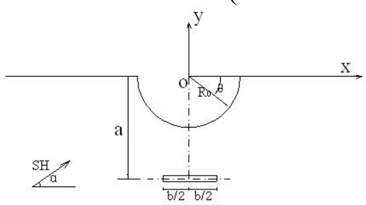
$$\tau_2 = \left|\frac{\tau_{\theta z}|_{\bar{r}=R_0}}{\tau_0}\right| \tag{35}$$

5 Example

In this paper, we pay attention to three representative kinds of models, which are shown as Fig. 5.



(a) Model of Case one (vertical crack)



(b)Model of Case two (horizontal crack)

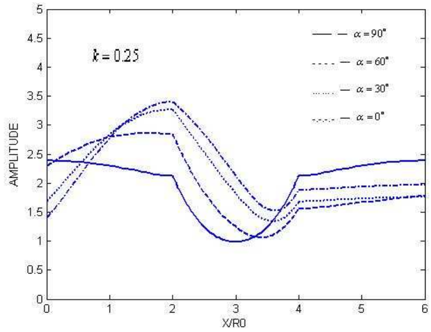
Fig.5 Model of Example

6 Results and Analysis

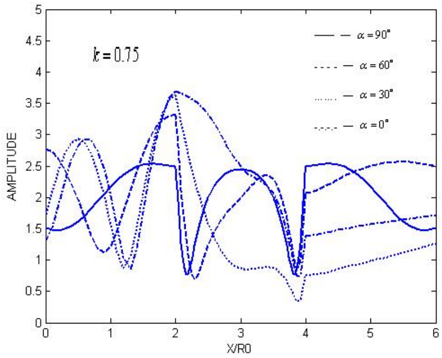
From the earthquake engineering, structural analysis, and design to strong-motion seismology, an important aspect of the above analysis is to describe and analyze the displacement amplitudes and the relative phases of motions at points on the surface. This information can then be used to understand and interpret the effects of the surface and subsurface topographic features similar to the model studied here.

Firstly, we make the radius of the semi-cylindrical as 1 and the length of crack tend to be zero. This condition correspond to the case of a half elastic space with a semi-cylindrical canyon but no crack. The incident angle are 90^0 □ 60^0 □ 30^0 and 0^0. Fig.6 interpret the displacement amplitude of the surface, it can be seen that they are consistent to the classical results[1] of the scattering for a half space with a semi-cylindrical canyon.

1□For the condition of vertical crack (Fig.5(a)) . The ground motion of the horizontal surface will be changed greatly if there is crack exists under the same conditions comparing with Fig.7. Here, we consider the distance between the semi-cylindrical canyon and the crack $a = 2$,the length of the crack $b = 2$. We can conclude that : the existence of the crack take the resisting effect to ground motion. Therefore, the surface motions in the semi-cylindrical canyon are weakened, but the surface motions in other region are intensified.

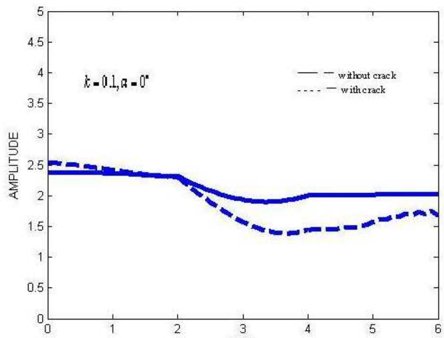


(a) $k = 0.25$

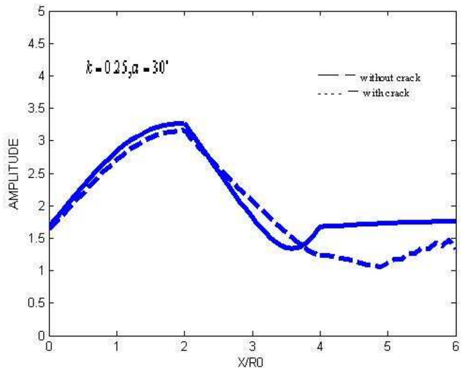


(b) $k = 0.75$

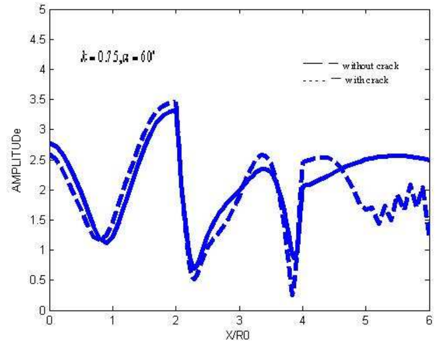
Fig.6 Surface amplitude due to the length of crack tend to be zero



(a) $k = 0.1$



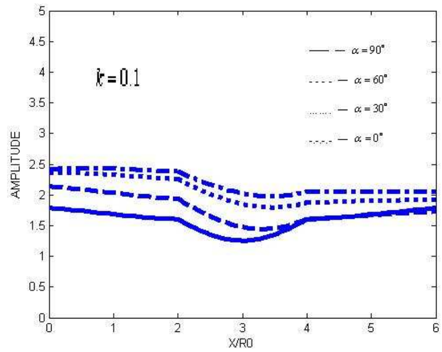
(b) $k = 0.25$



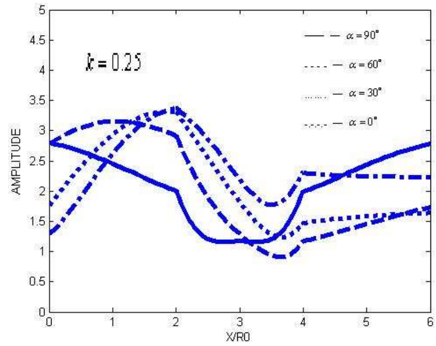
(c) $k = 0.75$

Fig.7 The amplitude comparing of crack and no-crack

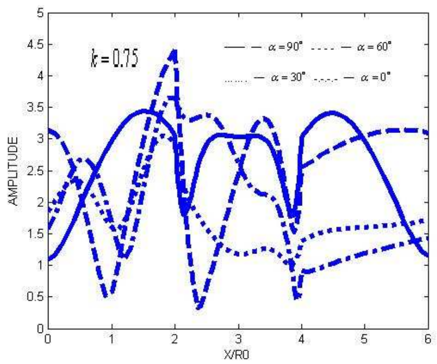
2 □ Consider the condition of Fig.5(b). a was the distance from ground surface to the center of the crack, b was the length of the crack .we take $a = b = 2$ and the incline angle of the crack horizontal crack. We gave the solution of being impacted by SH-wave with different wave numbers $k = 0.1,\ 0.25,\ 0.75$, different incident angles $\alpha = 90^0,\ 60^0,\ 30^0,\ 0^0$,showed in Fig.8. As you see , the existence of the crack also give the influence to the surface motions near the semi-cylindrical canyon.



(a) $k = 0.1$



(b) $k = 0.25$



(c) $k = 0.75$

Fig.8 Surface displacement under the condition of horizontal crack

Secondly, the DSCF of the semi-cylindrical canyon is studied:

3）For the condition of vertical crack.. Either the incident SH-wave impact crack vertically or aslant, the existence of the crack will influence the distribution of the stress field around the semi-cylindrical canyon greatly. For the incident wave number $k = 0.1$, 0.5, 1.0, 2.0, the existence of the crack enhance the DSCF around the canyon $20\% - 50\%$,showed as Fig 9.

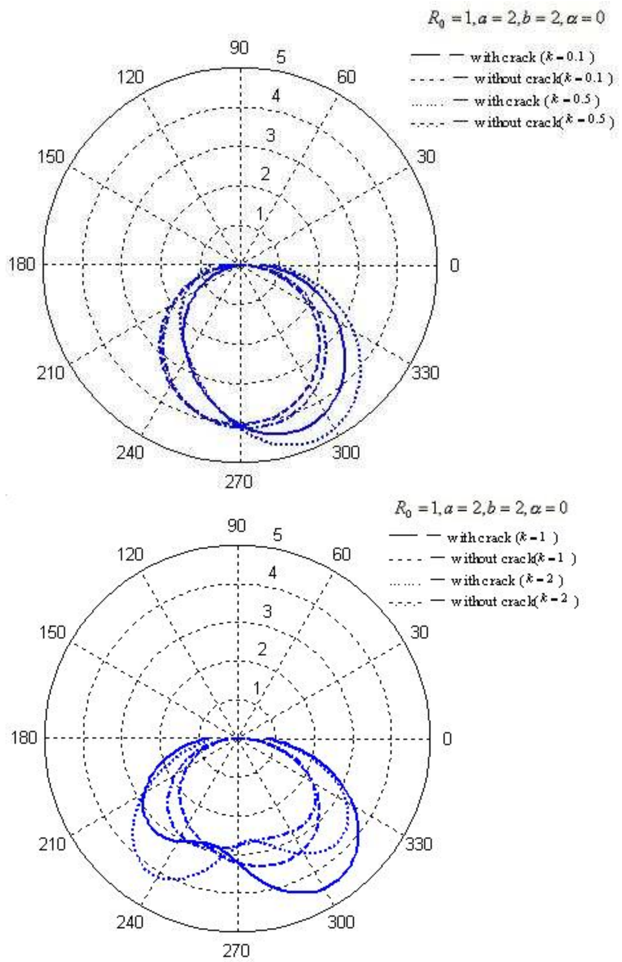


Fig.9 Variation of DSCF in the circle edge vs. k ($\alpha = 0^0$)

7 Summary

In this paper, by using the technique of crack-division, a new method is given to solve the problem of dynamic stress concentration factor (DSCF) of the semi-cylindrical canyon, the ground motion of the horizontal surface for incident SH wave. By using the method an example is solved, and some new conclusion is given. The method in the paper could be used to study some other correlative problem.

Acknowledgment

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