A Physically-based Model for Chinese Tops

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Abstract: The *Chinese top*, a toy propeller that hovers around when its shaft is spun between the palms, was created thousand years ago. In this paper, we present a physically based interactive simulation model for Chinese tops. Motions of Chinese tops are influenced by thrust, lift, drag and gravity forces. These four kinds of forces are computed in real time, based on the aero-dynamics equations. We expect that our works are expandable to simulate helicopter motions in virtual reality and computer games.

Key–Words: Computer Graphics, Physically-based Modeling, Chinese Top, Helicopter Dynamics.

1 Introduction

Reproduction of natural phenomena is one of interesting things in the filed of computer graphics. Many researchers have studied physically-based methods to increase the reality of these simulations. In this paper, we simulate the motion of the *Chinese top* which is a toy propeller, as shown in Figure 1. Its two wings are called as rotor blades or airfoils. The Chinese top hovers around when its shaft is spun between the palms[1]. Their motion equations are derived from aero-dynamics, and implemented using physically-based techniques.

A man can spin the shaft of a Chinese top to make torque that is converted to thrust and lift forces. Starting to fly upwards, the Chinese top hovers for a while, and then goes down when the lift force decreased.

Helicopters can hover straight up and down just like these Chinese tops, and thus, our Chinese top simulation is based on the helicopter dynamics. We will derive the force equations for the rotor blades from the helicopter dynamics equations.

This paper is organized as follows. In the next section, we provide an overview of previous works in aerodynamics area. The way of computing forces acting on rotor blades is described in Section 3. Some result images from our implementation is followed and we conclude this paper in Section 5.

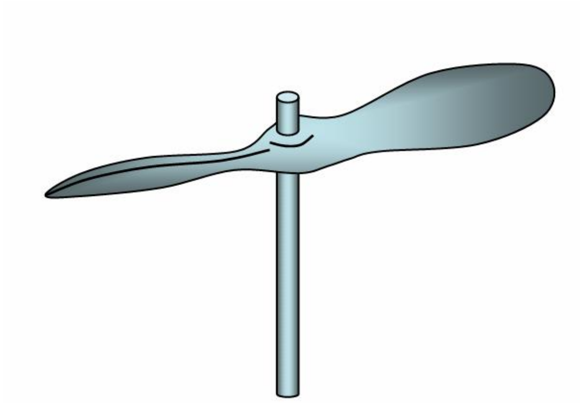

Figure 1: The Chinese top: a toy propeller.

2 Previous Works

To describe the motions of Chinese tops from the viewpoint of aero-dynamics, we need to construct equations of motions to finally calculate forces acting on the Chinese tops. Unfortunately, at least to the best of our knowledge, there has been no research result on the Chinese tops in the area of computer

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graphics. In contrast, many aerodynamicists have studied on various aircrafts, especially helicopters which have rotary-wing airfoils similar to the Chinese tops[2, 3, 4, 5, 6, 7, 8]. Hence our approach to simulate the motion of the Chinese tops had better to be based on helicopter dynamics.

An airfoil is a structure to obtain a useful reaction upon itself in its motion through the air. Wings of aircrafts, propellers and rotor blades are a kind of airfoils. Its unique shape produces lift forces when it passes through the air.

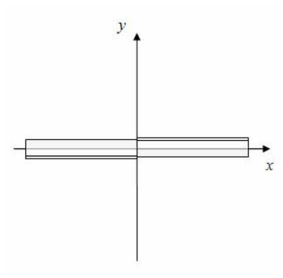
Blade Element Theory(BET) is the foundation of most analyses of helicopter aerodynamics because it deals with the detailed flow and loading of the blades[4]. This theory gives basic insights into the rotor performance as well as other characteristics. BET is very similar to the *Strip Theory* for the fixed wings in aerodynamics. The blade is assumed to be composed of numerous, minuscule strips which are connected from tip to tip. The lift and drag forces are estimated at the strip using the 2D airfoil characteristics of the section. Also, the local flow characteristics are calculated with respect to the climb speed, inflow velocity, and angular velocity. The section lift and drag forces are integrated over the whole blade span.

In contrast to BET, *Momentum Theory* is a global analysis which gives useful results but can not be used as a stand-alone tool to design the rotor[4]. Momentum Theory is also well known as *Disk Actuator Theory*. Momentum Theory assumes that the flow is inviscid and steady, and that the rotor is regarded as an actuator disk with an infinite number of blades, each with an infinite aspect ratio.

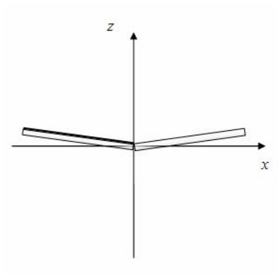
Combining these two theories, we are possible to evaluate a field of induced velocity around the rotor or propeller, and thus, correctly arrange the inflow conditions assumed in the pure blade element theory. The finally induced velocities are not calculated until the blade loads are computed. When the loads are available, we can re-compute the field of induced velocities, more accurately.

These theories have been used to describe the dynamic behavior of *boomerangs*. Hess measured aerodynamic forces and moments in a wind tunnel for several spinning boomerang models, simulated their flight characteristics, and compared them to experimentally measured flight paths[9]. An iterative method for calculation of initial conditions leading to exact return was proposed by Hubbard[10].

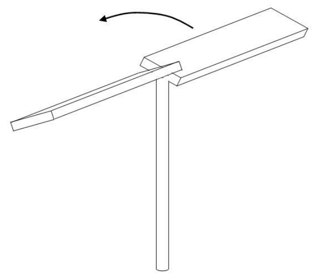
These theories, however, require an extremely large amount of computer resources, since they are designed to exactly calculate the physical behaviors of helicopters. In the case of Chinese tops, however, we can simplify equations from these theories. In this paper, we represent the simplified equations of helicopter dynamics to simulate the motion of the Chinese tops.



(a) top view



(b) front view



(c) perspective view

Figure 2: The Chinese top model.

3 Physically-based Model

Before calculating forces acting on the Chinese tops, we construct our Chinese top model as shown in Figure 2. Each airfoil is composed of 32 pieces of a mesh structure.

The airfoil shape and its area are two of the most important design factors. They are actually the primary elements that determine how much lift and drag forces a blade will produce[5]. Any changes in these design factors will affect the finally produced forces. Normally an increase in the lift force will also produce an increase in the drag force. Therefore, the airfoil is designed to produce the most lift and the least drag within normal speed ranges.

The *angle of attack* is the angle at which the relative wind meets an airfoil. It is also the angle formed by the chord of the airfoil and the direction of the relative wind, or between the chord line and the flight path. Since the airfoil of a Chinese top is simpler than that of a helicopter, we can focus airfoil's angle of attack which is one of the most important factors to determine airfoil shape in the modeling of the Chinese top. We use the angle of attack from 15 degrees to 20 degrees for our Chinese top models.

Dissymmetry of the lift force is the difference in the lift force that exists between the advancing half of the rotor disk and the retreating half. It is caused by the fact that the aircraft relative wind is added to the rotational relative wind on the advancing blade, and subtracted on the retreating blade, in a directional flight. This lift differential should be compensated, or the helicopter would not be controllable. Dissymmetry is corrected by a flapping hinge action in helicopter dynamics. But it is not applicable for our Chinese tops.

Hence we use another solution which partly corrects the dissymmetry. The *coning angle* is the angle of the blades relative to the shaft of Chinese top as shown in Figure 2 (b) and Figure 3. Therefore the coning angle is another important design factor for our Chinese tops.

Forces acting on the airfoil are four: *thrust*, *lift*, *drag* and *gravity*. Thrust is a mechanical force generated by the engines to move the aircraft through the air. Lift acts on the airfoil in a direction perpendicular to the relative wind. Drag is the resistance force that opposes to the motion of the airfoil through the air. It acts on the airfoil in a direction parallel to the relative wind.

The blade element provides a means to determine forces and moments by assuming the blade as composed of a number of aerodynamically independent cross-sections, whose characteristics are the same as a blade at a proper angle of attack. Therefore the operation of a cross-section is indirectly related to that of a two-dimensional airfoil.

Now it is necessary to calculate forces acting on the blade element. Since our Chinese top is constructed by mesh, we assume that a mesh is a blade element.

Using blade element theory and momentum theory, the force equatins can be expressed as follows[6, 7, 8, 11]:

$$\begin{aligned} dL &= \frac{1}{2}\rho V_i^2 A_i C_L, \\ dD &= \frac{1}{2}\rho V_i^2 A_i C_D, \\ dT &= dL\cos\phi - dD\sin\phi, \end{aligned} \tag{1}$$

where dL, dD and dT are the lift, drag and thrust forces acting on the mesh, respectively. C_L and C_D are the lift and drag coeffients, respectively. V_i and A_i are the velocity and the area of the i-th mesh, respectively. ρ and ϕ are the density of air, and the angle between dL and dD, respectively.

The velocity $\mathbf{V}_i$ is calculated as:

$$\mathbf{V}_i = \mathbf{V}\cos\alpha \cdot \sin\varphi + r\cos\beta \cdot \mathbf{\Omega}, \tag{2}$$

where $\mathbf{V}$, α and β are the relative velocity, the angle of attack and the coning angle, respectively. φ, $\mathbf{\Omega}$ and r are an angle of direction of the i-th mesh, anglur veloctiy of the rotor blade and the distance from blade root to the mesh, respectively. Figure 3 shows the relationship of these vectors in Equation (2).

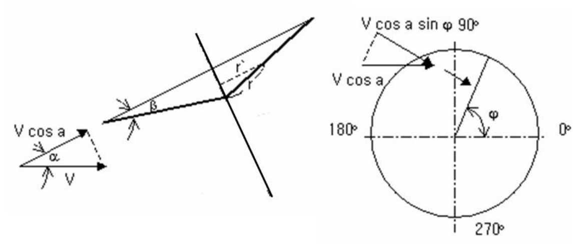


Figure 3: Velocity vectors in blades of the Chinese top.

Gravity force can be computed easily if we know weight of the Chinese top. Total forces acting on the Chinese top can be calulated to sum up forces acting on meshs.

4 Experimental Results

Our prototype system is implemented in C++ programming language with OpenGL library, and is tested on a personal computer with Intel Pentium IV 2.3 GHz CPU and 2G Byte main memory. Our system has a GeForce FX5700-based graphics card with 128M video memory, and runs on Windows XP operating system.

Snapshots from simulation of our Chinese top are shown in Figures 4 and 5. We spin the Chinese top with 90 degree/sec and assume that its coning angle is 3 degrees and its weight is 100g.

At its starting time, the Chinese top flied forward and upward due to the thrust and lift force, then it hovered for a while. After hovering, it dropped down slowly with tilting to the left due to result of dissymmetry of lift.

Our simulations of Figures 4 and 5 show more than 40 frames per second. Hence our system allows real-time visualization and interaction.

5 Conclusion

In this paper, we represent a dynamics model of the Chinese top which is a simple type of the rotary wings. Our approach to simulate the motion of the Chinese top is based on the helicopter dynamics and implemented as a real-time system.

User can generate various results with handling parameters such as the angle of attack, the weight of the Chinese top, the coning angle, and an angular ve-

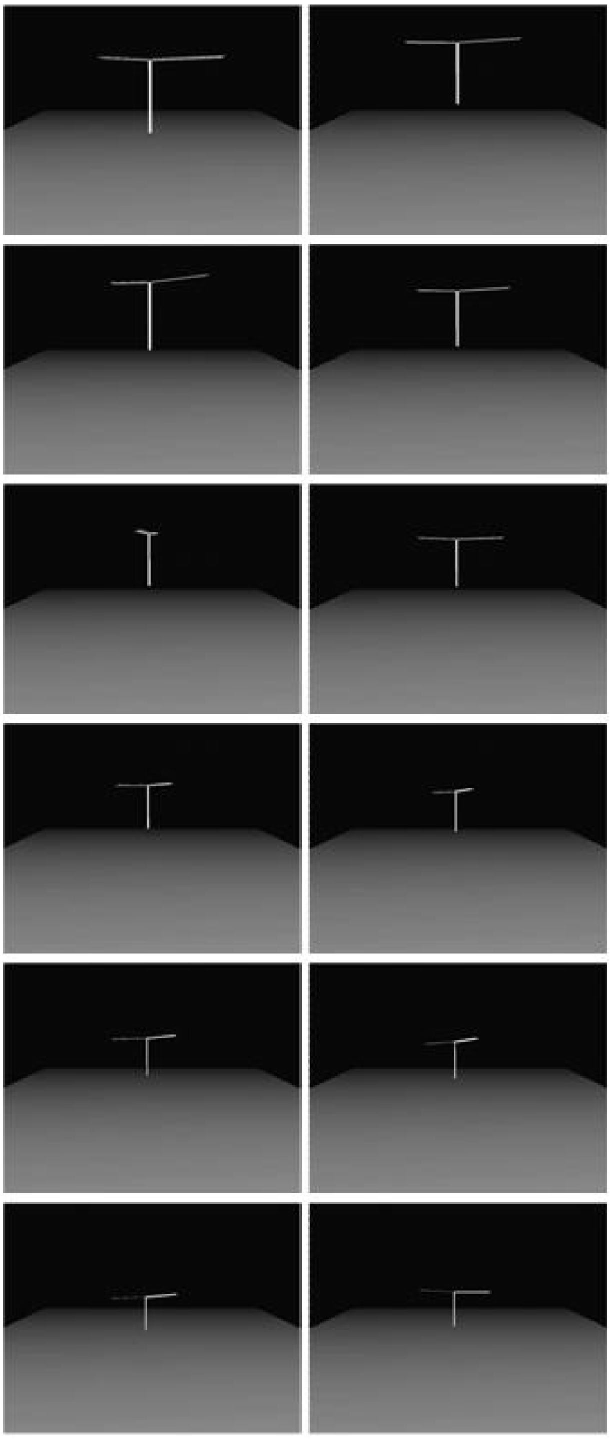


Figure 4: A flight of the Chinese top: front view.

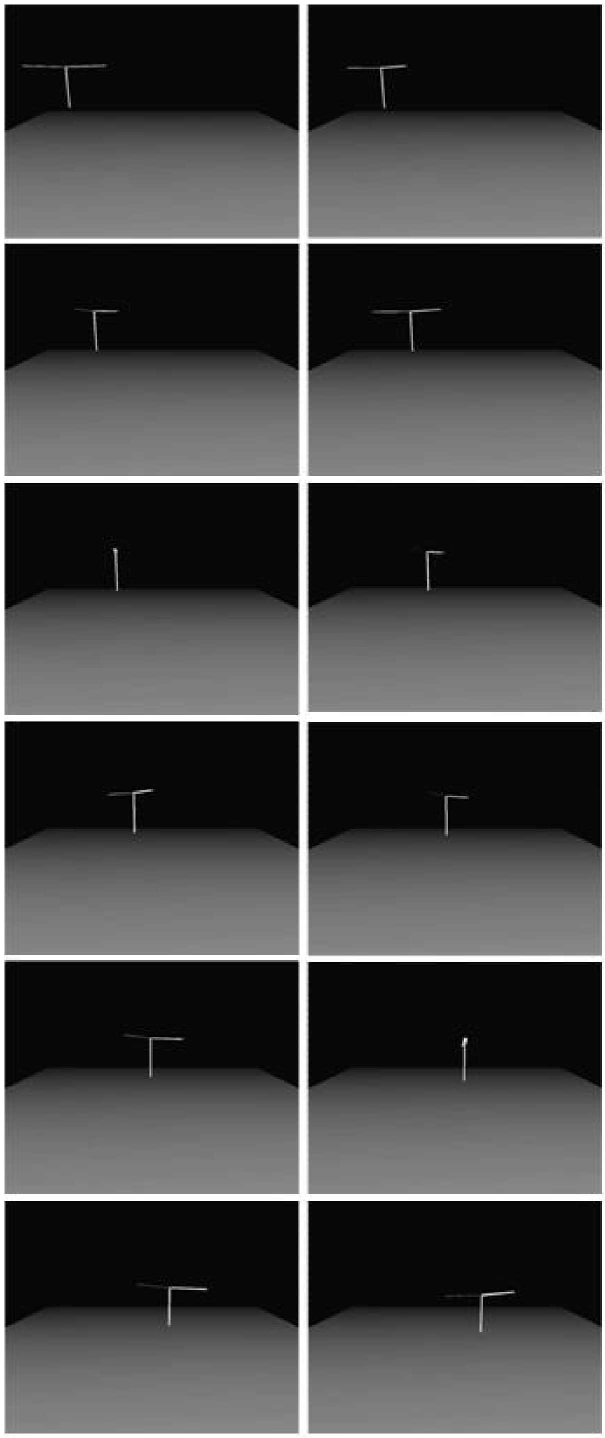


Figure 5: A flight of the Chinese top: side view.

locity. And also our method can be applicable to calculate helicopter motions in virtual reality and computer games.

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