Study on Gearing Theory of Logarithmic Spiral Bevel Gear

Li qiang[1,2] Wang guoping[1] Weng haishan[2]
(1.School of Mechanical , Inner Mongolia University of Science and Technology, Inner Mongolia, Baotou, CHINA 014010; 2. School of Mechanical, Beijing University of Science and Technology, Beijing, CHINA 100083)

Abstract: -A new logarithmic spiral transmitting form of bevel gear was putted up with in this article, analyzed the formation of the tooth area, established the reference frame of gearing analysis via analyzing space transmission theory, educed equations of the tooth area and conjugated profiles, proved that the gearing line is spherical surface involutes and the conjugated profiles is logarithmic spiral bending surface, demonstrated the research values of logarithmic spiral bevel gear.

Keywords: -Logarithmic spiral, Conjugated profiles, Tooth area, Pitch cone, Pitch curve, Base cone

1 Introduction

Spiral bevel gears usually consist of circinal arc tooth bevel gear, cycloid tooth bevel gear and quasi involutes tooth bevel gear, they define helical angle, lying in the midpoint of pith curve, as the nominal helical angle in gearing process. And in fact the helical angles, lying in the pitch curve, don't have the same degree in different points. So in machining process they all need to be adjusted many times and corrected complexly between whiles, which bring lots of troubles such as complex machining process, rigid assemblage, poor machining precision and so on[1]. Logarithmic spiral bevel gear, as a new gear transmission, can achieve equal helical angles in different points of the pitch curve and assure the most rational gearing transmission via improving transmission stability, carrying capacity and using life.

2 Analysis of Tooth Area

2.1 Tooth area formation and equation

As a plane do pure rolling around a base cylinder, a line, unparellelling with generatrix of the cylinder and lying on the plane, leaves a track, which is the tooth area of a bevel wheel[2]. In the similitude of bevel wheel, spiral bevel gears have a base cone but base cylinder, and a curve on the pure rolling plane. Now replacing the curve as a logarithmic spiral and repeating the pure rolling as the bevel wheel, the track, leaved by logarithmic spiral, is the logarithmic helical tooth area, shown in Fig.1.

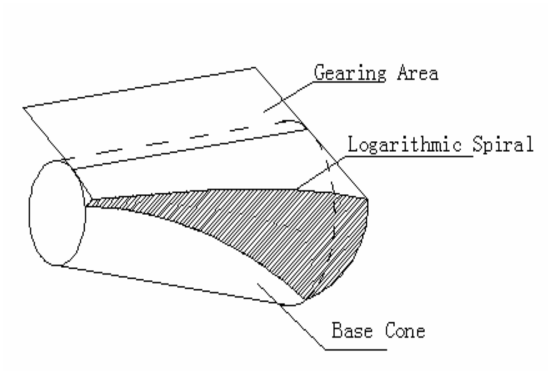

Fig.1.Tooth area Formation of logarithmic spiral bevel gear

2.2 Equation of tooth area

As shown in Fig.2, plane 'A' keeps tangency with the base cone along line OP, angle of base cone is θ. When plane 'A' does do pure rolling around the base cone, logarithmic spiral, making Point O as rolling center, leaves a track which is called logarithmic spiral tooth area. Making base cone peak O as origin, establishing the coordinate system [O; x, y, z] fastened with the base cone B, and the coordinate system [O; $x^{'}, y^{'}, z^{'}$] fastened with plane A. when plane A makes pure rolling around the base cone, generatrix OP is the instantaneous axis. Axis $x^{'}$ is attached to plane A. so the coordinate transform formula from [O; $x^{'}, y^{'}, z^{'}$] to [O; x, y, z] just as following:

$$\begin{cases} x' = x\sin\phi - y\cos\phi \\ y' = x\cos\theta\cos\phi + y\cos\theta\sin\phi - z\sin\theta \\ z' = x\sin\theta\cos\phi + y\sin\theta\sin\phi + z\cos\theta \end{cases} \quad (1)$$

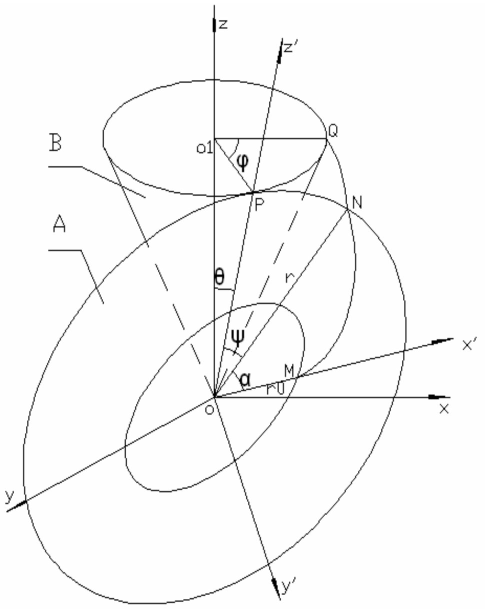

Fig.2. Coordinate system for confirming tooth area equation

Polar coordinate equation of the logarithmic spiral MN in the plane A is written as follows [3]:

$$r = r_0 e^{\beta\alpha} \quad (2)$$

（r-polar radius；r_0-base circle radius；β-spiral angle；α-polar angle）

So the equation of the logarithmic spiral MN in the plane O$x'z'$ shows as follows:

$$\begin{cases} x' = r_0 e^{\beta\alpha}\cos\alpha \\ y' = 0 \\ z' = r_0 e^{\beta\alpha}\sin\alpha \end{cases} \quad (3)$$

And known to all, base cone angle θ is content with a equation as follows:

$\sin\theta = \sin\beta\sin\gamma$

(β-decided by designer and engineering status, γ-calculated according to the gearing theory[4]).

So the tooth area equation of the logarithmic spiral bevel gears is as follows:

$$\begin{cases} r_0 e^{\beta\alpha}\cos\alpha = x\sin\phi - y\cos\phi \\ 0 = x\cos\theta\cos\phi + y\cos\theta\sin\phi - z\sin\theta \\ r_0 e^{\beta\alpha}\sin\alpha = x\sin\theta\cos\phi + y\sin\theta\sin\phi + z\cos\theta \\ \theta = \arcsin(\sin\beta\sin\gamma) \end{cases} \quad (4)$$

Supposing a vector, corresponding with a random point of the tooth area, names R' and R in the coordinate system [O; x', y', z'] and [O; x, y, z]. According to the transformation theory of coordinates, knowing that:

$$R' = \tilde{K}_{(-\phi+\frac{\pi}{2})}\tilde{I}_\theta R$$

So the vector equation of logarithmic spiral bevel gears is showed as follows:

$$\begin{aligned} R = {} & [r_0 e^{\beta\alpha}\cos\alpha\sin\phi + r_0 e^{\beta\alpha}\sin\alpha\cos\phi\sin\theta]\cdot i \\ & +[r_0 e^{\beta\alpha}\sin\alpha\sin\phi\sin\theta - r_0 e^{\beta\alpha}\cos\alpha\cos\phi]\cdot j \\ & +[r_0 e^{\beta\alpha}\sin\alpha\cos\theta]\cdot k \end{aligned} \quad (5)$$

（$\theta = \arcsin(\sin\beta\sin\gamma)$）

2.2 Gearing line analysis

Known to all that circinal arc tooth bevel gear and cycloid tooth bevel gear using sphere involutes as the gearing line[2], so it can also be used as gearing line of logarithmic spiral bevel gear, the difference between them is the gearing line equations. Conterminous line of intersecting tooth area with spherical surface that its origin lies in point O, is the sphere involutes. So its equation in coordinate [O; x, y, z] expresses as follows:

$$\begin{cases} x = r_0 e^{\beta\alpha}\cos\alpha\sin\phi + r_0 e^{\beta\alpha}\sin\alpha\cos\phi\sin\theta \\ y = r_0 e^{\beta\alpha}\sin\alpha\sin\phi\sin\theta - r_0 e^{\beta\alpha}\cos\alpha\cos\phi \\ z = r_0 e^{\beta\alpha}\sin\alpha\cos\theta \\ \sqrt{x^2+y^2+z^2} = l \end{cases} \quad (6)$$

（l-length of the base cone generatrix）

Then a pair of conjugated gear will mesh each other along the sphere involutes.

3 Eduction of Gearing Equation and Conjugated Profiles Equation

3.1 Relative speed analysis

Transmission of logarithmic spiral bevel gear is attached to space gearing theory. As gears mesh, two rotational axis keep certain space location and two gears only rotate around their axis but move along their axis. Their rolling speed has certain relations. In terms of conjugated gears, their moving parameter only is rolling speed of a gear among them, so the transmission belongs to space single parameter gearing theory and the instantaneous axis area becomes the pitch cone. The meshing belongs to intersectant axis transmission. Consequently, established following coordinates.Fig.3.

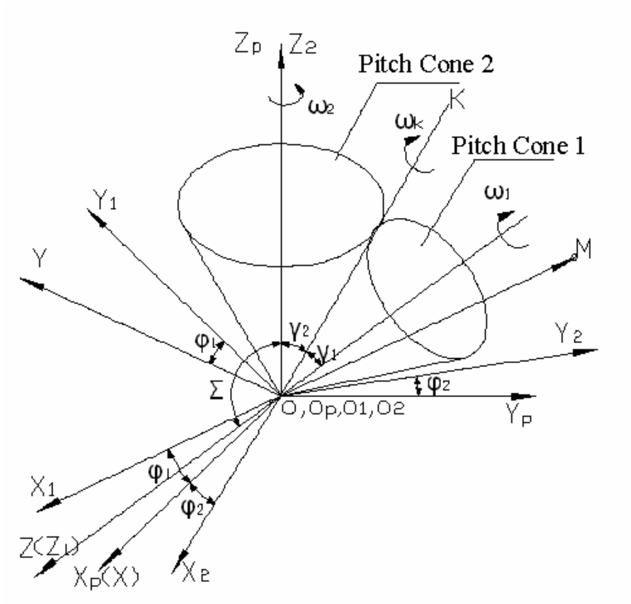

Fig.3. Gearing analysis coordinates

At the beginning, Coordinates $S_p(o_p,x_p,y_p,z_p)$ and $S(o,x,y,z)$ fasten with pitch cone 2 and pitch 1 separately, axis z_p share the same line with pitch cone 2, axis z shares the same line with pitch cone 1, and the intersectant angle names Σ .supposing rotate speed of gear 1 and gear 2 as ω_1 and ω_2 separately, if gear 1 rotates a angle ϕ_1, gear 2 will rotate a angle ϕ_2 , here the coordinate S_p transforms into coordinate $S_2(o_2,x_2,y_2,z_2)$ and the coordinate S transforms into coordinate $S_1(o_1,x_1,y_1,z_1)$.

Supposing the unit vector of coordinates S_p,S,S_1,S_2 as $i_p,j_p,k_p;i,j,k;i_1,j_1,k_1;i_2,j_2,k_2$ separately, then rotate speed vector ω^{I} of gear 1 and rotate speed vector ω^{Π} of gear 2 can show as follows:

$$\omega^{\mathrm{I}} = \omega_1 \cdot k \,,$$

$$\omega^{\Pi} = \omega_2 \cdot k_p = \omega_2 |k_p| \sin\Sigma \cdot j + \omega_2 |k_p| \cos\Sigma \cdot k$$
$$= \omega_2 \sin\Sigma \cdot j + \omega_2 \cos\Sigma \cdot k$$

Supposing a point M's coordinate is (x,y,z) in space coordinate S, then:

$$\overline{OM} = x\cdot \mathrm{i} + \mathrm{y}\cdot \mathrm{j} + \mathrm{z}\cdot \mathrm{k}$$

Therefore, with the rolling of gear 1 the rotate speed vector V^{I} of point M can show as follows:

$$V^{\mathrm{I}} = \omega^{\mathrm{I}} \times \overline{OM} = \omega_1 \cdot k \times (x\cdot \mathrm{i} + \mathrm{y}\cdot \mathrm{j} + \mathrm{z}\cdot \mathrm{k})$$
$$= \omega_1 x \cdot \mathrm{j} - \omega_1 \mathrm{y} \cdot \mathrm{i} \qquad (7)$$

And with the rolling of gear 2 the rotate speed vector V^{Π} of point M can show as follows:

$$V^{\Pi} = \omega^{\Pi} \times \overline{OM} = (\omega_2 \sin\Sigma \cdot \mathrm{j} + \omega_2 \cos\Sigma \cdot \mathrm{k})$$
$$\times (x\cdot \mathrm{i} + \mathrm{y}\cdot \mathrm{j} + \mathrm{z}\cdot \mathrm{k})$$
$$= (z\omega_2 \sin\Sigma - y\omega_2 \cos\Sigma)\cdot \mathrm{i}$$
$$+ x\omega_2 \cos\Sigma \cdot \mathrm{j} - x\omega_2 \sin\Sigma \cdot \mathrm{k} \qquad (8)$$

Then relative speed $V^{\Pi\mathrm{I}}$ of gearing point M of conjugative gears shows as follows:

$$V^{\Pi\mathrm{I}} = V^{\Pi} - V^{\mathrm{I}} = \omega^{\Pi} \times \overline{OM} - \omega^{\mathrm{I}} \times \overline{OM}$$

Viz.

$$V^{\Pi\mathrm{I}} = (z\omega_2 \sin\Sigma + \omega_1 y - y\omega_2 \cos\Sigma)\cdot i$$
$$+ (-x\omega_1 + x\omega_2 \cos\Sigma)\cdot \mathrm{j} - x\omega_2 \sin\Sigma \cdot \mathrm{k} \qquad (9)$$

3.2 Eduction of gearing equation

As logarithmic spiral bevel gears mesh in space, two conjugated tooth areas contact in a point in which there is a common normal **n** and a common tangent plane. If two tooth areas can mesh glossily and continuously, it's necessary to meet that common normal **n** has to plumb relative speed $V^{\Pi\mathrm{I}}$ of common contact point. So the generic gearing equation shows as follows:

$$V^{\Pi\mathrm{I}} \cdot n = 0$$

In order to convenient solve the problem, it's important to transform equation (5) form left hand coordinate system to right hand coordinate system, as follows:

$$R = [-r_0 e^{\beta\alpha}(\cos\alpha\cos\phi - \sin\alpha\sin\theta\sin\phi)]\cdot i$$
$$+ [r_0 e^{\beta\alpha}(\cos\alpha\sin\phi + \sin\alpha\sin\theta\cos\phi)]\cdot j \qquad (10)$$
$$+ (r_0 e^{\beta\alpha} \sin\alpha\cos\theta)\cdot k$$

According to Fig.3, suppose that the tooth area Σ^{Π} equation of gear 2 is shown as follows:

$$r^{\Pi}(\alpha,\phi) = x_2(\alpha,\phi)\cdot i_2 + y_2(\alpha,\phi)\cdot j_2 + z_2(\alpha,\phi)\cdot k_2$$

Thereinto:

$$\begin{aligned} x_2(\alpha,\phi) &= -r_0 e^{\beta\alpha}(\cos\alpha\cos\phi - \sin\alpha\sin\theta\sin\phi) \\ y_2(\alpha,\phi) &= r_0 e^{\beta\alpha}(\cos\alpha\sin\phi + \sin\alpha\sin\theta\cos\phi) \\ z_2(\alpha,\phi) &= r_0 e^{\beta\alpha}\sin\alpha\cos\theta \end{aligned} \qquad (11)$$

（note: $\theta = \arcsin(\sin\beta\sin\gamma)$ ）;

Suppose that normal vector of tooth area Σ^{Π} names n^{Π} and shows as follows:

$$n^{\Pi} = n^{\Pi}_{x_2} i_2 + n^{\Pi}_{y_2} j_2 + n^{\Pi}_{z_2} k_2$$

So:

$$n^{\Pi}=\frac{\partial r^{\Pi}}{\partial\alpha}\times\frac{\partial r^{\Pi}}{\partial\phi}=\begin{vmatrix} i_2 & j_2 & k_2 \\ \dfrac{\partial x_2}{\partial\alpha} & \dfrac{\partial y_2}{\partial\alpha} & \dfrac{\partial z_2}{\partial\alpha} \\ \dfrac{\partial x_2}{\partial\phi} & \dfrac{\partial y_2}{\partial\phi} & \dfrac{\partial z_2}{\partial\phi} \end{vmatrix}$$

$$n_{x_2}^{\Pi}=\begin{vmatrix} \dfrac{\partial y_2}{\partial\alpha} & \dfrac{\partial z_2}{\partial\alpha} \\ \dfrac{\partial y_2}{\partial\phi} & \dfrac{\partial z_2}{\partial\phi} \end{vmatrix};\ n_{y_2}^{\Pi}=\begin{vmatrix} \dfrac{\partial z_2}{\partial\alpha} & \dfrac{\partial x_2}{\partial\alpha} \\ \dfrac{\partial z_2}{\partial\phi} & \dfrac{\partial x_2}{\partial\phi} \end{vmatrix};\ n_{z_2}^{\Pi}=\begin{vmatrix} \dfrac{\partial x_2}{\partial\alpha} & \dfrac{\partial y_2}{\partial\alpha} \\ \dfrac{\partial x_2}{\partial\phi} & \dfrac{\partial y_2}{\partial\phi} \end{vmatrix} \quad (12)$$

According to Fig.3.coordinates transformative matrix M_{SS_2}, showing the conversion from coordinate system S_2 to coordinate system S, put as follows:

$$M_{SS_2}=\begin{pmatrix} \cos\phi_2 & -\sin\phi_2 & 0 \\ \cos\Sigma\sin\phi_2 & \cos\Sigma\cos\phi_2 & \sin\Sigma \\ -\sin\Sigma\sin\phi_2 & -\sin\Sigma\cos\phi_2 & \cos\Sigma \end{pmatrix} \quad (13)$$

Suppose that relative speed in coordinate system S_2 names $V_{S_2}^{\Pi I}(V_{x_2}^{\Pi I},V_{y_2}^{\Pi I},V_{z_2}^{\Pi I})$, according to equation (3), giving conclusion as follows:

$$V_{S_2}^{\Pi I}=V_{x_2}^{\Pi I}i_2+V_{y_2}^{\Pi I}j_2+V_{z_2}^{\Pi I}k_2 \quad (14)$$

$$V_{x_2}^{\Pi I}=(-\omega_2\sin\phi_2+\omega_1\cos\Sigma\sin\phi_2)x_2$$
$$+(\omega_1\cos\phi_2\cos\Sigma-\omega_2\cos\phi_2)y_2+(\omega_1\sin\Sigma)z_2$$
$$V_{y_2}^{\Pi I}=(\omega_2\cos\phi_2\cos\Sigma-\omega_1\cos\phi_2)x_2$$
$$+(\omega_1\sin\phi_2-\omega_2\sin\phi_2\cos\Sigma)y_2$$
$$V_{z_2}^{\Pi I}=(-\omega_2\cos\phi_2\sin\Sigma)x_2+(\omega_2\sin\phi_2\sin\Sigma)y_2$$

So the gearing equation, describing in coordinate (x_p,y_p,z_p), can shows as follows:

$V_{S_2}^{\Pi I}\cdot n^{\Pi}=0$, Viz.

$$V_{x_2}^{\Pi I}n_{x_2}^{\Pi}+V_{y_2}^{\Pi I}n_{y_2}^{\Pi}+V_{z_2}^{\Pi I}n_{z_2}^{\Pi}=0 \quad (15)$$

simultaneous equations of equation (11),(12), (14) and (15) is the gearing equation of logarithmic spiral bevel gear in coordinate system S_2. Ejusd. It's easy to educe the gearing equation which is described in coordinate (x,y,z), (x_1,y_1,z_1) and (x_p,y_p,z_p).

3.3 Eduction of conjugated profiles equation

Although the theory of logarithmic spiral bevel gear is attached to space single parameter theory, as two conjugated tooth areas mesh instantaneously, the tangent part of them is a point but a line, so it's necessary to choose a method which can give attention to two aspects. Here use the space wrapping theory to solve the problem. According to coordinates transformative theory[4] and linking Fig.3, giving a coordinates transformative matrix $M_{S_1S_2}$ as follows:

又 $M_{S_1S_2}=M_{S_1S}M_{SS_2}$

$$=\begin{pmatrix} \cos\phi_1\cos\phi_2+\sin\phi_1\sin\phi_2\cos\Sigma & \sin\phi_1\cos\phi_2\cos\Sigma-\cos\phi_1\sin\phi_2 & \sin\phi_1\sin\Sigma \\ \cos\Sigma\sin\phi_2\cos\phi_1-\sin\phi_1\cos\phi_2 & \cos\Sigma\cos\phi_2\cos\phi_1+\sin\phi_2\sin\phi_1 & \cos\phi_1\sin\Sigma \\ -\sin\Sigma\sin\phi_2 & -\sin\Sigma\cos\phi_2 & \cos\Sigma \end{pmatrix} \quad (16)$$

Using r_2^{I} to describe the vector of tooth area Σ^{Π} in coordinate system S_1 and supposing its coordinate value are described in $x_1^{'},y_1^{'},z_1^{'}$. So equation of tooth area Σ^{Π} in the coordinate system S_1 shows As follows:

$$x_1^{'}=-r_0e^{\beta\alpha}(\cos\alpha\cos\phi-\sin\alpha\sin\theta\sin\phi)$$
$$\bullet(\cos\phi_1\cos\phi_2+\sin\phi_1\sin\phi_2\cos\Sigma)$$
$$+r_0e^{\beta\alpha}(\cos\alpha\sin\phi+\sin\alpha\sin\theta\cos\phi) \quad (17)$$
$$\bullet(\sin\phi_1\cos\phi_2\cos\Sigma-\cos\phi_1\sin\phi_2)$$
$$+r_0e^{\beta\alpha}\sin\alpha\cos\theta\sin\phi_1\sin\Sigma$$

$$y_1^{'}=-r_0e^{\beta\alpha}(\cos\alpha\cos\phi-\sin\alpha\sin\theta\sin\phi)$$
$$\bullet(\cos\Sigma\sin\phi_2\cos\phi_1-\sin\phi_1\cos\phi_2)$$
$$+r_0e^{\beta\alpha}(\cos\alpha\sin\phi+\sin\alpha\sin\theta\cos\phi) \quad (18)$$
$$\bullet(\cos\Sigma\cos\phi_2\cos\phi_1+\sin\phi_2\sin\phi_1)$$
$$+r_0e^{\beta\alpha}\sin\alpha\cos\theta\cos\phi_1\sin\Sigma$$

$$z_1^{'}=r_0e^{\beta\alpha}(\cos\alpha\cos\phi-\sin\alpha\sin\theta\sin\phi)\sin\Sigma\sin\phi_2$$
$$-r_0e^{\beta\alpha}(\cos\alpha\sin\phi+\sin\alpha\sin\theta\cos\phi)\sin\Sigma\cos\phi_2 \quad (19)$$
$$+r_0e^{\beta\alpha}\sin\alpha\cos\theta\cos\Sigma$$

Here simultaneous equations of (17), (18), (19) shows different locations of tooth area Σ^{Π} in coordinate system S_1. They are also the wrapping areas group C of tooth area Σ^{I}, which is the conjugated profiles of Σ^{Π}. So the wrapping area of C is the tooth area Σ^{I}. Knowing the parameters of equations (17), (18), (19) are $\alpha,\phi,\phi_1,\Sigma$, therefore suppose that the equation of tooth area Σ^{I} shows as follows:

$$r_2^{I}(\alpha,\phi,\phi_1,\Sigma)=x_1^{'}(\alpha,\phi,\phi_1,\Sigma)\cdot i_1+y_1^{'}(\alpha,\phi,\phi_1,\Sigma)\cdot j_1$$

$$+z_1'(\alpha,\phi,\phi_1,\Sigma)\cdot k_1 \tag{20}$$

According to equations (17), (18), (19), (20), knowing that the areas group is with two parameters areas group, parameters α,ϕ affect the curving areas of areas group and parameters ϕ_1,Σ affect areas group. if fastening a parameter of curving areas group, such as ϕ_1 or Σ, coordinates x_1', y_1', z_1' will be the functions about parameter Σ or ϕ_1, and also be coordinates value of the instantaneous contacting line C^{ϕ_1} or C^{Σ}. All C^{ϕ_1} and C^{Σ} bestrew two conjugated tooth areas as a net, which they have wrapping relations each other. When changing both of parameters ϕ_1 and Σ, two conjugated tooth areas will contact in a point that is the intersecting point of C^{ϕ_1} and C^{Σ} ,then it's obvious that α,ϕ are also the functions about ϕ_1,Σ. Writing them as follows:

$$\begin{cases}\alpha=\alpha(\phi_1,\Sigma)\\ \phi=\phi(\phi_1,\Sigma)\end{cases} \tag{21}$$

And tangent plane of Σ^{I} can shows as follows:

$$\begin{cases}\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}\dfrac{\partial\alpha}{\partial\phi_1}+\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}\dfrac{\partial\phi}{\partial\phi_1}+\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi_1}\\ \dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}\dfrac{\partial\alpha}{\partial\Sigma}+\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}\dfrac{\partial\phi}{\partial\Sigma}+\dfrac{\partial r_2^{\mathrm{I}}}{\partial\Sigma}\end{cases} \tag{22}$$

If fastening parameters ϕ_1 and Σ, the tangent plane of tooth area $\Sigma^{\mathrm{I}}_{\phi_1\Sigma}$ of gear 2 can be defined by vector $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}$ and $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}$ in coordinate system S_1. In order to make wrapping area Σ^{I} and tooth area $\Sigma^{\mathrm{I}}_{\phi_1\Sigma}$ have commonly tangent plane, vectors group (22) and vectors $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}$, $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}$ must be in the same plane. Viz. vectors $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}$、$\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}$、$\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi_1}$ are in the same plane, and vectors $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha}$、$\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}$、$\dfrac{\partial r_2^{\mathrm{I}}}{\partial\Sigma}$ are also in the same plane. Describing as follows:

$$\begin{cases}[\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha},\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi},\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi_1}]=0\\ [\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha},\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi},\dfrac{\partial r_2^{\mathrm{I}}}{\partial\Sigma}]=0\end{cases} \tag{23}$$

Suppose that coordinates values of vector $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\alpha},\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi}$, $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\Sigma}$ and $\dfrac{\partial r_2^{\mathrm{I}}}{\partial\phi_2}$ are $(x_\alpha,y_\alpha,z_\alpha)$, (x_ϕ,y_ϕ,z_ϕ), $(x_\Sigma,y_\Sigma,z_\Sigma)$ and $(x_{\phi_2},y_{\phi_2},z_{\phi_2})$ so equation (23) can show as follows according to the space analytic geometry:

$$\begin{cases}\begin{vmatrix}x_\alpha & y_\alpha & z_\alpha\\ x_\phi & y_\phi & z_\phi\\ x_{\phi_2} & y_{\phi_2} & z_{\phi_2}\end{vmatrix}=0\\ \begin{vmatrix}x_\alpha & y_\alpha & z_\alpha\\ x_\phi & y_\phi & z_\phi\\ x_\Sigma & y_\Sigma & z_\Sigma\end{vmatrix}=0\end{cases} \tag{24}$$

Obviously equations (24) expound the equation (21), viz. equations (24) describes all intersecting points of curve C^{ϕ_1} and C^{Σ} based on the equation (20), so simultaneous equations of (20) and (24) is the equation of conjugated tooth area Σ^{I} of tooth area Σ^{Π} of gear 2.

4 Touching Track and Shape of Tooth Areas

Known to all that gear flank curve is the intersecting curve of tooth area and pitch cone, and thinking that touching track curve is equidistant with gear flank curve. Because the equidistant curve is congruent with the original curve, the touching track of logarithmic spiral bevel gear is a logarithmic spiral. Because of the analytic complexity of conjugated tooth area, here using the conjugated pitch curve on the pitch cone to analyze conjugated specialities[5]. Firstly, establishing the coordinates and drawing a spiral curve on a cone, refering to Fig. 4.

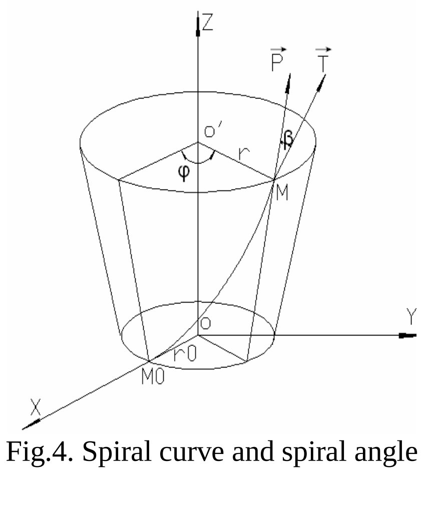

Fig.4. Spiral curve and spiral angle

In the Fig.4, vector $\vec{P}$ is the tangent vector of generatrix that is through a point M, vector $\vec{T}$ is the tangent vector of spiral curve on the point M. Point M_0 is the jumping-off point of spiral curve and the distance is r_0 form point O to point M_0. The coordinates value of point M is $(x(\phi), y(\phi), z(\phi))$, and the distance is r from point O' to point M. α is the self-cone angle. Suppose that i, j and k are the unit vectors of coordinates axis x, y and z. As well as:

$$\overrightarrow{OM} = x(\phi)\, i + y(\phi)\, j + z(\phi)\, k$$
$$x(\phi) = r(\phi)\cos\phi$$
$$y(\phi) = r(\phi)\sin\phi$$
$$z(\phi) = r(\phi)\cot\alpha$$

Viz.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r(\phi) \\ 0 \\ r(\phi)\cot\alpha \end{bmatrix} \tag{25}$$

Well then:

$$\vec{P} = r'(\phi)\cos\phi i + r'(\phi)\sin\phi j + r'(\phi)\cot\alpha k \tag{26}$$

$$\left|\vec{P}\right|^2 = r'^2(\phi) + r'^2(\phi)\cot^2\alpha \tag{27}$$

$$\begin{aligned} \vec{T} &= [r'(\phi)\cos\phi - r(\phi)\sin\phi]\bullet i \\ &+ [r'(\phi)\sin\phi + r(\phi)\cos\phi]\bullet j \\ &+ r'(\phi)(\cot\alpha)\bullet k \end{aligned} \tag{28}$$

$$\left|\vec{T}\right|^2 = r'^2(\phi) + r^2(\phi) + r'^2(\phi)\cot^2\alpha \tag{29}$$

As well as:

$$\cos\beta = \frac{\vec{T}\bullet\vec{P}}{\left|\vec{T}\right|\left|\vec{P}\right|} \tag{30}$$

Taking equations（26）、（27）、（28）、（29） into equation（30）can obtain the results as follows:

$$\tan\beta = \frac{r(\phi)}{r'(\phi)}\sin\alpha \tag{31}$$

Because logarithmic spiral has equal spiral angle β in every point, (viz. β is a constant), making quadrature operation to the equation (31), the result shows as follows:

$$r(\phi) = r_0 e^{\phi\sin\alpha\cot\beta} \tag{32}$$

Taking equation (32) into equation (25), the result shows as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_0 e^{\phi\sin\alpha\cot\beta} \\ 0 \\ r_0 e^{\phi\sin\alpha\cot\beta}\cot\alpha \end{bmatrix} \tag{33}$$

Well then the equation (33) describes the logarithmic spiral taper curve. If two pitch cones make absolute rolling, there is a curve on the pitch cone 2 at all time, which meshes with the logarithmic spiral taper curve on the pitch cone 1. According to basic meshing condition of spatial curves, if two curves mesh each other, they must have equal spiral angle and the direction of them is just opposite. Therefore conjugated curve of logarithmic spiral taper curve must be the logarithmic spiral taper curve too, the only difference is that they have different rotary direction. Suppose that intersecting angle of two axis of two pitch cone is Σ, establishing the coordinates system as follows(Fig.5):

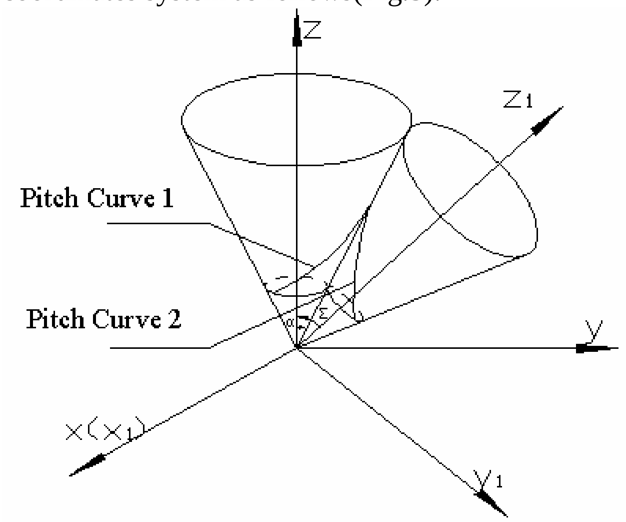

Fig.5. Conjugated pitch curve

Well then transformative equation of two coordinates shows as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Sigma & \sin\Sigma \\ 0 & -\sin\Sigma & \cos\Sigma \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \tag{34}$$

So the equation of logarithmic spiral taper curve 2 in coordinates system ($oxyz$) shows as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Sigma & \sin\Sigma \\ 0 & -\sin\Sigma & \cos\Sigma \end{bmatrix} \times \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} r_0 e^{\phi\sin\alpha\cot\beta} \\ 0 \\ r_0 e^{\phi\sin\alpha\cot\beta}\cot\alpha \end{bmatrix}$$

According to above paragraphs, knowing that two pitch cones mesh along a pair of conjugated logarithmic spiral pitch curve, and the speciality of them is equal to a pair of conjugated bending areas, so logarithmic spiral bevel gears can assure equal spiral angle when two gears are meshing.

5. Conclusion

According to transmitting particularity of logarithmic spiral bevel gears, text researches their basic gearing theory thoroughly based on classical research method and improved them properly, educed the equations of tooth area and conjugated profiles and gearing equation, analyzed the gearing curve, contacting curve and the shape of them, demonstrated the equal spiral angle merits when conjugated gears meshed.

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