A NEW APPROACH OF SLIDING MODE CONTROL FOR INVERSE RESPONSE SYSTEMS

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Abstract: This Paper shows the synthesis of a Sliding Mode Controller using the concepts of the internal model approach. The controller designed comes from the invertible part of the model, which represents the model free of RHZ. The resulting controller is a discontinuous feedback one, presenting a fixed algorithm, with tuning parameters as a function of the characteristics parameters of the model of the process.

Keywords: Inverse response, Non minimum phase systems. Sliding mode.

1 Introduction

Inverse response processes are a kind of process where the initial step response is contrary to the final direction, and it is a type of non-minimum phase system. As is well known from linear control theory that non-minimum phase systems result in a limitation of the feasible closed-loop performance. This leads inevitably to reduce performance. The non-minimum-phase behavior limits the frequency bandwidth of the controller and thus makes the plant response slow [1,2].

The identification of this kind of processes is not an easy task [3]. Therefore, the lack of a precise knowledge of model parameters, and the ever presence of disturbances affecting the performance of the regulation process makes the sliding modes an useful tool that can overcome these problems [4,5,6,7].

Recent papers[7,8,9,10] have shown the possibility to combine other controller structures with Sliding Mode Control (SMC), to improve the performance characteristic of SMC and the robustness of the other scheme for processes with long deadtime, integrating, and inverse response systems.

By other side, the use of SMC for controlling chemical processes has gained a great attraction. Camacho and Smith (2000) proposed a SMC based on the FOPDT model for controlling open loop stable chemical processes [11]. Camacho *et al.* (2003) gave the use of the SMC in the internal model control [9]. Rojas *et al.* (2004) extended the use of SMC to control open loop unstable processes [12]. Camacho and De La Cruz (2004) presented Smith predictor based sliding mode control for integrating processes [10]. Cheng and Peng (2005) have also given the design of a SMC system for chemical processes [13].

Therefore, the proposal facilitates the controller synthesis and makes possible using SMC for different kind of processes that can contain non-invertible terms, such as inverse response systems.

In this paper is synthesized, from a reduced order model of the inverse response process, a variable structure controller. The controller designed comes from the invertible part of the model, which represents the model free of Right Half Zeros (RHZ). The resulting controller is of fixed algorithm, with tuning parameters as a function of the characteristic parameters of the model, which can be used for inverse response processes in a general way. The paper is organized as follows: Section two makes a brief description of Internal Model Structure Control (IMC) and SMC. Section 3 describes the controllers' synthesis, the Section 4 shows computer simulations and finally some conclusions are presented.

2 Basic Concepts

2.1 Internal Model Structure Control

The Internal Model Structure is shown in Figure 1. The idea behind this scheme is firstly to obtain a model of the process, and then decompose the model into two components, an invertible one, and other noninvertible. From the invertible model, the controller can be designed

[1].

Therefore, the model can be represented in the following way

$$G_m(s) = G_m^+(s)G_m^-(s) \qquad (1)$$

where $G_m^+(s)$ corresponds to the noninvertible term of the model, and $G_m^-(s)$ is the invertible part. The noninvertible part has an inverse that is not causal or is unstable, such as deadtime, RHZ or unstable poles. By other side, the invertible component is causal and stable, which make its use easy for controller design.

Therefore, the IMC procedure eliminates all elements in the process model that can produce an unrealizable controller.

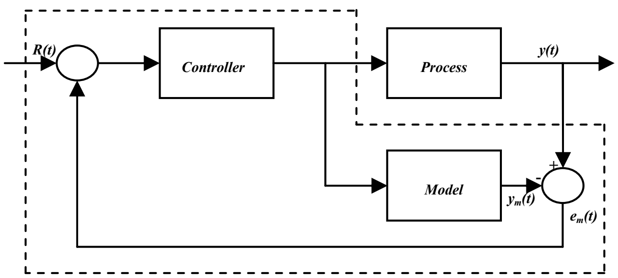

Figure 1. Internal Model Structure

2.2 Sliding Mode Control

Sliding mode control (SMC) (Utkin, 1977; Edwards and Spurgeon, 1998) is well known for its robustness to modeling errors, insensitivity to parameter variations and disturbances, which are expected in practice [4], [6]. It is the property of the SMC that made it useful for many successful practical applications, such as, in robotics (Slotine and Sastry, 1983) [14] and electric drives (Utkin, 1993, Rojas et al, 2004) [15], [12]. There are two parts in the SMC, namely the reaching part and the sliding mode part. In the reaching stage, the system state is derived onto a specified and user chosen surface, which is called sliding surface, in a finite time. Once in the sliding mode, the system dynamics are strictly determined by the dynamics of the sliding surface and therefore the closed loop system becomes insensitive to parameter changes and disturbances (Edwards and Spurgeon, 1998) [6]. However, no such insensitivity to parameter variations and disturbances can be possessed during the reaching phase. For that reason, to guarantee a good closed loop system response, the control system should be designed in such a way that the initial reaching phase is as short as possible (Edwards and Spurgeon, 1998) [6].

To design a SMC controller, the first step is choosing the sliding surface that is usually formulated as a linear function of the system states. The proposed sliding equation is composed of the reference signal, the model output, and the modeling error. Therefore, *s(t)* can be represented

$$s(t) = f\big(R(t), ym(t), e_m(t)\big) \qquad (2)$$

where *R(t)* is the reference, *ym(t)* is the model output, $e_m(t)$ is the modeling error.

Filippov's construction of the equivalent dynamics is the method normally used to generate the equivalent sliding mode control law [5]. It consists of satisfying the following sliding condition

$$\frac{ds(t)}{dt} = 0 \qquad (3)$$

And substituting it into the system dynamic equations, the control law is thereby obtained.

To design the reaching mode control law, the signum function of *s(t)* affected by a constant gain can be used [5,6]. However, this produces the undesirable effect of chattering, normally not tolerated by the actuators. A more appropriate solution is to use the sigmoid-like function, instead of the signum one, to smooth the discontinuity and to obtain a continuous approximation to the surface behavior and avoid chattering [4,5,6] in the control signal when the surface is (pseudo)reached. In a general way, let us propose a general discontinuous control part.

$$U_D(t) = K_D \Psi(t) \qquad (4)$$

Where K_D is the tuning parameter responsible for the speed with which the sliding surface is reached, and $\Psi(t)$ is a nonlinear function of *s(t)*.

3 Synthesis of the controller for inverse response systems.

To design the controller a model of the process should be obtained, as it is known an inverse response system posses RHZ. A second order model with inverse response will be used for design purposes, equation 5 shows the resulting model and the way that it can be divided. The model has two parts connected in cascade. The first part of the model contains the time constant closest to the dominant process time constant, and the overall process gain, while the second model part has the other time constant and the inverse response term a RHZ. Therefore, the model for synthesis purposes can be written as follows:

$$G_m(s) = \frac{K}{(\tau_1 s+1)} \frac{(-\zeta_1 s+1)}{(\tau_2 s+1)} \quad (5)$$

$G_m^-(s)$ does not consider the RHZ term from the model, then it facilitates the SMC design. Hence, the model that will be used is:

$$G_m^-(s) = \frac{K}{\tau_1 s+1} \quad (6)$$

the previous equation can be described in differential equation form, as follows:

$$\tau_1 \frac{dy_m^-(t)}{dt} + y_m^-(t) = Ku(t) \quad (7)$$

Let us use a Proportional-Integral Sliding Surface[16], the resulting sliding surface is as follows:

$$s(t) = K_p e(t) + \lambda \int_0^t e(t)dt \quad (8)$$

Where *e(t)* is the error between the reference, *R(t)*, the model output without RHZ, and e_m(t). Where e_m(t) is the error between the process output and the complete model output. It is observed, that the sliding surface is given indirectly as a function of the reference or set point, the model output of the invertible part and the modeling error. This representation is very important because the controlled variable is given as feedback indirectly through the model output response.

Following the equivalent control procedure [4],

$$\dot{s} = K_p \frac{de(t)}{dt} + \lambda e(t) = 0 \quad (9)$$

Since the tracking error is:

$$e(t) = R(t) - y_m^-(t) - e_m(t) \quad (10)$$

The derivative of the sliding surface can be rewritten as follows:

$$K_p\left(\frac{dR(t)}{dt} - \frac{dy_m^-(t)}{dt} - \frac{de_m(t)}{dt}\right) + \lambda e(t) = 0 \quad (11)$$

Extensive simulation examples have shown that the derivative of the set point variable can be eliminated without affecting the closed loop system performance, Camacho and Smith (2000) [11]. In addition, it is supposed that initially a perfect model is obtained; hence the modeling error is zero and also its derivative is zero. Therefore, the resulting continuous part of the controller is obtained as follows:

$$K_p\left(\frac{y_m^-(t)}{\tau_1} - \frac{K}{\tau_1}u(t)\right) + \lambda e(t) = 0 \quad (12)$$

Replacing (7) in the previous equation, can be obtained

$$u_c(t) = \frac{\tau_1}{KK_p}\left(\frac{K_p y_m^-(t)}{\tau_1} + \lambda e(t)\right) = \frac{y_m^-(t)}{K} + \frac{\tau_1 \lambda}{K_p} e(t) \quad (13)$$

The discontinuous part $\Psi(t)$ is chosen [11] as follows:

$$\Psi(t) = K_D \frac{s(t)}{|s(t)| + \delta} \quad (14)$$

Finally, the complete controller can be written:

$$u(t) = \frac{y_m^-(t)}{K} + \frac{\tau_1 \lambda}{KK_p} e(t) + K_D \frac{s(t)}{|s(t)+\delta|} \quad (15)$$

With a Sliding Surface:

$$s(t) = K_p e(t) + \lambda \int_0^t e(t)dt \quad (16)$$

The scheme of the controller appears in the next figure

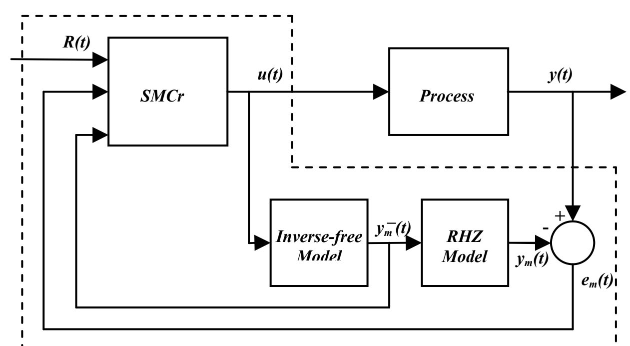

Figure 2 . Controller structure approach

To complete the -Sliding Mode Controller, it is necessary to have a set of tuning equations. For the tuning equations as first estimates, using the Nelder-Mead searching algorithm [12], the following equations were obtained.

$$0 \le \lambda \le (\tau_1)^{-1} \qquad [\text{time}^{-1}] \qquad (17)$$

$$K_D = \frac{0.51}{K}\left(\frac{\tau_1}{\zeta_1}\right)^{0.76} \qquad [\text{fraction CO}] \qquad (18)$$

$$\delta = 0.68 + \frac{0.12}{\tau_1}\left|KK_D\right| \qquad [\text{fraction TO}] \qquad (19)$$

$$K_p = \left(\frac{\tau_1}{\zeta_1}\right) \qquad [\text{dimensionless}] \qquad (20)$$

Eqs. 17,18, 19 and 20 are used when the signals from the transmitter and controller are in fractions (0 to 1). Sometimes, the control systems work in percentages that is, the signals are in % (0 to 100) of range. In these cases the values of K_D and δ are multiplied by 100

The characteristic parameters of the model K, τ_1, τ_2, and ζ_1 are obtained from process identification [3]

4 Simulation Results

This section simulates the control performance of the SMCr designed and given in Eqs.13 and 14. Firstly, a linear model is used to see how the controller tunings work, and secondly a third order nonlinear systems is used to compare the performance of the proposed controller and the SMCr presented by Camacho et al,1999 [13].

The first example can be represented by the following transfer function:

$$G_1(s) = \frac{-0.4(s-0.5)}{(s+1)(s+0.2)} \qquad (21)$$

Figure 3 depicts the closed and open loop responses for a step change. The idea behind this figure is to show that the closed loop response is faster than the open loop without increasing the inverse response effect, and speeding up the direct transient response, therefore, the tuning equations can be used for the designed controller to increase the system's performance.

Figure 4 shows the λ effect over the process response, as can be observed as this parameter increases, both the direct and inverse overshoots increases. If λ continues increasing the integral term grows and the system response becomes oscillatory.

Figure 5 portraits the process response to set point and disturbances changes. In both cases, for set point and disturbance changes the system`s response offers a good behavior, therefore both changes are well adjusted by the controller.

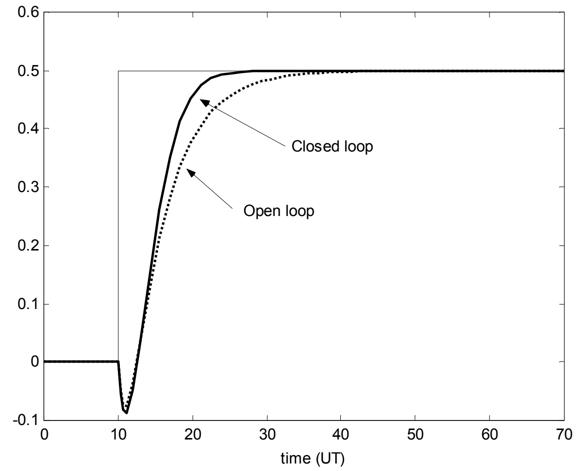

Figure 3. Open and Closed loop responses

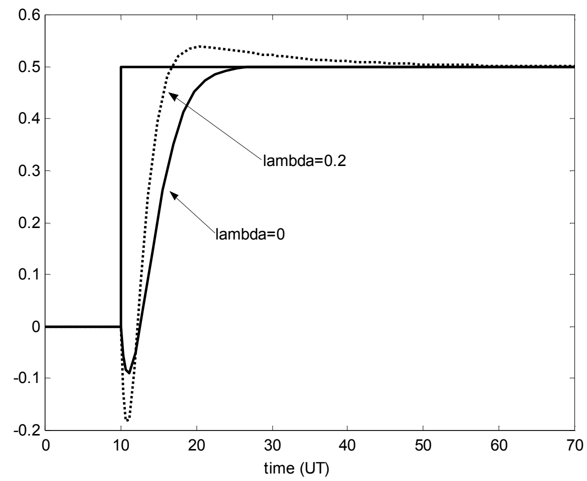

Figure 4. Closed loop response for λ=.2 and λ =0

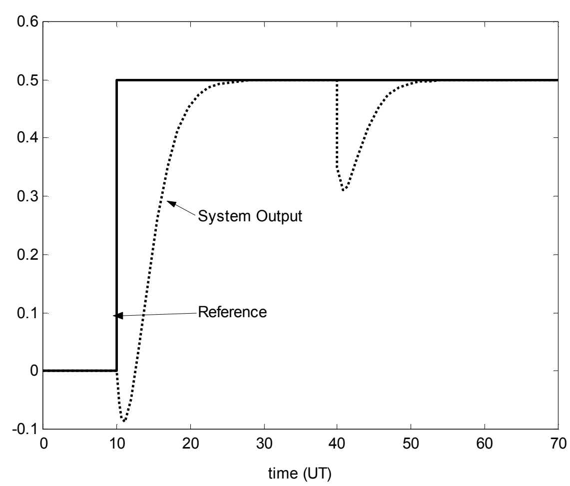

Figure 5. Process response to set point and disturbances changes

The second example is a nonlinear third order system. This example is interesting since from a first view the nonminimun efect does not appears directly, but when the identification procedure is utilized, the resulting models show the presence of RHZs.

The third order model is described as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 + 0x_3 \\ x_1^2 - 3x_2 + 0x_3 \\ x_1 + 0x_2 - 2x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u \tag{22}$$

$$y = x_1 - 3x_3$$

The reaction curve procedure is used to get the linear models. From this procedure, two linear models can be obtained, one with different poles (linear model 1), Eq. (23), and the other one with repeated poles (linear model 2), Eq. (24).

The transfer functions that define the two linear models are:

Linear Model 1

$$G_1(s) = -\left(\frac{0.5}{2.37s+1}\right)\left(\frac{-1.163s+1}{0.332s+1}\right) \tag{23}$$

Linear Model 2

$$G_2(s) = -\left(\frac{0.5}{1.78s+1}\right)\left(\frac{-1.154s+1}{1.78s+1}\right) \tag{24}$$

Figure 6 illustrates the open loop responses, for a unit step change, of the nonlinear and the two linear systems. From that figure, it is observed that the model 1 has a closer behavior (lower ISE) than the linear model 2. In consequence, the linear model 1 is selected, and its characterisct parametrs used for tuning considerations.

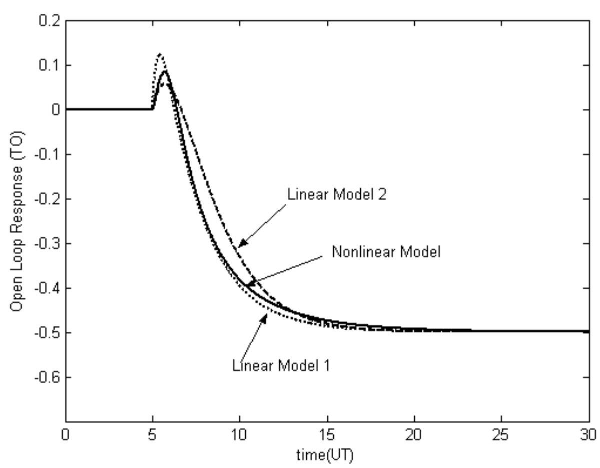

Figure 6. Open loop responses of nonlinear and linear models.

Figure 7 shows the system response for set point and disturbance changes. A set point change of -0.4 at t= 10 units of time and a disturbance of -0.2 at t=30 units of time are used in this simulation. The proposed approach is compared against the SMCr as was proposed by Camacho et al, 1999[13]. The simulation results portray that the new approach presents better performance than the old one. Therefore, the internal model approach is smoother and faster than the previous approach, hence the internal model improves the response of the system.

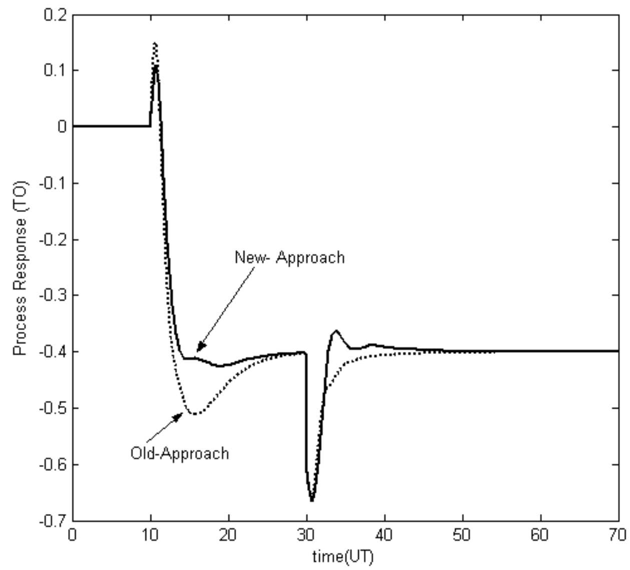

Figure 7. Process responses for set point and disturbance changes, using both controllers.

Figure 8 depicts the controllers responses for the changes mentioned above, the new approach presents a smoother controller response than the old one, thus hard changes are avoided in the final control element. Therefore, the internal model increases the overall performance of sliding mode control to inverse response systems.

Conclusions.

A way to design a SMCr of an inverse response system was shown. The concepts of internal model structure were used to separate the inverse response systems into two linear components, and from the invertible part of the model the controller was sinthesysed. A PI sliding surface has been utilized for designing purpouses.

The paper has shown, a new control scheme, that mixes two control concepts, the internal model approach and sliding mode control concept, for processes with inverse response has been proposed.

The new control scheme was simulated and its performance compared against the approach presented by Camaho et al 1999 [17]. Simulation results showed that the proposed approach outperforms the old approach.

The controller presents a fixed algorthm, which allows a unique controller of adjustable parameters that can easily be implemented using DCS.

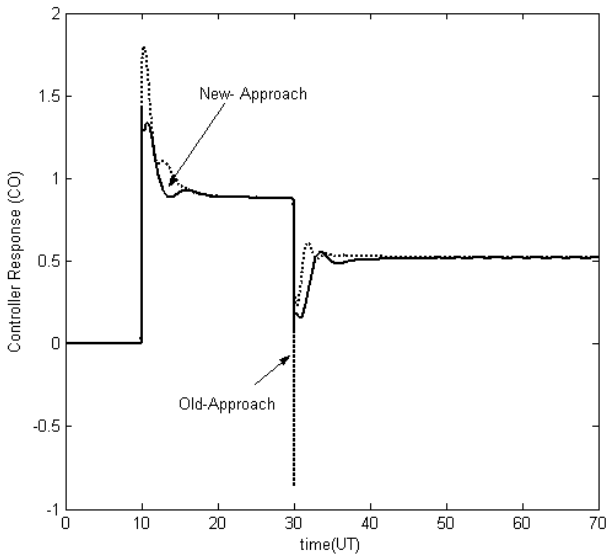

Figure 8. Controller responses for set point and disturbance changes, using both controllers.

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