Optimum Design of Partial Double-layer Reticulated Shells

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Abstract: - The determination of optimum design variables of partial double-layer reticulated shells is a problem of multistage optimization with multi-variables, complex load combinations, and complex constraint conditions. The variables have different physical property or different quantitative attribute. Moreover, the variables interact. In the paper, a kind of uniform design method is proposed to make a quick optimization. It is a cascade cycle approach. At each updating step, a small number of new experimental points are arranged in the vicinity of the latest optimum solution to better the results. New regression equations are fitted by all of historical data. Design verifications demonstrate the reliability and efficacy of the improvement iteration method.

Key-Words: - Structural optimization, response surfaces, uniform design, partial double-layer reticulated shells

1 Introduction

The configuration of partial double-layer reticulated shells is based on the theory of uniform strength shells. In a partial double-layer reticulated shell, the single-layer area corresponds to the thin part of uniform strength shell. The double-layer area with equal curvature of its top and bottom chord is the mapping of thick part. Between the single-layer and double-layer areas, stiffness transition region is established. The orderly configuration of partial double-layer reticulated shells provides positive aesthetic quality of indoor.

Optimization of partial double-layer reticulated shells is one of the chief problems that affect their engineering application. The determination of optimum combination of variable levels of the structures is a problem of multistage optimization with multi-variables, complex load combinations, and complex constraint conditions. They are subjected to stress, local buckling, overall buckling, frequency, displacement constraints, and etc. The variables have different physical property or different quantitative attribute. Moreover, the variables interact.

Due to the complexity of the problem, current approaches and software packages can not efficiently meet the need of the optimization of space trusses, especially partial double-layer reticulated shells.

Sake et al [1] employed optimality criteria approach and nonlinear finite element method for the optimization of elastic framed domes. Gil et al [2] proposed a methodology combining a full stressed design optimization with a conjugates gradient optimization for the identification of the optimum shape and cross-sections of a plane truss structure. Wang et al [3] presented an evolutionary optimization method for weight minimum problem of a space truss structure, where nodal coordinates and element cross-sectional areas are optimized by using full stressed design and evolutionary node shift method.

Mathematical programming techniques, such as linear programming, nonlinear programming, geometric programming, dynamic programming, stochastic programming, multi-objective programming, network methods, game theory, simulated annealing, genetic algorithms, and neural networks, are useful in finding the minimum of a function of several variables under a prescribed set of constraints. Stochastic process techniques, such as statistical decision theory, Markov processes, queuing theory, renewal methods, simulation methods, and reliability theory, can be used to analyze problems described by a set of random variables having known probability distributions. Statistical methods, such as regression analysis, cluster analysis, experimental design, and discriminate analysis, enable one to analyze the experimental data and build empirical models to obtain the most accurate representation of the physical situation.

It draws attention to obtaining optimum variables simply, quickly and efficiently for design of partial double-layer reticulated shells. In the paper, cascaded uniform design is proposed to make a quick optimization.

2 Presentation of Cascaded Uniform Design

Structural optimization can be divided into sizing, shape, topology, and structural type optimization. The benefits increase with the enhancement of optimization stage. Due to the different physical property or different quantitative attribute of design variables, and the interaction of variables, the degree of difficulty of multistage optimization is relatively greater.

One of the aims of experimental design is to find the best combination of variables under certain constraint conditions. The methods have not demands for variables with identical physical property or quantitative attribute. The interactions among variables can also be considered.

Uniform design, one of experimental design methods, was put forward by Fang [4]. The method is suitable for the design of experiments with multi-variables. By regression to the experimental data, the internal relationship between objective values and variables, as well as the approximate optimum results, can be obtained. It is easy for uniform design method to make a quick optimization.

Many software packages for spatial structures have the function of full stressed optimization. PLAScad, developed by us, is one of such software packages. PLAScad has many inline procedures, such as modeling, structural static analysis, cross-section full stressed optimization, modal analysis, and nonlinear stability analysis. Currently, professional software packages can not undertake the optimization that meets the requirements of strength, stiffness, frequency and stability at the same time.

For a uniform design with s variables, q levers for each variable the observation points are (Fang [4]):

$$P_n(k) = (ka_1, ka_2, \cdots, ka_s)(\bmod q) \qquad (1)$$

where k=1, 2,…, q and a_1, a_2, …, a_s are natural numbers. If q is a prime number, the observation points are:

$$P_q(k) = (k, ka, ka^2, \cdots, ka^{s-1})(\bmod q) \qquad (2)$$

where a is a natural number.

Usually, we use only one uniform design table for structural optimization. Since the observation points are uniformly distributed in design space of wide range, the response surface of local area may disagree with reality. Some researchers (Xiao et al [5]) regarded the results with uncertainty for optimization of spatial structures with overall buckling constraint.

Besides uniform design, Xiao et al [5] used rotational second order design to improve the precision of optimization within the local range of the approximate results. Here further efforts are exerted to approach to the real response surface by using cascade cycle. At each updating step, a small number of new experimental points are arranged in the vicinity of the latest optimum solution to better the results. New regression equations are fitted by all of historical data.

3 Description and Process

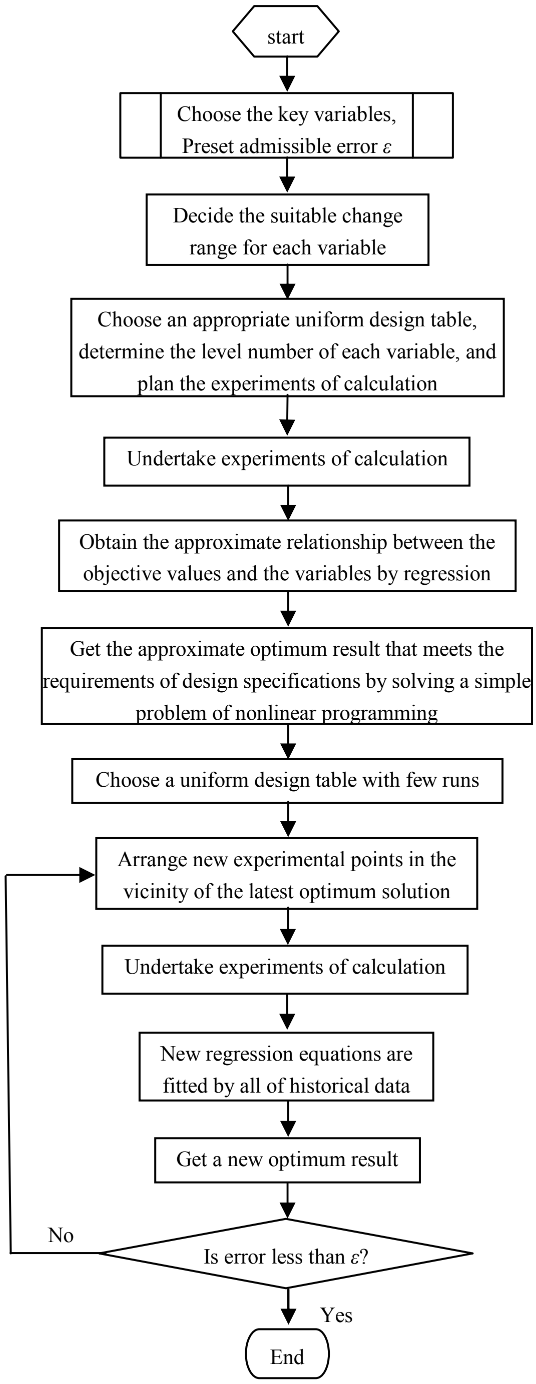


Fig. 1 Process of analysis

Partial double-layer reticulated shells are economical types of spatial structures, thus structural type optimization is not considered.

In the analysis, full stressed optimization and objective values is achieved by professional design software packages for spatial structures. Regression analysis uses MS-Excel. Nonlinear programming problem is solved by Matlab.

A great deal of modeling is induced in the experiments of calculation. Professional software packages build FEM models at a fast rate by utilizing the constructional rule of grids, changing initial parameters, or adding/erasing members.

Structural optimization is the act of obtaining the least cost of a structure under various constraints, such as strength, stiffness, frequency, stability, and etc. The results that are obtained by full stressed optimization usually meet the requirements of strength and stability of members. Perhaps other constraints, such as stiffness, frequency, and total stability, are ignored. In this instance, cost, displacements, ultimate load, basic frequency, and other objective values should be calculated. Referenced structures use the results of full-stressed optimization.

Function analogues are adopted for discrete properties of members and joints with the least square approximation.

The process of analysis is given in Fig. 1.

4 A Reticulated Shell with Projected Plane of Equilateral Hexagonal

4.1 Description of Problem

For totally double-layered reticulated shells, structural design is mainly dominated by strength. However, in the design of totally single-layered reticulated shells, stability is the main dominated condition.

For partial double-layer reticulated shells, the single-layer area and material design stress ratio for single-layer members significantly influence the objective values. To simplify the analysis, the single-layer members use the same material design stress ratio ρ in the process of full-stressed optimization. While, the material design stresses for double-layer members do not discount.

Example 1 is a reticulated shell with a projected plane of equilateral hexagonal. Its 6 pin-joint supports are 10 m above the ground. The length of one side of the projected plane is 40 m. The maximum diagonal length of the projected plane L is 80 m. The equation of top-chord surface of the reticulated shell is:

$$z = -\frac{x^2 + y^2}{a} \tag{3}$$

where z is the vertical coordinate value. The origin of the space rectangular coordinates is at the centre of the top-chord surface.

The grids of reticulated shell are in three-way. One side of the projected plane is divided into 12 segments. The thickness of the double-layer region is 2.5m.

Assume that that supporting structures are identical and the supporting structures satisfy the design requirements of all calculation models. Thus, we substitute the cost of a reticulated shell for the engineering cost. Considering steel consumption is the dominating factor in the cost of a reticulated shell, use steel consumption as the index of construction cost.

Design loads are: (i) deck load 0.2kN/m^2, (ii) live load 0.5kN/m^2, and (iii) basic wind pressure 0.35kN/m^2. The weight of structure is automatically calculated by PLAScad. Uniformed live load is exerted vertically on the top-chord surface of the shell, and wind load being perpendicular to the top-chord tangent surface. Loads are equivalently acted on the nodes. Ignore the effects of snow, seeper and earthquake. Load combinations include: (i) 1.2 Dead Load, (ii) 1.2 Dead Load + 1.4 Live Load (global), (iii) 1.2 Dead Load + 1.4 Live Load (half span) (iv) 1.2 Dead Load +1.4 Wind Load, (v) 1.2 Dead Load + 1.4 Live Load (global) +0.8 Wind Load. Members use round hollow steel. Nodes are welded hollow spherical balls.

4.2 Possible Positions of Single-layer Regions

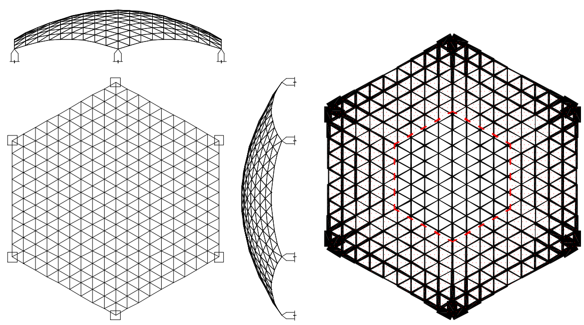


Fig. 2 Possible position of single-layer region

The possible positions of single-layer regions R can be determined according to the stress distribution of the corresponding continuum shell or the corresponding single-layer reticulated shell. Fig. 2 shows the stress distribution detail under load combinations. In the figure, line thickness represents the magnitude of member design stress. The regions of less stress, where are the possible positions of single-layer regions, are enclosed by a broken line.

4.3 Optimization Design and Results

Design variables include ρ, R, and a. For convenience, R is represented by the number of members on the outskirts of single-layer k. Fig. 3 is a calculation model.

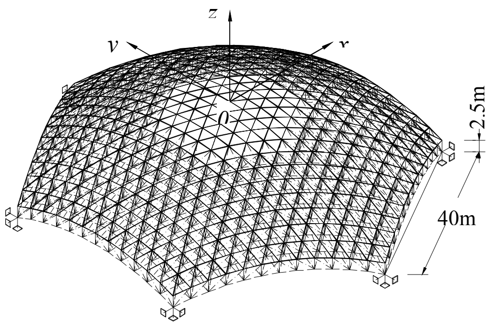


Fig. 3 A calculation model

Element types are chosen by using full stressed design. The stability of a compressive element is controlled by the following formula:

$$\frac{C_f}{C_r}+\frac{0.85U_{1x}M_{fx}}{M_{rx}}+\frac{\beta U_{1y}M_{fy}}{M_{ry}}\le 1.0 \qquad (4)$$

where C_f is compressive force under factored load. M_{fx} and M_{fy} are bending moment under factored load in two main axils separately. C_r is factored compressive resistance. M_{rx} and M_{ry} are factored moment resistances in two main axils separately. U_{1x} and U_{1y} are amplification factors for stability analysis.

The levels of variables are given by Table 1.

Table 1 Levels of design variables

Number	Design variables ρ	k	a
1	1.0	2	60
2	0.8	4	80
3	0.6	6	100
4	0.4	8	120
5	0.2	10	140

Uniform design table $U_{15}(5^3)$ is used, which given by http://www.math.hkbu.edu.hk/UniformDesign. The deviation value of $U_{15}(5^3)$ is 0.013149.

The scheme and results of experiments of calculation is shown as Table 2, where $w_{up,max}$ and $w_{down,max}$ are the maximum vertical displacement and the minimum vertical displacement separately. λ is the minimum geometrically nonlinear buckling load factor. λ=1 corresponds to 1.2 Dead Load + 1.4 Live Load (half span). f is the basic frequency. The value in parentheses corresponds to the number of design variable.

The regression of the results gives the approximate equations (5). Here for convenience, k is looked as a real number temporarily.

The basic data of regression analysis is shown as Table 3. According to JGJ 61-2003 [6] the minimum value of λ is 5. The standard error of λ is about 36.62% of the minimum value, which overruns the

Table 2 Scheme and results of initial experiments of calculation

Number of experiments	Combination of design variables			Results				
	ρ	k	a	Steel consumption G/t	$w_{up,max}$/mm	$w_{down,max}$/mm	λ	f/Hz
1	1.0 (1)	6 (3)	140 (5)	132.071	7.4	51.2	2.6546	1.6554
2	1.0 (1)	2 (1)	60 (1)	184.883	2.1	23.5	17.4270	1.2473
3	0.6 (3)	4 (2)	80 (2)	158.558	1.1	27.9	8.5546	1.5303
4	0.6 (3)	10 (5)	60 (1)	97.808	3.4	23.8	7.0345	1.3798
5	0.8 (2)	4 (2)	100 (3)	152.118	1.3	35.7	5.9919	1.6296
6	0.4 (4)	8 (4)	80 (2)	121.217	0.9	25.9	9.2156	1.6480
7	0.2 (5)	6 (3)	60 (1)	163.377	1.7	20.1	27.6010	1.2988
8	0.4 (4)	4 (2)	120 (4)	152.199	0.2	37.3	9.6606	1.6553
9	0.8 (2)	8 (4)	80 (2)	118.031	1.5	26.7	6.2624	1.6589
10	0.2 (5)	10 (5)	140 (5)	137.214	3.9	51.5	10.6640	1.6364
11	0.4 (4)	6 (3)	120 (4)	138.227	0.5	37.0	7.8579	1.6763
12	1.0 (1)	10 (5)	100 (3)	83.565	5.1	45.0	3.6057	1.8431
13	0.8 (2)	8 (4)	120 (4)	110.336	4.8	46.3	3.5253	1.7511
14	0.2 (5)	2 (1)	100 (3)	164.187	0.4	32.3	15.9680	1.5946
15	0.6 (3)	2 (1)	140 (5)	158.774	4.6	48.4	4.1095	1.6105

$$
\begin{cases}
G = 178.6263329908 - 164.5071731726\ \rho + 12.0696096066\ k + 1.1232869916\ a \\
\quad + 328.4938318760\ \rho^2 - 3.1829977544\ k^2 - 0.0209088876\ a^2 - 3.8836716091\ \rho k \\
\quad + 0.0379648411\ ka - 0.4055356562\ \rho a - 155.3264756513\ \rho^3 + 0.1352773774\ k^3 \\
\quad + 0.0000949046\ a^3 \\
w_{up,\max} = 26.1093667139 - 8.9357153305\ \rho - 2.9866358280\ k - 0.3478698430\ a \\
\quad - 1.0091660810\ \rho^2 + 0.2172011959\ k^2 + 0.0009713169\ a^2 + 0.3063825777\ \rho k \\
\quad + 0.0089153409\ ka + 0.1517650331\ \rho a - 2.6489925895\ \rho^3 - 0.0038250712\ k^3 \\
\quad + 0.0000014337\ a^3 \\
w_{down,\max} = 50.5300116245 - 23.7724969965\ \rho - 4.4414468375\ k - 0.5521755067\ a \\
\quad + 15.1329999905\ \rho^2 - 0.1603538414\ k^2 + 0.0061148410\ a^2 + 0.0263746034\ ka \\
\quad + 0.3134421307\ \rho a - 15.4419215572\ \rho^3 + 0.0354763381\ k^3 - 0.0000211505\ a^3 \\
\lambda = 119.3189885923 - 93.7518414699\ \rho - 2.3103600457\ a + 107.3535944712\ \rho^2 \\
\quad + 0.0206006597\ a^2 + 0.0041231513\ ka - 0.0252634317\ \rho a - 36.5108900975\ \rho^3 \\
\quad - 0.0087977724\ k^3 - 0.0000621412\ a^3 \\
f = -1.3182803973 + 0.7042167440\ \rho - 0.0708726426\ k + 0.0697882012\ a \\
\quad - 1.3705781160\ \rho^2 + 0.0167968361\ k^2 - 0.0005393577\ a^2 + 0.0156822812\ \rho k \\
\quad - 0.0000886978\ ka + 0.7551478292\ \rho^3 - 0.0009215711\ k^3 + 0.0000013508\ a^3
\end{cases}
\tag{5}
$$

design requirements of JGJ 61-2003. One of the main reasons leading to the errors is that there are more polynomial items of higher orders. Another is that nonlinear calculation results in significant errors. The other is that R represented by k also affects the results. Among the 15 times of calculation experiments, the maximum of λ is about 10.4 times of the minimum of λ. The presence of big change extents of objective values is another reason. Notwithstanding, all the correlative coefficients of regression equations are good enough. It shows that the equations (5) give correct relations between objective values and design variables still. The interaction effects of variables are included in the equations (5).

Table 3 Basic data of regression analysis

	G	$w_{\text{up,max}}$	$w_{\text{down,max}}$	λ	f
Correlative coefficients	0.99893	0.99988	0.99804	0.98622	0.99613
Standard error σ	3.407	0.088	1.463	1.831	0.031
F	77.6903	687.5518	69.2973	19.7422	35.0633
Significance **F**	0.0128	0.0015	0.0025	0.0022	0.0069
Significance order of design variables	k^2, ρ^2, ρ ρ^3, k^3 ρa, ρk ka, k, a^3 a^2, a	ρa, k, ka a, k^2, ρ ρk, k^3 a^2, ρ^3 a^3, ρ^2	ρa, ka k, k^3, ρ a^3, a^2 a, ρ^3 k^2, ρ^2	k^3, ρ, a a^2, a^3 ρ^2 ρ^3, ka ρa	a, a^2, a^3 k^3, k^2 ρ^3, ρ^2, ρ k, ρk, ka

For a certain k, the determination of the optimum combination of design variables comes down to the following problem:

$$
\text{Minimize} \quad G(\rho, a)
$$

$$
\text{s.t.} \quad
\begin{cases}
w_{\text{up,max}} \le L/400 = 200 \\
w_{\text{down,max}} \le L/400 = 200 \\
\lambda \ge 5 \\
0.2 \le \rho \le 1.0 \\
60 \le a \le 140
\end{cases}
\tag{6}
$$

where the constraint conditions of $w_{\text{up,max}}$, $w_{\text{down,max}}$ and λ are decided according to JGJ 61-2003. JGJ 61-2003 makes no demands on structural frequency. By solving the minimization problem when k is 2, 3, 4, 5, 6, 7, 8, 9 and 10 separately, the approximate condition solutions for preliminary design are obtained. Comparison of these results gives the approximate optimum solution of $k = 6$, $a = 119.0$, $\rho = 1.000$.

At first updating step, a uniform design table with 4 runs is used. The scheme of experiments of calculation is shown in Table 4.

Experiments of calculation give 4 groups of data additionally. There are 19 groups of data totally. New regression equations are fitted by the 19 groups of data. Similarly, nonlinear programming problem give a new approximate optimum solution of $k = 5$, $a = 120.4$, $\rho = 0.993$. This solution is very close to the result given by Xiao et al [5]. However the number of experiments of calculation is 2 less.

Table 4 Scheme of experiments at first updating step

Number of experiments	Combination of design variables		
	ρ	k	a
1	0.8 (3)	7 (4)	120 (3)
2	0.9 (2)	4 (1)	130 (4)
3	1.0 (1)	6 (3)	100 (1)
4	0.7 (4)	5 (2)	110 (2)

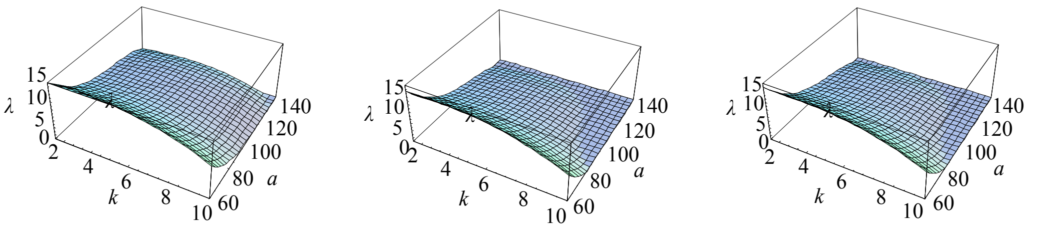
At second updating step, the same uniform design table with 4 runs is used. The scheme of experiments of calculation is shown in Table 5. At this time, optimization produces the result of $k = 5$, $a = 120.3$, $\rho = 0.992$. It is very close to the result of the first updating step.

Table 5 Scheme of experiments at second updating step

Number of	Combination of design variables		
	ρ	k	a
1	0.8 (3)	6 (4)	120 (3)
2	0.9 (2)	3 (1)	130 (4)
3	1.0 (1)	5 (3)	100 (1)
4	0.7 (4)	4 (2)	110 (2)

4.4 Verification

Fig. 4 gives the change of response surface of $\lambda(k, a)$ when ρ=0.6. Table 6 gives the comparison of initial optimization with updating solutions. In the table verification of optimization results are also attached. It can be seen that the solutions at second updating step are the closest to the values of verification design.



(a) Initial solution (b) First updating step (c) Second updating step

Fig. 4 Change of response surface of $\lambda(k, a)$ when ρ=0.6

Table 6 Optimization solutions and verification design

Step of calculation	Combination of design variables			Comparison of results							
				Steel consumption G/t		$w_{up,max}$/mm		$w_{down,max}$/mm		λ	
	k	a	ρ	Optimization solution	Verification design	Optimization solution	Verification design	Optimization solution	Verification design	Optimization solution	Verification design
Initail solution	6	119.0	1.000	127.392	130.201	3.621	3.571	43.043	44.379	6.5243	5.057
First step	5	120.4	0.993	120.912	124.790	3.429	3.385	40.547	42.739	6.0839	5.109
Second step	5	120.3	0.992	120.799	124.352	3.426	3.384	40.548	42.741	6.0989	5.117

5 A Prestressed Column-supported Partial Double-layer Reticulated Shell

The column-supported prestressed partial double-layer shallow reticulated shell is a new type of long-span roof structure. Proper prestressing of the column-supported shallow reticulated shell over part of the shell or along its edges will make the distribution of the interior shell forces more uniform, will enlarge the membrane stress area, and will reduce the horizontal reaction forces on the shell supports. The theory of equal strength shells can then be used to lay single layer in the reduced membrane stress region. The layout not only reduces the cost, but also provides a clear interior view.

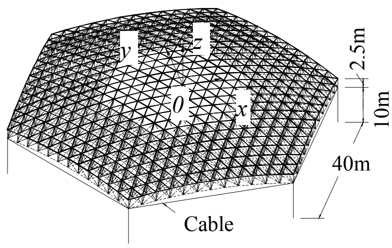


Fig. 5 A calculation model

Example 2 is a column-supported shallow reticulated shell with a projected plane of equilateral hexagonal. The length of one side of the projected plane is 40 m. The maximum diagonal length of the projected plane L is 80 m. The equation of top-chord surface of the reticulated shell adopts the formula (3), where a=250.

In prestressed shallow reticulated shells, it is suitable that the maximum initial tension of cable T is less than two times of the value that approximately makes the horizontal support deflections of the shell zero under dead load (Mbakogu et al [7]). Other design conditions are the same as those of Example 1. Design variables include ρ, k, and T. Fig. 5 is a calculation model.

The initial uniform design table uses $U_{15}(5^3)$. The results gives the approximate optimum solution of $\rho = 0.439$, $k = 6$, $T = 1888.62$. At updating steps, a uniform design table with 4 runs is used. Table 7 is the comparison of initial optimization with updating solutions. The table also demonstrates the reliability and efficacy of the improvement iteration method.

Table 7 Optimization solutions and verification design

Step of calculation	Combination of design variables			Comparison of results							
				Steel consumption G/t		$w_{up,max}$/mm		$w_{down,max}$/mm		λ	
	ρ	k	T/kN	Optimization solution	Verification design	Optimization solution	Verification design	Optimization solution	Verification design	Optimization solution	Verification design
Initail solution	0.439	6	1888.62	132.017	140.660	80.5	84.2	94.6	90.7	5.000	4.486
First step	0.549	5	1529.58	133.980	143.647	78.4	80.5	89.9	87.4	5.538	5.047
Second step	0.546	5	1491.34	130.802	137.899	77.6	79.7	98.7	93.9	5.495	5.033

6 Conclusions

The structural design of partial double-layer shallow reticulated shells involves size, shape, and topology optimization.

To obtain optimum variables simply and efficiently, cascaded uniform design is proposed to make a quick optimization. This is a kind of response surface method, which requires fewer times of computational experiment.

Besides the initial optimization, further efforts are exerted to approach to the real response surface by adding updating calculation. At each updating step, a small number of new experimental points are arranged in the vicinity of the latest optimum solution. Then, new regression equations are fitted by all of historical data. Design verifications demonstrate the reliability and efficacy of the improvement iteration method.

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