Global Practical Output Tracking of Uncertain Nonlinear Systems By Smooth Output Feedback

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Abstract: This paper considers a global practical output tracking problem for a family of uncertain nonlinear systems whose Jacobian linearization is neither controllable nor observable. It is shown that under some mild conditions on such a system there is a smooth output feedback achieving global practical output tracking and such a smooth output controller is explicitly constructed by a new design method proposed. The usefulness of our result is illustrated by a numerical example.

Key-Words: Global practical output tracking, smooth output feedback, uncertain nonlinear systems, rescaling transformation

1. Introduction and Problem Statement

In this paper, we consider the output tracking problem of a class of nonlinear systems of the form

$$\begin{aligned} \dot{x}_i &= x_{i+1}^p + \phi_i(t,x,u),\ i=1,\dots,n-1, \\ \dot{x}_n &= u + \phi_n(t,x,u), \\ y &= x_1 - y_r \end{aligned} \tag{1}$$

where $x=[x_1,x_2,\cdots,x_n]^T \in \boldsymbol{R}^n$, $u \in \boldsymbol{R}$ and $y \in \boldsymbol{R}$ are the system state, input and measurement output, respectively, and $p \geq 1$ is an odd integer. For $i=1,\dots,n$, $\phi_i : R^i \to R^1$, are unknown nonlinear C^1 functions that involve uncertainty and may not be precisely known and y_r is a reference signal. The goal is to regulate the output y to zero.

The nonlinear the problem of nonlinear output regulation is to obtain a feedback law making the controlled output of a system asymptotically track a prescribed smooth reference signal. The major approach to solving of the problem originally introduced by Davison, Francis, and Wonham in [1-2] for linear systems. The study of the corresponding problem in a nonlinear setting has been extensively investigated by a number of researchers over the past twenty years;for instance, see [3-6]. However, these papers considered only a simple case where the reference signals produced by the exosystem are constants. Later on, the problem for nonlinear systems with time-varying reference signals was addressed by Isidori and Byrnes [7].

Most of the aforementioned contributions to the nonlinear regulator theory require that the Jacobian linearization of a controlled nonlinear systems be stabilizable and detectable(at least partially) [12]. Stabilizability and detectability of the the linearized system are two key assumptions for solving the problem of nonlinear output regulation by either state or error feedback [7].

However, in the case when the system under consideration is genuinely nonlinear, the problem becomes more complicated and difficult to solve. For nonlinear systems with uncontrollable/unobservable linearization, there are very few results in the literature, here we review only [8-9] and [11] that are closely related to this paper. In [8], studied the local output regulation problem for a class of triangular systems whose linearized system may be not stabilizable nor detectable and a local continuous controller was designed, forcing the tracking error within a prior given bound and in [11] considered the problem of global output tracking for a special case of a considerably general class of single input single output nonlinear systems (1) with differents $p_i \geq 1$. It was proved in [11] that the problem of global asymptotic output tracking of a constant signal is solvable by smooth state feedback.

However, output asymptotic tracking is usually not be possible (even locally) for such nonlinear systems (1) (even by smooth state feedback) [9]. In [9], Qian and Lin even gave a counter-example to show that even if the stabilization problem is solvable, the corresponding asymptotic output tracking problem is not possible.

To deal with such a problem, in the literature, a more suitable concept was introduced, that is practical output tracking [8], [9].

For system (1), the global practical output tracking via state feedback has been solved [9]. However, if the controller is required to only depend on the output, which is meaningful in the practical implementation, the problem is still open. Here, we first give a precise definition of the problem.

Practical output tracking via Output Feedbak: Let $y_r(t)$ be a bounded C^1 reference signal whose derivative $\dot{y}_r(t)$ is also bounded. For any given $\varepsilon > 0$, design a output feedback controller of the form

$$\dot{\zeta} = \alpha(\zeta, y),\ \zeta \in R^m, \quad u = u(\zeta, y) \tag{2}$$

such that

i) Every state of the closed-loop system (1)-(2) is well-defined on $[0, +\infty)$ and globally bounded.

ii) For every $x(0) \in R^n$, there is a finite-time $T > 0$ such that the output of the closed-loop system (1)-(2) satisfies

$$|y| = |x_1(t) - y_r(t)| < \varepsilon, \quad \forall t \geq T > 0. \tag{3}$$

In order to find such an output feedback control, we need to construct an observer to recover the state information. Recently a novel nonlinear observer design is introduced in [10]. In [10], it has been shown that the global stabilization of the system (1) can be solved by smooth output feedback under suitable growth conditions. These growth conditions guarantees, the existence of smooth output controllers.

In this paper, we extend the result in [10] to solve the global output tracking problem and indicates that the problem of global practical output tracking is solvable. To this end, the following conditions are introduced.

Assumption 1: For the nonlinear system (1), there exists a real number $C > 0$ such that for $i = 1, 2, \ldots, n$,

$$|\phi_i(t, x, u)| \leq C\left(|x_1|^p + \cdots + |x_i|^p\right) + C \tag{4}$$

Assumption 2: There is a known constant D such that

$$|y_r(t)| \leq D, \quad |\dot{y}_r(t)| \leq D \tag{5}$$

Remark 1. Assumption 1 is slightly more general than the assumption imposed in [10] which contains no constant term. Due to the lack of this constant term in growth-conditions of [10], the smooth output feedback control schemes developed in [10] cannot be applied to such a class of genuinely nonlinear systems such as

$$\dot{x}_1 = x_2^3 + x_1,\ \dot{x}_2 = u, \quad y = x_1 \tag{6}$$

Notably, (6) involve an uncontrollable/unobservable linearization. Moreover, the uncontrollable mode has an eigenvalue on the open right-half plane. As a result, there are no smooth state/output feedback control laws stabilizing either (6) at the origin, even locally. Hence, needless to say, the problem of global output tracking is, more challenging and difficult than the feedback stabilization. Due to the growth condition (4), the family of nonlinear systems considered in this paper is larger than those considered in [10]. In this sense, the result in this paper is a generalization of the result in [10].

2. Global Practical Output Tracking by Output Feedback

In this section we show how to extend the output feedback stabilization results in [10]to achieve practical tracking of system (1)

Theorem 1: Under Assumptions 1-2, the problem of global practical output tracking for system (1) is solvable by a smooth dynamic output feedback controller of the form (2).

Proof: Define $e_1 = y,\ e_i = x_i,\ i = 2, \ldots n.$ Note that, in the definition of the error signal $e_i,\ i = 1, \ldots, n$, we only change the coordinate of the first state x_1. It is different to the common definition used in solving asymptotic tracking, where the error is defined as the difference between all the states and steady values. Then

$$\begin{aligned} \dot{e}_i &= e_{i+1}^p + \varphi_i(t, e, u),\ i = 1, \ldots, n-1, \\ e_n &= u + \varphi_n(t, e, u), \quad y = e_1 \end{aligned} \tag{7}$$

where

$$\varphi_1(t, e, u) = \phi_1(e_1 + y_r(t), e_2, \ldots, e_n, u) - \dot{y}_r(t),$$
$$\varphi_i(t, e, u) = \phi_i(e_1 + y_r(t), e_2, \ldots, e_n, u), \quad i = 2, \ldots, n$$

By assumptions 1-2, it is readily to show that, for $i = 1, 2, \ldots, n$,

$$|\varphi_i(t, e, u)| \leq C_1\left(|e_1|^p + |e_2|^p + \cdots + |e_i|^p\right) + C_1 \tag{8}$$

where $C_1 > 0$ is a constant only depending on $C > 0$, D and p.

Next, introduce the following rescaling transformation

$$z_1 = e_1, \quad z_i = e_i \Big/ M^{\frac{1}{p} + \cdots + \frac{1}{p^{i-1}}}, \quad i = 2, \ldots, n,$$
$$v = u \Big/ M^{1 + \frac{1}{p} + \cdots + \frac{1}{p^{n-1}}} \tag{9}$$

where $M \geq 1$ is a rescaling factor to be assigned later.

In the rescaling coordinates z_i's, the uncertain system (7) can be transferred to

$$\begin{aligned} \dot{z}_i &= M z_{i+1}^p + \psi_i(t, z, v), \quad i = 1, \ldots, n-1, \\ \dot{z}_n &= M v + \psi_n(t, z, v), \quad y = z_1 \end{aligned} \tag{10}$$

where

$$\psi_1(t,z,v)=\varphi_1(t,e,u),\ \psi_i(t,z,v)=\frac{\varphi_i(t,e,u)}{M^{1/p+\cdots+1/p^{i-1}}},2\le i\le n$$

Using (8) and the fact that $M\ge 1$, it is easy to see that

$$|\psi_1(t,z,v)|=|\varphi_1(t,e,u)|\le C_1|z_1|^p+C_1,$$

$$|\psi_i(t,z.v)|\le C_1M^{1-1/p^i}\left(|z_1|^p+\cdots+|z_i|^p\right)+\frac{C_1}{M^{1/p^{i-1}}} \quad (11)$$

for $i=2,\ldots,n$.

In this way, a new parameter-the rescaling factor M –is introduced for the design of dynamic output compensators. It creates an extra freedom and plays an important role in dealing with the system uncertainty, i.e., $\psi_i(\cdot)$, $1\le i\le n$, in (10).

i) State Feedback Design:

For the rescaled system (10) satisfying the growth condition (11), global practical output tracking is achievable by smooth state feedback.

Let $\xi_1=z_1-z_1^*$ with $z_1^*=0$ and choose the Lyapunov function $U_1(\xi_1)=\xi_1^2/2$. Then it is easy to deduce from (11) that

$$\dot U_1(\xi_1)\le M\left[\xi_1z_2^{*P}+a_1\xi_1^{p+1}+\xi_1\left(z_2^p-z_2^{*p}\right)\right]+\alpha_1M^{-1/p} \quad (12)$$

where $\alpha_1>0,a_1>0$ are known and independent of M. Thus the virtual controller $z_2^*=-a_1\xi_1$ is such that

$$\dot U_1(\xi_1)\le M\left[-(n+5)\xi_1^{p+1}+\xi_1\left(z_2^p-z_2^{*p}\right)\right]+\alpha_1M^{-1/p} \quad (13)$$

Using an inductive argument similar to the one in [9], one can find a set of virtual controllers, transformations, and Lyapunov functions

$$\begin{aligned}&z_1^*=0,\ z_i^*=-a_{i-1}\xi_{i-1},\quad i=2,\ldots,n,\\&\xi_i=z_i-z_i^*,\ i=1,\ldots,n, \qquad (14)\\&U_i=U_{i-1}+\xi_i^2/2,\ i=1,\ldots,n,\quad where\ U_0=0\end{aligned}$$

and the smooth state feedback control law

$$v^*=z_{n+1}^*=-(a_n\xi_n)^p=-[b_1z_1+\cdots+b_nz_n]^p \quad (15)$$

such that

$$\dot U_n(\xi_1,\ldots,\xi_n)\le M\left[-6\sum_{i=1}^n\xi_i^{p+1}+|\xi_n||v-z_{n+1}^*|\right]+\sum_{i=1}^n\alpha_iM^{-1/p^i} \quad (16)$$

where all constants a_i, b_i, $i=1,\ldots,n$ and α_1 are known and independent of M.

ii) Output Feedbak Design:

Since $(z_2,\ldots,z_n)$ of the rescaled system (10) are unmeasurable but $y=z_1$ is measurable, we need only to design an (n-1)-dimensional observer for (10). The unmeasurable variables $(z_2,\ldots,z_n)$ defined by

$$\eta_i=z_i-L_i\ldots L_2z_1,\quad i=2,\ldots,n \quad (17)$$

where the parameters $L_n,\ldots,L_2\ge 1$ are gain constants to be determined later.

From (17), it follows that

$$\begin{aligned}\dot\eta_i&=Mz_{i+1}^p+\psi_i(\cdot)-L_i\cdots L_2\left(Mz_2^p+\psi_1(\cdot)\right),i=2,\ldots,n-1\\\dot\eta_n&=Mv+\psi_n(\cdot)-L_n\cdots L_2\left(Mz_2^p+\psi_1(\cdot)\right)\end{aligned} \quad (18)$$

In view of (18), one can construct the $(n-1)$ dimensional observer

$$\begin{aligned}\dot{\hat\eta}_i&=M\left(\hat\eta_{i+1}+L_{i+1}\cdots L_2z_1\right)^p-ML_i\cdots L_2\left(\hat\eta_2+L_2z_1\right)^p\\\dot{\hat\eta}_n&=Mv-ML_n\cdots L_2\left(\hat\eta_2+L_2z_1\right)^p,i=2,\ldots,n-1,\end{aligned} \quad (19)$$

which does not involve the uncertain functions $\psi_i(\cdot)$, $i=1,\ldots,n$ in (10). Moreover, the estimates of z_i's can be obtained based on the relationships

$$\hat z_i=\hat\eta_i+L_i\ldots L_2z_1,\qquad i=2,\ldots,n \quad (20)$$

Let $\varepsilon_i=z_i-\hat z_i=\eta_i-\hat\eta_i$, $i=2,\ldots,n$, be the estimate errors. Then, the error dynamics is given by

$$\begin{aligned}\dot\varepsilon_i&=M\left(z_{i+1}^p-\hat z_{i+1}^p\right)+\psi_i(\cdot)-ML_i\cdots L_2\left(z_2^p-\hat z_2^p\right)\\&\quad-L_i\cdots L_2\psi_1(\cdot),\quad i=2,\ldots,n-1\\\dot\varepsilon_n&=\psi_n(\cdot)-ML_n\cdots L_2\left(z_2^p-\hat z_2^p\right)-L_n\cdots L_2\psi_1(\cdot)\end{aligned} \quad (21)$$

By the certainty equivalence principle, the unmeasurable state $(z_2,\ldots,z_n)$ in the controller (15) can be replaced by its estimate $(\hat z_2,\ldots,\hat z_n)$ generated by the nonlinear observer (19)-(20). In this way, one obtains the implementable feedback controller

$$v=-\left[b_1z_1+b_2\hat z_2\cdots+b_n\hat z_n\right]^p \quad (22)$$

Substituting (22) into (16), we have

$$\dot U_n\le M\left[|\xi_n|K_1\sum_{i=2}^n|\varepsilon_i|\left(\sum_{i=1}^nz_i^{p-1}+\sum_{i=2}^n\varepsilon_i^{p-1}\right)-6\sum_{i=1}^n\xi_i^{p+1}\right]+\sum_{i=1}^n\alpha_iM^{-1/p^i}$$

where $K_1>0$ is real constant related to b_i's and independent of L_i, $2\le i\le n,M$.

By (14), the aforementioned inequality can be simplified as

$$\dot{U}_n \le M\left[-5\sum_{i=1}^{n}\xi_i^{p+1} + K_2\sum_{i=2}^{n}\varepsilon_i^{p+1}\right] + \sum_{i=1}^{n}\alpha_i M^{-1/p^i} \quad (23)$$

where $K_2 > 0$ is a constant independent of $L_i,\ 2 \le i \le n, M$.

To determine the observer gains $L_i,\ i = 2,\ldots,n$, consider the change of coordinates

$$\tilde{\varepsilon}_2 = \varepsilon_2,\ \tilde{\varepsilon}_3 = \varepsilon_3 - L_3\varepsilon_2,\ldots,\tilde{\varepsilon}_n = \varepsilon_n - L_n\varepsilon_{n-1}. \quad (24)$$

In the coordinates of ξ and $\tilde{\varepsilon}$, (23) can be represented as

$$\begin{aligned}\dot{U}_n \le M\Big[&-5\sum_{i=1}^{n}\xi_i^{p+1} + c_2\left(L_3,\ldots,L_n\right)\tilde{\varepsilon}_2^{p+1} \\ &+\cdots + c_{n-1}\left(L_n\right)\tilde{\varepsilon}_{n-1}^{p+1} + c_n\tilde{\varepsilon}_n^{p+1}\Big] + \sum_{i=1}^{n}\alpha_i M^{-1/p^i}\end{aligned} \quad (25)$$

The error dynamics (21) in the coordinate $\tilde{\varepsilon}$ can be rewritten as

$$\begin{aligned}\dot{\tilde{\varepsilon}}_i &= M\left(z_{i+1}^p - \hat{z}_{i+1}^p\right) + \psi_i(\cdot) - ML_i\left(z_i^p - \hat{z}_i^p\right) - L_i\psi_{i-1}(\cdot) \\ \dot{\tilde{\varepsilon}}_n &= \psi_n(\cdot) - ML_n\left(z_n^p - \hat{z}_n^p\right) - L_n\psi_{n-1}(\cdot),\ i = 2,\ldots,n-1,\end{aligned} \quad (26)$$

Now, consider the Lyapunov function

$$W_n\left(\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n\right) = \frac{1}{2}\left(\tilde{\varepsilon}_2^2 + \cdots + \tilde{\varepsilon}_n^2\right)$$

A direct computation gives

$$\begin{aligned}\dot{W}_n = M\Big\{&4\sum_{i=1}^{n}\xi_i^{p+1} - \frac{1}{2^{p-1}}\sum_{i=2}^{n}L_i\tilde{\varepsilon}_i^{p+1} \\ &+ \sum_{i=2}^{n-1}\left[\tilde{c}_i\left(L_{i+1},\ldots,L_n\right) + \left(K_i+1\right)M^{-1/p^{i-1}}L_i^{p+1}\right]\tilde{\varepsilon}_i^{p+1} \\ &+\left[\tilde{c}_n + \left(K_n+1\right)M^{-1/p^{n-1}}L_n^{p+1}\right]\tilde{\varepsilon}_n^{p+1}\Big\} + 2\alpha_1\sum_{i=2}^{n}M^{-1/p^i}\end{aligned} \quad (27)$$

Finally, choose the Lyapunov function

$$V_n\left(\xi,\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n\right) = U_n\left(\xi_1,\ldots,\xi_n\right) + W_n\left(\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n\right)$$

For the closed-loop system in the coordinate $\left(\xi,\tilde{\varepsilon}\right)$.

Then, it follows from (25) and (27) that

$$\begin{aligned}\dot{V}_n \le -M\Big\{&\left[\xi_1^{p+1} + \cdots + \xi_n^{p+1}\right] + \sum_{i=2}^{n-1}\Big[\frac{L_i}{2^{p-1}} - C_i(L_{i-1},\ldots L_n) \\ &-\left(K_i+1\right)M^{-1/p^{i-1}}L_i^{p+1}L_i^{p+1}\Big] \\ &+\left[\frac{L_n}{2^{p-1}} - C_n - \left(K_n+1\right)M^{-1/p^{n-1}}L_i^{p+1}{}_n^{p+1}\right]\tilde{\varepsilon}_n^{p+1}\Big\} \\ &+ 2\alpha_1\sum_{i=2}^{n}M^{-1/p^i} + \sum_{i=1}^{n}\alpha_i M^{-1/p^i}\end{aligned} \quad (28)$$

Where $C_2(L_3,\ldots L_n),\ldots,C_{n-2}(L_{n-1},L_n),C_{n-1}(L_n)$ are positive constants independent of M , while $C_n > 0$ and $K_i > 0,\ 2 \le i \le n$ are positive constants independent of L_i 's and M .

From (28), it is easy to conclude that if the gain pa rameters L_i 's and M are assigned one by one.

We have

$$\dot{V}_n\left(\zeta\right) \le -\left[\sum_{i=1}^{n}\xi_i^{p+1} + \sum_{i=2}^{n}\tilde{\varepsilon}_i^{p+1}\right] + F(M) \quad (29)$$

Where $\zeta = \left(\zeta_1,\ldots,\zeta_{2n-1}\right)^T = \left(\xi_1,\ldots,\xi_n,\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n\right)^T$, $F(M) = \sum_{i=1}^{n}\left(2\alpha_1 + \alpha_i\right)M^{-1/p^i}$. It is easy to see $F(M)$ is positive and monotone decreasing to zero as M increases.

Next, we will show that (29) implies the existence of a gain M to achieve the global practical tracking of system (1).

By [9], there is a finite $T > 0$

$$\zeta_i^2/2 \le V(\zeta(t)) \le nF(M)^{2/(p+1)},\quad \forall t \ge T, i = 1,\ldots,n. \quad (30)$$

This implies that all the solutions $\zeta = \left(\zeta_1,\ldots,\zeta_{2n-1}\right)^T = \left(\xi_1,\ldots,\xi_n,\tilde{\varepsilon}_2,\ldots,\tilde{\varepsilon}_n\right)^T$ of the closed-loop system are globally bounded and well defined over $[0,\infty)$. This, in turn, leads to the conclusion that the state $\left(x_1,\ldots,x_n\right)$ are globally bounded, because of the relation (9),(14) and boundedness of y_r .

To achieve pratical output tracking, we must show that by choosing M appropriately, the output error

$$\left|\xi_1(t)\right| = \left|y(t)\right| = \left|x_1(t) - y_r(t)\right|$$

can be made arbitrarily small in a finite time. To this end, we choose $M \ge \max\left\{L_2^{(p+1)p}, L_3^{(p+1)p^2},\ldots,L_n^{(p+1)p^{n-1}}\right\} \ge 1$, so there is a finite time $T(\zeta(0),M) > 0$, such that

$$\left|x_1(t) - y_r(t)\right| = \left|\xi_1(t)\right| \le \left[nF(M)^{2/(p+1)}\right]^{1/2},\ \forall t \ge T > 0.$$

Now, from monotone decreasing property of $F(M)$, for any given $\varepsilon > 0$ there is a sufficiently large M such that

$$\left[nF(M)^{2/(p+1)}\right]^{1/2} \le \varepsilon.$$

That is $\left|y(t)\right| = \left|x_1(t) - y_r(t)\right| \le \varepsilon$, for all $\forall t \ge T > 0$. This completes the proof of Theorem 1.

3. An Illustrative Example

Example: We work out a simple numerical example to illustrate the result described in Theorem 1. The example we consider is a 2-dimensional system of the following form:

$$\dot{\eta}_1 = \eta_2^3 + \eta_1^2 \eta_2 \ln\left(1+\eta_1^2\right) \big/ \left(1+\eta_1^2\right)$$
$$\dot{\eta}_2 = u, \quad y = \eta_1 - y_r \tag{31}$$

The control objective is to force the state η_1 to track the reference $y_r = \sin(t)$ using the measurement $y(t)$ only.

It is easy to verify that system (31) satisfies Assumption 1 and Assumption 2. According to (19) and (22), we can construct a dynamic output feedback controller as

$$\dot{\hat{\eta}}_2 = \frac{u}{M^{1/3}} - ML_2\left(\hat{\eta}_2 + L_2\left(\eta_1 - y_r\right)\right)^3$$
$$u = -M^{4/3}\left(24\left(\hat{\eta}_2 + L_2\left(\eta_1 - y_r\right)\right) + 24\left(\eta_1 - y_r\right)\right)^3 \tag{32}$$

By Theorem 1, with properly chosen M, following the design procedure above, the tracking error can be made arbitrarily small. In Fig.1 we plot out the simulation results.

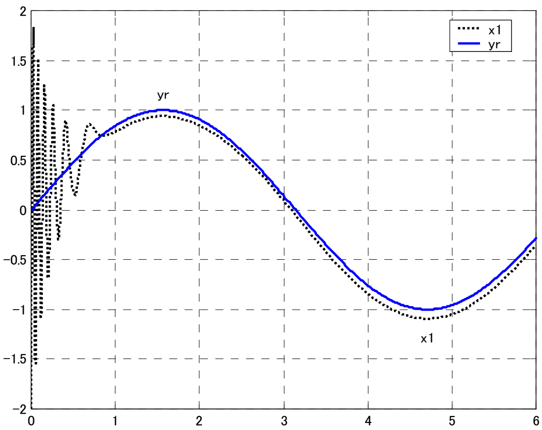

Fig.1. Simulation result for the closed-loop system (31)-(32). (Where $L_2 = 4$, $M = L_2^{12} + 1$, $y_r = \sin(t)$)

4. Conclusions

For a class of nonlinear systems having nonstabilizable linear approximation, a method for the practical output regulation using a smooth output feedback has been developed and successfully tested by numerical simulations.

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References

[1] E. J. Davison, "The robust control of a servomechanism problem for linear time-invariant multivariable systems," *IEEE Trans. Automat.Contr.*, vol. AC-21, pp. 25–34, Jan. 1976.

[2] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, pp. 457–465, 1976.

[3] Anantharam, V. and Desoer, C. A. "Tracking and disturbance rejection of MIMO nonlinear systems with a PI or PS controller," Proc. of the 24th IEEE CDC,pp.1367-1368(1985).

[4] J. S. A. Hepburn andW. A.Wonham, "Error feedback and internal model on differentiable manifolds," IEEE Trans. Automat. Contr., vol. AC-29, pp. 397–403, May 1984.

[5] M. D. Di Benedetto, "Synthesis of an internal model for nonlinear output regulation," *Int. J. Control*, vol. 45, pp. 1023–1034, 1987.

[6] J. Huang and W. J. Rugh, "On a nonlinear multivariable servomechanism problem," *Automatica*, vol. 26, pp. 963–972, 1990.

[7] A. Isidori and C. I. Byrnes, "Output regulation of nonlinear system," *IEEE Trans. Automat. Contr.*, vol. 35, pp. 131–140, Feb. 1990.

[8] S. Celikovsky and J. Huang, "Continuous feedback practical output regulation for a class of nonlinear systems having nonstabilizable linearization," in Proc.38th IEEE Conf. Decision Control, Phoenix, AZ, 1999, pp. 4796–4801.

[9] C. Qian and W. Lin, "Practical output tracking of nonlinear systems with uncontrollable unstable linearization," *IEEE Trans. Automat. Contr.*, vol. 47, pp. 21–36, Jan. 2002.

[10] Yang, B. and Lin, W. "Robust output feedback stabilization of uncertain nonlinear systems with uncontrollable and unobservable linearization", IEEE Trans. Automat. Contr., Vol. 50, pp. 619-630, 2005.

[11] W. Lin and C. Qian, "Robust regulation of a chain of power integrators perturbed by a lower-triangular vector field," *Int. J. Robust Nonlinear Control*, vol. 10, pp. 397–421, 2000.

[12] C. I. Byrnes, F. Delli Piscoli, and A. Isidori, *Output Regulation of Uncertain Nonlinear Systems*. Boston, MA: Birkhäuser, 1997.