Generic Theoretical Formulae for Estimating Site Effects

HING-HO TSANG
Department of Civil Engineering
The University of Hong Kong
Pokfulam Road
HONG KONG

NELSON T. K. LAM
Department of Civil and Environmental Engineering
The University of Melbourne
Victoria 3010
AUSTRALIA

MICHAEL W. ASTEN
Centre for Environmental and Geotechnical Applications of Surface Waves (CEGAS)
School of Geosciences, Monash University
Melbourne, Victoria
AUSTRALIA

S. H. LO
Department of Civil Engineering
The University of Hong Kong
Pokfulam Road
HONG KONG

Abstract: - Significant modification of the seismic ground shaking may be developed in sites pertaining to soil resonance behaviour. Generic theoretical formulae have been developed to estimate the amplification for soil sites, and directly address potential soil resonance behaviour. The development of these formulae has taken into account the effects of soil damping, stiffness degradation, impedance contrast at soil-bedrock interface and soil plasticity. The microtremor array method with the spatial auto-correlation (SPAC) processing technique has been recommended for obtaining the shear wave velocity (SWV) model of the site.

Key-Words: - site effects, resonance, amplification, soil, damping, period shift, microtremor, spatial auto-correlation

1 Introduction

The ground motion characteristics of a soil site can be highly dependent on conditions of the overlying Quaternary sediments. Engineering design spectra stipulated by contemporary codes of practices specify site factors for different site classes and hence enable site effects to be predicted without calculations, or with simple manual calculations. Site classification schemes adopted by major codes of practices typically parameterize soil dynamic properties on the basis of the shear wave velocity (SWV) averaged over a certain depth in the sediment. With this approach, which is based on the statistical analyses of abundant empirical data, parameters representing details of the soil layers have been averaged. Consequently, the effects of multiple reflections at the boundaries of the soil medium (pertaining to resonance behaviour) have not been parameterized.

The significance of soil resonance phenomenon depends on soil conditions, level of seismic hazard, and so forth. The resonance phenomenon deserves special attention with flexible soil sediments with high impedance contrast at the interface with bedrock, and more so in regions of low and moderate seismicity typified by infrastructures with limited ductility which accentuates the effects of resonance.

A range of analytical software has been developed to model site effects at varying levels of sophistication. Whilst 1-D non-linear wave analysis is the lowest tier approach and the most widely used analytical tool, e.g. program *SHAKE* [1], however it is still not well known to practising professionals in low and moderate seismic regions. The main difficulty with the use of these time-history analysis programs is the lack of knowledge on the ground motion time-histories and hence uncertainties as to what accelerogram data is considered representative and suitable for input into the analysis.

This paper presents the development of a simple (hand-calculation) model for predicting site effects characterized by soil resonance behaviour as described above. Importantly, the impedance ratio between the bedrock and the overlying soil has been introduced as a parameter in the calculation along with damping parameters. It is noted that expressions used in developing the proposed formulae are based on well-established wave theories. The predicted amplification has been shown to be very consistent with results obtained from analyses using *SHAKE*. The proposed calculation procedure which is in its early stage of development is based on modelling the soil sediment as a homogenous material overlying bedrock. Intuitively, non-homogenous soil layers may also be analysed using this method by weighted averaging the soil SWV and density. Further study is now underway to further develop this method for handling complex layering conditions.

The microtremor array method with the spatial auto-correlation (SPAC) processing technique has been discussed in later part of the paper. The method appears to be extremely well suited in urban areas due to its non-invasiveness and high speed of data acquisition. It is recommended for obtaining the SWV model of the site.

2 Theoretical Development

2.1 Nonlinear Peak Displacement Ratio

Modelling of the nonlinear peak displacement ratio (*PDR*) is based on three principal mechanisms: (i) transmission of seismic waves across the interface between two media (bedrock and soil), (ii) reflection of seismic waves at the two boundaries of the soil medium (i.e. boundary with rock and that with air), and (iii) hysteretic energy dissipation during wave transmission within the soil medium.

Consider a soil layer of thickness H with weighted average SWV of V_S, the fundamental natural period of the site (T_g) can be computed using the well-known quarter-wavelength principle as represented by Equation (1).

$$T_g = \frac{4H}{V_S} \tag{1}$$

As upward propagation seismic waves reach the interface between the bedrock and the soil, as shown in Fig. 1, only part of the wave energy is transmitted into the soil whilst the rest is reflected back into the half-space of the bedrock. The displacement amplitude of the transmitted wave (A_T) and the reflected wave (A_R) can be calculated using Equations (2) and (3) for zero angle of incidence (approach the interface at 90° angle).

$$A_R = \frac{\alpha - 1}{\alpha + 1} A_i \quad \text{and} \quad A_T = \frac{2\alpha}{1+\alpha} A_i \qquad (2) \text{ and } (3)$$

where A_i is the amplitude of the incident wave and α the impedance ratio as defined by Equation (4).

$$\alpha = \frac{\rho_R V_R}{\rho_S V_S} \tag{4}$$

where ρ and V are the weighted-average of the density and the SWV (the subscripts R and S represent the rock and soil layers respectively).

Equation (3) can also be used to model the amplification of seismic waves reaching the soil surface, based on considering the soil and air as two media separated by the interface. This surface amplification factor is accordingly equal to 2. Meanwhile, there are waves reflecting back down into the soil medium. The amplitude of the downward propagating reflected waves is accordingly equal in amplitude and sign to the incident wave based on Equation (2).

The reflected seismic waves will then reach bedrock for the second time when reflection will again occur. Equation (2) may, yet again, be used for modelling seismic waves reflecting from the bedrock-soil interface back up into the soil medium, but the value of α is reciprocal to that defined by Equation (4) due to the change in direction of the wave transmission. The ratio of the amplitude of the

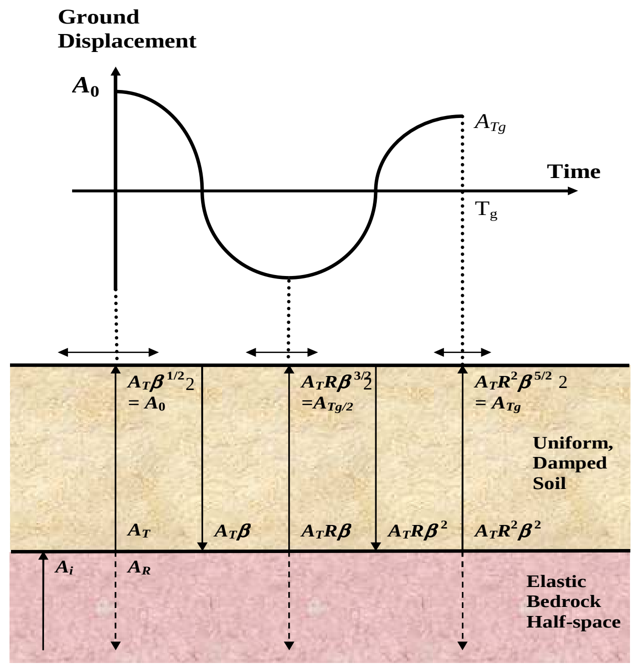


Fig. 1. Illustration of the concept of the site fundamental natural period, multiple wave reflections, material and radiation damping.

reflected and incident waves, which is defined as the wave reflection coefficient (R), can be calculated using Equation (5).

$$R = \frac{1/\alpha - 1}{1/\alpha + 1} = \frac{1-\alpha}{1+\alpha} \quad (5)$$

From Equation (5), R varies between 0 and 1 and with a change in sign which means that the polarity of the waves will also change. The de-amplification of the seismic waves ($R < 1$) reflected back up from the bedrock surface is sometimes described as radiation damping.

Unlike boundary mechanisms, hysteretic damping occurred within the soil medium modifies wave amplitude continuously. The de-amplification of the wave amplitude can be expressed as an exponential function of the number of wave cycles experienced during the wave transmission. The de-amplification factor β for half wave-cycle is given by Equation (6).

$$\beta = \exp(-\pi\zeta) \quad (6)$$

where ζ is the damping ratio (as a proportion of critical damping). The dependence of the shaking level (nonlinearity) in site response is accounted for by this soil damping ratio. A model for estimating soil damping ratio for given intensity of shaking has been developed [2], and has been illustrated in Section 3. From Equation (1), seismic wave components possessing the site natural period (T_g) will experience quarter-of-a-cycle periodic motion during the transmission of the waves through the thickness of the soil medium. The reduction in the wave amplitude is accordingly represented by:

$$A_0 = 2\beta^{\frac{1}{2}} A_T \quad (7)$$

where A_0 is the wave amplitude reaching the soil surface. The upwardly propagating S-waves after reflecting from the soil-bedrock interface will reach the soil surface to complete half a cycle of wave motion. The displacement amplitude is defined by:

$$A_{\frac{Tg}{2}} = R\beta A_0 \quad (8)$$

The same modifications will be experienced by the waves when undergoing another half a cycle of motion (with yet another change in the wave polarity). On completion of the two half-cycles, the displacement amplitude of the wave reaching the soil surface is defined by:

$$A_{Tg} = R\beta A_{\frac{Tg}{2}} = R^2\beta^2 A_0 \quad (9)$$

Equations (7) and (9) represent the displacement amplitudes of the wave when reaching the soil surface at time $T = 0$ and $T = T_g$ (i.e. n = 0 and 1), respectively. The polarity of the wavefront at both instances have the same polarity.

Wavefronts with time-lag will superpose as they are reflected onto the soil surface repetitively. The amplitude of two wave components, as defined by Equations (7) and (9), corresponding to $n = 0$ and 1 respectively, can be aggregated as shown by Equation (10) which satisfies the principle of the conservation of energy.

$$\tilde{A}_{Tg} = \sqrt{A_0^{\ 2} + A_{Tg}^{\ 2}} = A_0\sqrt{1+R^4\beta^4} \quad (10)$$

The superposition of infinite number of wave components (i.e. n = infinity) can also be represented by the algebraic relationship of Equation (11) which features the summation of a geometric series with infinite number of terms.

$$A_{soil-surface} = \sqrt{\sum_{n=0}^{\infty} A_{nTg}^{\ 2}} = A_o\sqrt{\sum_{n=0}^{\infty}\left(R^{2n}\beta^{2n}\right)^2} \quad (11)$$

where n is the number of wave cycles (of period T_g).

Given that the value of $R^{2n}\beta^{2n}$ is less than unity, Equation (11) can be re-written as:

$$A_{soil-surface} = A_0\sqrt{\frac{1}{1-R^4\beta^4}} \quad (12)$$

In comparison, the amplitude of ground motions experienced by structures founded directly on the rock surface can be represented by Equation (13).

$$A_{rock-surface} = 2A_i \tag{13}$$

where the factor of 2 represents the surface effects at the interface between rock and air.

The peak displacement ratio (*PDR*) which is the ratio of the wave amplitude as calculated from Equations (12) and (13) is hence represented by:

$$PDR = \frac{A_{\text{soil-surface}}}{A_{rock\text{-surface}}} = \frac{2\alpha}{1+\alpha}\sqrt{\frac{\beta}{1-R^4\beta^4}} \tag{14}$$

2.2 Spectral Ratio

It is a common practice to use site-dependent coefficients for different soil classes to scale the code design spectrum. In this section, a simple technique for constructing site-dependent design spectra has been described. The key element of this technique is the use of the spectral ratio (*SR*) at the site natural period T_g (considering the period-shift effect). It is defined as the ratio between the maximum spectral displacement of soil spectrum ($RSD_{\max}$ at period T_g) and the spectral displacement of bedrock spectrum at period T_g (RSD_{Tg}), and has been proposed to be composed of two components: (1) the amplification of the peak ground displacement at the bedrock surface to that at the soil surface as represented by the nonlinear *PDR* developed in the previous section, and (2) the response amplification of an elastic single-degree-of-freedom (SDOF) system when subject to periodic motion at the soil surface and is represented by a resonance factor $f(\alpha)$:

$$SR = PDR \cdot f(\alpha) \quad \text{where} \quad f(\alpha) = \alpha^{0.3} \leq 2.3 \tag{15}$$

The empirical function of Equation (15) was developed by the authors [3] in a parametric study to study the trends. The upper limit of 2.3 is to reflect the observation that f becomes insensitive to changes in the value of α when $\alpha > 16$. It is noted that *SR* is basically a function of (1) the impedance ratio α; and (2) the half-period damping factor β (a function of soil damping ratio ζ). R is also a function of α.

3 Verification and Application

3.1 Verification

Shear wave analyses using program *SHAKE* have been undertaken on some twenty soil columns to analyse the values of *PDR* and *SR* for comparison with results obtained using Equations (14) and (15) [3]. The analyses covered the following parameter values: (i) bedrock spectral velocity (RSV_{Tg} = 20 – 400 mm/s) (ii) initial soil SWV (V_s = 100 – 500 m/s), (iii) initial site natural period (T_i = 0.12 – 2.4 s), (iii) soil plasticity index (*PI* = 0, 15, 30 and 50 %) and (iv) SWV of the bedrock half-space (V_R = 500 – 3500 m/s). The correlation between results obtained from Equation (15) and that obtained from *SHAKE* provides support for the proposed model (refer Figure 2). Further verification has been provided in Tsang *et al.* [3], by comparing the model with 1994 Northridge earthquake recordings [4].

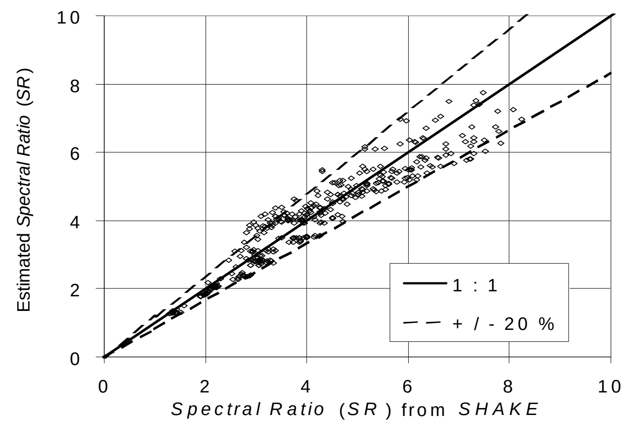


Fig. 2. Correlation of the estimated spectral ratio (*SR*) and the computed values from *SHAKE*.

3.2 Displacement-based Design Spectra

A simple procedure for constructing displacement-based (DB) design spectra is described as follows:

1. Obtain the basic parameters from normal site investigation: initial V_{si} and *PI*, thickness H of soil layer; bedrock V_R. Initial site natural period T_i can then be computed by Equation (1). RSV_{Ti} or RSD_{Ti} can be obtained from bedrock spectrum.
2. Calculate the soil damping ratio, by Equation (16) [2], with $\lambda = 1$; and then β by Equation (6), and the reduction factor λ, by Equation (17).

$$\zeta = 12.5 + 6.5\log(R_\gamma \lambda \psi) - 0.13PI$$
$$\text{where} \quad \psi = \frac{RSV_{Tg}}{V_s} = \frac{RSD_{Tg}}{H}\frac{\pi}{2} \tag{16}$$

R_γ is the ratio of the effective shear strain to maximum shear strain, which has been empirically found to vary between about 0.5 to 0.7 (0.6 has been used in this study). The reduction factor λ is needed to account for the bedrock rigidity effect:

$$\lambda = \frac{\alpha}{1+\alpha}\sqrt{\frac{1-\beta^4}{1-R^4\beta^4}} \quad (17)$$

Equation (16) may be bounded by a "practical" minimum damping ratio ζ_{pi} and an upper bound damping ratio ζ_{ub}:

$$\zeta_{pi}(\%) = 2.5 + 0.03 \cdot PI(\%) \le 6.8 \quad (18)$$

$$\zeta_{ub}(\%) = 17.5 - 0.07 \cdot PI(\%) \ge \zeta_{pi} \quad (19)$$

3. Calculate the degraded soil V_s, by Equation (20):

$$\frac{V_s}{V_{si}} = \frac{T_i}{T_g} = \frac{1}{1+R_\gamma \lambda \psi \mu} \quad (20)$$

The actual shifted site natural period T_g, can then be computed by Equation (21), using the degraded soil V_s and the revised RSV_{Tg} or RSD_{Ti}.

$$T_g / T_i = 1 + R_\gamma \lambda \psi \mu \quad (21)$$

where μ is the plasticity factor which has the values of 1.6 (for sand with PI = 0%), 0.9 (PI = 15%), 0.4 (PI = 30 %) and 0.2 (PI = 50%).

4. The impedance ratio α [Equation (4)], reflection coefficient R [Equation (5)] soil damping ratio ζ [Equation (16)], and damping factor β [Equation (6)] are now known and hence the spectra ratio (*SR*) can be calculated using Equation (15).
5. The bi-linear DB design spectrum can then be constructed for any given site based on the calculated value of SR as shown in Figure 3.

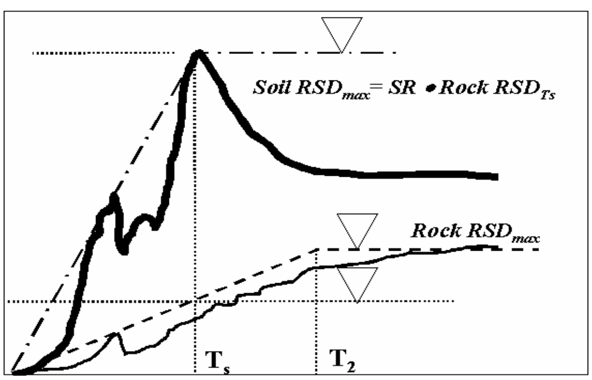


Fig. 3. Idealised bi-linear displacement-based design spectrum model.

3.3 Shear Wave Velocity Measurement

In a recent study undertaken by the second and third authors [5], the importance of accurately modelling the SWV profile (down to bedrock level) at a soil site has been ascertained. However, the commonly adopted site classification schemes in major codes of practices are based on SWV averaged over a certain depth in the sediment (20m in the Chinese Code and 30m in the International Building Code).

The microtremor array method with the spatial auto-correlation (SPAC) processing technique has been used widely in estimation of the SWV profile of Quaternary sediments (Asten [6] and references therein). The method appears to be well suited in urban areas. Its advantages include its non-invasiveness (no drilling required), speed of data acquisition, low cost and the ability to be able to provide SWV information over a wide range of depths (up to several hundreds metres). It is hence highly recommended that the SWV model of a site be obtained by field measurements.

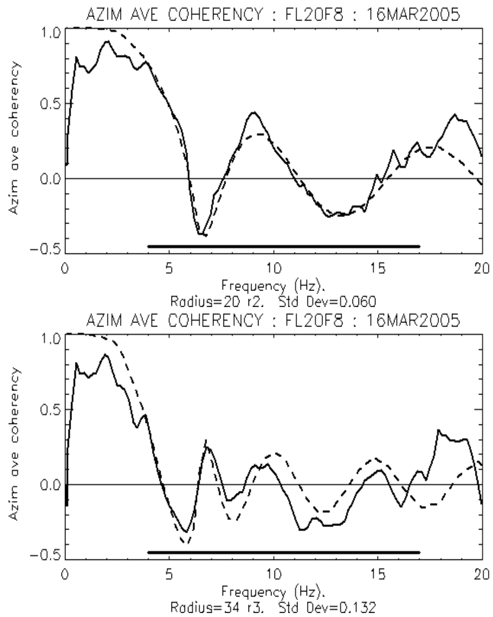
A sensitivity study for the proposed model revealed that the site response is the most sensitive to the SWV of both soil and bedrock materials [3]. This is equivalently addressing the importance of the seismic impedance contrast at the soil-bedrock interface [Equation (4)]. Hence, not only the soil SWV has to be accurately measured, the bedrock SWV is also important to be parameterised in estimating the seismic hazard of a site.

An example site in north-west Melbourne is used herein as case-study to illustrate how the SWV profile of a site can be obtained by the SPAC method which involves the use of an array of geophones in capturing synchronised signals. This method enables the SWV profile of a site to be de-lineated, and must be distinguished from the more commonly used, and simpler, method of estimating only the site natural periods based on the measurement of the horizontal/vertical spectral ratios (HVSR) of transmitted signals received by only one geophone.

With the SPAC method, the SWV profile of a site is obtained by calibration. First, a (model) coherency spectrum is generated analytically for an assumed SWV profile. The model spectrum is then compared with the averaged coherency spectrum as measured from the array of geophones. The model SWV profile is refined iteratively until the measured coherency spectrum matches with the modelled spectrum. Examples of the model-measured coherency spectra of the case-study site obtained from the hexagonal array of seven geophones are shown in Figures 4(a) and 4(b) for different array configurations (with array radius of 20 m). The SWV profile determined iteratively by the calibration procedure is summarised by Table 1.

(a) array radius of 20 m; and station separation of 20 m

(b) array radius of 20 m; and station separation of 34.6 m



Solid line – field measured coherency spectrum
Broken line – modelled coherency spectrum

Fig. 4. Model and measured coherency spectra in SPAC method

Table 1. SWV model of case-study site

Layer	H (m)	V_P (m/s)	V_S (m/s)	ρ (t/m^3)	Geology
1	2	800	190	1.8	sand/silt
2	3	1600	190	2.0	sand/silt
3	6.5	1600	140	2.0	Coode silt
4	95	2100	600	2.4	gravels
5		3100	1500	2.4	basement

The resolution of the SWV measurements at different depth ranges can be optimised by configuring the geophones with different array dimensions. For example, measurements from the smaller array [Figures 4(a) and 4(b)] were best used in constraining the SWV in the upper 5 m of the sand/silt sediments whereas measurements from the larger array (not shown) were used in constraining the SWV of the underlying (soft) Coode Island silt sediments and the gravel sediments. The SWV velocity of the Silurvian (basement) mudstone was estimated from similar surveys undertaken for other sites in Melbourne [7].

4 Conclusions

1. The effects of soil resonance on the seismic hazard can be represented by PDR and SR which are defined by Equations (14) and (15). The model proposed enables the site effects to be predicted by a hand-calculation procedure.
2. In the proposed procedure, both PDR and SR can be estimated as a simple function of the impedance contrast ratio α [Equation (4)] and the hysteretic damping factor β [Equation (6)].
3. Verification analyses based on comparison with results obtained from program *SHAKE* have been undertaken to support the proposed model.
4. The microtremor array method with SPAC processing technique has been recommended for obtaining SWV model of both soil and bedrock materials of the site.

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