A NSGAII approach to the fault detection filter design

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Abstract- In this paper, we propose to design a fault detection filter for a linear time invariant system using the non-dominated sorting genetic algorithm II. The fault detection filter is an observer with a set of projectors that map each fault in a specific residual direction. The design of the fault detection filter is formulated as a multi-objectives optimisation problem in the frequency domain. The non-dominated sorting genetic algorithm II is utilised to tune the filter gain and each projector in order to minimise the sensitivity of the fault signals to be blocked and maximise the sensitivity of each fault signal to be identified in each residual direction. With this approach, different fault isolation problems can be formulated; simultaneous faults or one fault at a time. Furthermore, there is a large freedom in the way the observer gain and the projectors can be designed. Finally, the viability of the approach is demonstrated through the detection and the isolation of sensor and actuator faults for a linear aircraft model.

Keywords: Genetic algorithm, fault detection filter, multi-objective optimisation, residual generation, aircraft.

1 Introduction

A traditional technique to remedy to the problem of automatic control system failures is known as hardware redundancy. Several hardware modules are multiplexed to increase the level of integrity of the underlying system. However, this technique is often criticised for its complexity, cost and the weight it adds to the system. A method that addresses the drawback of hardware redundancy is known as model based fault diagnosis. Model based fault diagnosis relies usually upon a mathematical model of the system to generate residual signals that are indicators of the presence of faults. Observer based approaches [1][2][3][4][5][6][7] are some of the common approaches that are found in the fault diagnosis literature. This paper focuses on one observer based approach known as the fault detection filter. Beard [6] proposed first the fault detection filter. It was later refined by several investigators using eigenstructure assignment [3] and geometric theory [4]. Moreover the robustness of the fault detection filter was addressed using different formulation such as left eigenstructure assignment [7] and H-infinity theory [5].

In this paper the problem of the design of the fault detection filter is formulated as a multi-objective optimisation problem. This paper is organised as follows. The fault detection filter background is provided in section 2. Section 3 gives a brief summary of the genetic algorithm utilised for the purpose of the multi-objectives optimisation problem. Section 4 presents the multi-objective optimisation problem formulation. Finally, section 5 demonstrates the effectiveness of the approach through an example.

2 Fault Detection Filter

Consider the linear time invariant (LTI) system:

$$\dot{x}(t) = Ax(t) + Bu(t) + F_1 f_1(t) + F_2 f_2(t) + \cdots + F_p f_p(t) \quad (1)$$

$$y(t) = Cx(t) + E_1 f_1(t) + E_2 f_2(t) + \cdots + E_p f_p(t) \quad (2)$$

$F_i f_i(t)$ are actuator faults, $E_i f_i(t)$ are sensor faults $,i=1,2,\ldots,p$.

A fault detection filter is a Luenberger observer of the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)) \quad (3)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (4)$$

The residual signal is given by:

$$r(t) = y(t) - \hat{y}(t) \quad (5)$$

And governed by the error dynamic:

$$\dot{e}(t) = (A - LC)e(t) + F_1 f_1(t) + F_2 f_2(t) + \cdots + F_p f_p(t) \quad (6)$$

The fault detection filter design consists of mapping each fault direction in a unique residual direction as follows:

$$r_1(t) = H_1 r(t) \quad (7)$$

$$r_2(t) = H_2 r(t) \quad (8)$$

$$\vdots$$

$$r_p(t) = H_p r(t) \quad (9)$$

Where H_i is a projector. $i=1,2,\ldots,p$

In geometric term [4], L and H_i are selected such that the *ith* actuator fault is confined in the *ith* residual subspace and the *jth* actuator faults are placed in unobservable subspace of the *ith* residual. This ensures that *ith* fault is uniquely identified from the *jth* faults in the residual. Moreover L must also be chosen such that A-LC is asymptotically stable. L can be designed using eigenstructure assignment [3] or geometric theory [4]. The fault detection filter can also accommodate the sensor fault problem. The sensor fault must be transformed into a pseudo actuator fault [7]. Finally, one important requirement for the existence of the fault detection filter is that the fault directions must be linearly independent at the output of the system. When this condition is not fulfilled another method should be utilised. In this paper this problem is addressed using a multi-objective optimisation evolutionary approach known as fast elitist non-dominated genetic algorithm II [8]. This algorithm is briefly discussed in the next section.

3 Non-dominated Sorting Genetic Algorithm II

Genetic algorithm was developed by J.Holland, his colleagues and his students at the University of Michigan [9] .Genetic algorithm is an optimisation algorithm which built its foundation from the evolutionary theory where the fittest species will survive and the weaker will be eliminated. For solving multi-objectives optimisation problem, advanced genetic algorithms were devised to find the best compromise between objective functions or Pareto optimality. In this research a fast elitist non-dominated genetic algorithm II (NSGAII) was chosen to solve the optimisation problem. NSGAII retains the three main operations of a basic genetic algorithm; selection, crossover and mutation. Additional mechanisms were added to help the algorithm to estimate the Pareto optimal front. In brief NSGAII works as follows. A population of individuals of size N is first randomly generated. The solutions from this populations are then organised in fronts (Front 1,2,3..etc). This ranking process is performed with a fast non-dominated algorithm. A tournament selection with a special feature called crowding operation is applied to the solutions to keep the fittest among the ranked solutions. New offspring are generated from the fittest solutions using simulated binary crossover and polynomial mutation. These two operators were devised to deal with real parameter solutions. The populations and the offspring are then combined to form a population with a size that is at utmost 2N. The process is reiterated until the maximum number of generations is reached.

4 Multiobjective Optimisation Problem Formulation

In this research work, a different approach from traditional techniques is adopted for the design of the fault detection filter. The fault detection filter design is formulated as multi-objectives optimisation problem and solved using NSGAII. To simplify the presentation, the problem is posed to identify only actuator faults. Using relations 5,6,7,8,9, it can be shown that residual signals can be expressed in the Laplace domain as follows:

$$r_1(s) = H_1G_1(s)f_1(s) + H_1G_2(s)f_2(s) + \cdots + H_1G_q(s)f_q(s) \quad (10)$$

$$r_2(s) = H_2G_1(s)f_1(s) + H_2G_2(s)f_2(s) + \cdots + H_2G_q(s)f_q(s) \quad (11)$$

$$r_p(s) = H_pG_1(s)f_1(s) + H_pG_2(s)f_2(s) + \cdots + H_pG_q(s)f_q(s) \quad (12)$$

Where $G_i(s) = C(sI - A + LC)^{-1}F_i$, $i=1,2,..,q$, $q \geq p$. p is the maximum number of residuals

These equations describe the influence of each fault signal on each residual $r_i(t)$. Note that these signals could also be assumed to be sensor faults, noise and external disturbances. To maximise the sensitivity of the *ith* fault and minimise the *jth* faults in the *ith* residual, the gain L and the projectors Hi must be found such as the following objective function is minimised:

$$J_i(H_i, L) = min \frac{\|H_i G_j(s)\|_\infty}{\|H_i G_i(s)\|_\infty} \tag{13}$$

Where $\|\cdot\|_\infty$ denotes the H-infinity norm. *min* means minimisation .i=1,2,3,….m, i≠j .

Using the objective function (13), a multi-objectives optimisation problem can be formulated to either address simultaneous fault isolation if $q \leq p$ or successive single faults at any one time if $q > p$.

Note that for the sensor faults problem, the following two objective functions are used:

$$J_i(H_i, L) = min \frac{\|H_i G_j(s)\|_\infty}{\|H_i G_i(s)\|_\infty} \tag{14}$$

$$J_i(H_i, L) = min \frac{\|H_i G_j(s)\|_\infty}{\|H_i G_i\|_-} \tag{15}$$

Where $G_i(s) = C(sI - A + LC)^{-1}(-KE_i) + E_i$ is the transfer function from sensor fault to the residual. $G_i = C(-A + LC)^{-1}(-KE_i) + E_i$ is the steady gain of the sensor. $\|G_i\|_-$ is minimum singular value.

Since the problem is a multi-objective optimisation problem where no unique solution can be found, we propose the design of the fault detection filter using NSGAII. The advantage of using genetic algorithms is that the end result is a set of optimal fault detection filters. The designer can assess each optimal solution and select the one that suites best the problem to solve.

Several ways were identified for the design of the fault detection filter parameters (L and Hi).

They are summarised as follows:

- Method 1: *L* and *Hi* can be designed both by NSGAII.
- Method 2: *L* can be predefined using a pole placement approach and *Hi* is found by NSGAII
- Method 3:*Hi* can be pre-defined by placing each fault to be blocked in the unobservable subspace and L is found by NSGAII to force *ith* fault direction onto the predefined *ith* residual subspace. A similar idea was proposed in reference [10]. But the algorithm is based on Lyaponov equations.
- Method 4: Similar to the previous method, *Hi* is first pre-defined. Moreover an additional freedom term can be added to each projector. Since the role of the projector Hi is to place the *jth* fault *Fj* in the unobservable subspace or equivalently or $H_i C F_j = 0$. This is the same as $N H_i C F_j = 0$ where *N* is a free parameter matrix or vector with appropriate dimension. Since anything time zero is equal to zero. This additional parameter could be used to achieve other requirements.

The design of the gain *L* is based on an eigenstructure approach proposed in references [2][11]. The gain *L* is defined as follows:

$$L = \left(WV^{-1}\right)^T \tag{16}$$

$W = \begin{bmatrix} w_1^T & w_i^T & \cdots & w_i^T \end{bmatrix}$ are left eigenvectors, $V = \begin{bmatrix} v_1 & v_2 & \cdots & v_i \end{bmatrix}$, are right eigenvectors., $v_i = -(\lambda_i I - A^T)^{-1} C^T w_i$, λ_i are real eigenvalues. (For complex conjugate see [11]).

The implementation of the NSGAII requires the selection of several parameters. The first one is the alphabet of the genetic encoding. Since the elements of W, Hi, λ contain real values, it was decided to encode all decisions variables with real values. The genetic coding (GC) is given by:

$$\begin{aligned} GC = [\, & w_1 \quad w_2 \quad \cdots \quad w_n\,, \\ & \lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_n\,, \\ & h_{11} \quad h_{12} \quad \cdots \quad h_{1n}\,, \\ & h_{n1} \quad h_{n2} \quad \cdots \quad h_{nn}\,] \end{aligned} \tag{17}$$

Where w_i, h_{ij} are the elements of the matrices *W, Hi,* respectively. λ_i are eigenvalues. In practise only the eigenvalues need to be in a predefined region [2]. This ensures that all eigenvalues of the matrix *A-LC* are asymptotically stable. The other parameters are defined in the next section.

Patton *et al* [2] proposed a multi-objectives approach using the inequality method [12] combined with a genetic algorithm to find the gain L only for the design of the robust fault detection. The selection process for the L and Hi parameters in the design of the detection filter is fundamentally different in the research work presented here. The fast elitist non-dominated genetic algorithm II is used to find L and a set of projectors Hi to isolate simultaneous faults. In case it is not possible to isolate the faults simultaneously, like for example the case where the number of faults and uncertainties is larger than the number of residuals, the objectives can be easily reformulated to consider the case where one fault at a time appears in the system. The faults can then be isolated successively rather than simultaneously.

5 Aircraft Application

5.1 Aircraft model

The perturbed lateral motion of an aircraft model was found in reference [13].

$$\dot{x}(t) = Ax(t) + B\delta(t) + F_1 f_1(t) + F_2 f_2(t) \qquad (18)$$
$$y(t) = Cx(t) + E_1 f(t)$$

Where the states are side velocity (ft/s), roll rate (deg/sec), yaw rate (deg/s), roll angle (deg) respectively. δ_a, δ_r are the aileron and rudder deflections respectively. $F_1 f_1(t)$ is the rudder actuator fault. $F_2 f_2(t)$ is the aileron actuator fault. $E_1 f(t)$ is the roll rate sensor fault. The aircraft matrices are as follows:

$$A = \begin{bmatrix} -0.1008 & 0 & -468.2000 & 32.2 \\ -0.0058 & -1.2320 & 0.3970 & 0 \\ 0.0028 & -0.0346 & -0.2570 & 0 \\ 0 & 1.0000 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 13.4842 & 0 \\ 0.3920 & -1.6200 \\ -0.8640 & -0.0187 \\ 0 & 0 \end{bmatrix}$$

F_1 and F_2 are equal to the first and the second column of B respectively. E_1 is a column vector with zeros but 1 in the third row.

5.2 Fault detection filter design

Two faults scenarios were considered to explore the design of the fault detection filter. Scenario1 consider the case when the fault directions are linearly independent. Scenario 2 considered the case where the fault directions are coupled.

Using relation (17) the genetic coding is formed. There is no general method for selecting the boundaries of the parameter values of the genetic coding except that the eigenvalues must be selected such that A-LC is asymptotically stable. The boundaries were defined as follows:

$$-10 < \lambda_1 < -3, -11 < \lambda_2 < -4,$$
$$-12 < \lambda_3 < -5, -30 < \lambda_4 < -6$$

NSGAII parameters are:

- Number of real solution variables: Scenario 1=44, Scenario 2=56
- Number of objectives: Scenario 1= 2 Scenario 2= 5
- Population Size: 100
- Number of generation: 100
- Crossover probability: 0.9
- Mutation probability: 0.001
- Crossover distribution index: 20
- Mutation distribution index: 20
- Lower and upper bound of the decision variables: -10<W, H, <10
- Random seed: 0.34

For the first scenario two objective functions were formulated to simultaneously identify the actuator faults *F1* and *F2*. They are given as follows:

$$J_1(H_1, L) = min \frac{\|H_1 G_2(s)\|_\infty}{\|H_1 G_1(s)\|_\infty} \qquad (19)$$

$$J_2(H_2, L) = min \frac{\|H_2 G_1(s)\|_\infty}{\|H_2 G_2(s)\|_\infty} \qquad (20)$$

After a number of generations, the NSGAII returns a set of optimal solutions. One optimal solution among this set was selected. Table 2 shows the magnitude of the two objective functions for this particular solution.

Table 2, scenario1 Pareto optimal solution

J_1	J_2
0.000276	0.028406

Since both objective functions in table 2 are inferior to 1, the two faults can be simultaneously identified. This is further confirmed by drawing the singular value plot from the faults $f_1(t)$ and $f_2(t)$ to each residual. Figure 1 shows the singular plot of the transfer function from the fault $f_1(t)$ and $f_2(t)$ to the residual r1(t). The singular value plot shows that the actuator fault F1 is easily identifiable from the fault F2 in the residual r1(t) since F2 is well attenuated. Similarly in figure 2, the fault F1 is well attenuated compared to the fault F2. The fault F2 can be identified from the fault F1 in the residual r2(t).

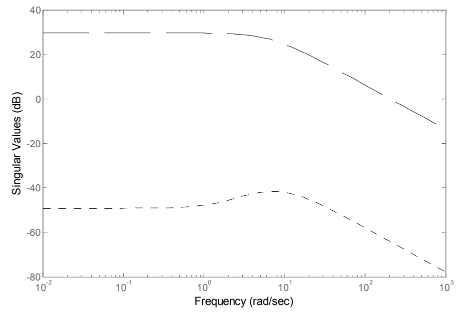

Fig1: Singular plot from actuator fault signals $f_1(t)$ (dashed line), $f_2(t)$ (dotted line) to the residual r1(t).

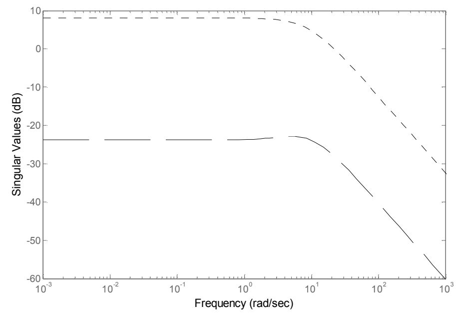

Fig 2: Singular plot from actuator fault signals $f_1(t)$ (dashed line), $f_2(t)$ (dotted line) to the residual r2(t).

For the second scenario 5 objective functions were formulated to isolate all faults in accordance to the truth table 3. The objectives 1 and 2 are already defined in equations 19 and 20. The other three objectives are given as follows:

$$J_3(H_3, L) = min \frac{\|H_3 G_1(s)\|_\infty}{\|H_3 G_3(s)\|_\infty} \quad (21)$$

$$J_4(H_3, L) = min \frac{\|H_3 G_2(s)\|_\infty}{\|H_3 G_3(s)\|_\infty} \quad (22)$$

$$J_5(H_3, L) = min \frac{\|H_3 G_3(s)\|_\infty}{\|H_3 G_3\|_-} \quad (23)$$

Table 3:Truth table

	Fault F1	Fault F2	Fault E1
Residual r1	1	0	1
Residual r2	0	1	1
Residual r3	0	0	1

A logic one in the table 3 means that the corresponding fault must affect the corresponding residual. A logic zero means that the corresponding fault must be minimised in the corresponding residual. Moreover, it is not possible to identify simultaneously all faults with this fault configuration.

After a number of generations, the NSGAII returns a set of optimal solutions. One optimal solution among this set was picked. Table 4 shows the magnitude of the 5 objective functions.

Table 4: Pareto optimal solution

J_1	J_2	J_3	J_4	J_5
0.0929	0.0574	0.0003	0.0001	0.0103

Table 4 indicates that all objectives were minimised to a sufficient level to identify all faults. Figure 3, 4 and 5 show the singular plots of the transfer functions from the actuator faults F1, F2 and the sensor fault E1 to each residual. These figures show that if the sensor fault E1 occurs, all residuals will be affected. If the actuator faults F1, F2 occur then this is the same as scenario 1.

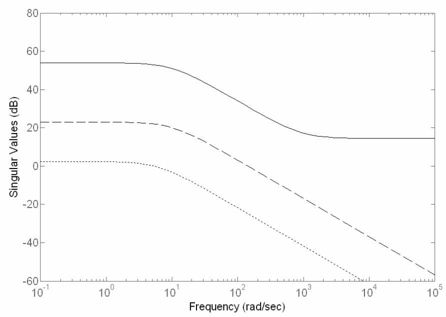

Fig 3: Singular plot from actuator fault signals $f_1(t)$ (dashed line), $f_2(t)$ (dotted line) and the sensor fault signal (solid line) to the residual r1(t).

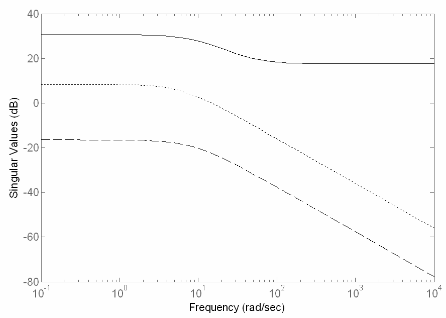

Fig 4: Singular plot from actuator fault signals $f_1(t)$ (dashed line), $f_2(t)$ (dotted line) and the sensor fault signal (solid line) to the residual r2(t).

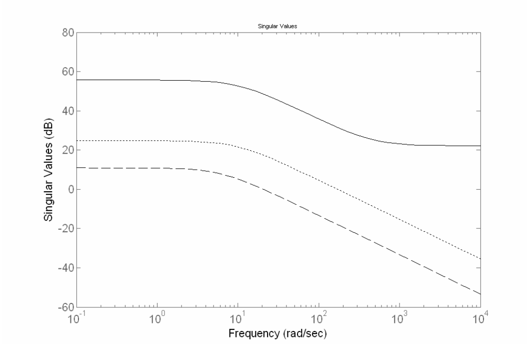

Fig 5: Singular plot from actuator fault signals $f_1(t)$ (dashed line), $f_2(t)$ (dotted line) and the sensor fault signal (solid line) to the residual r3(t).

Note that one may try different faults pattern in table 3 and reformulate the multi-objective optimisation problem accordingly in order to find a set of fault detection filters that performs better than this one. Furthermore, one must select appropriate thresholds in the residuals to perform the fault detection task.

6 Conclusion

In this paper the design of the fault detection filter based on the NSGAII was proposed. This approach provides enough flexibility to address different fault isolation problem formulation. Moreover the robustness problem to disturbances and modelling errors can also be addressed using the same method. The effectiveness of the approach was demonstrated using a linear time invariant aircraft model. Further research work would be to compare some other multi-objective genetic algorithms with the performance of the NSGAII. This research work is already undergoing.

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