HEMISPHERE, a Fully Decoupled Parallel 2-DOF Spherical Mechanism

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Abstract: - This paper presents a fully decoupled parallel 2-DOF spherical mechanism, named HEMISPHERE. First, the design idea is introduced based on a general 2-DOF spherical mechanism. Then, a practical architecture is presented, which employs only two kinds of 1-DOF kinematic pairs: revolute joint and prismatic pair. Next, following the discussion about the limitation from the friction circle of the revolute joint, the modified mechanisms are given, which can provide a hemisphere workspace.

Key-Words: - HEMISPHERE, Decoupled, Spherical mechanism, Friction circle, Workspace

1 Introduction

A typical parallel mechanism consists of a moving platform, a fixed base, and several kinematical chains (also called the legs) which connects the moving platform to its base. Only some kinematical pairs are actuated, whose number usually equals to the number of DOFs that the platform possesses with respect to the base. Frequently, the number of legs equals to that of DOFs. This makes it possible to actuate only one pair per leg, allowing all motors to be mounted close to the base. Such mechanisms show desirable characteristics, such as large payload and weight ratio, large stiffness, low inertia, and high dynamic performance. Though, compared with serial manipulators, the disadvantages include lower dexterity, smaller workspace, and more seriously, singularity, by which the functioning of the mechanism is disruptive.

In the past two decades, parallel mechanisms with fewer than 6-DOF have attracted much attention. These mechanisms have the advantages of simpler architecture and lower manufacturing cost. In particular, parallel spherical mechanisms allow the platform to rotate around a fixed point and may be used to orient an object. The object may be a telescope, an antenna, a solar panel, a camera, a tool, the end-effector of a robot, a human or humanoid artificial limb, and etc.

Spherical mechanisms may take various configurations. Some of them use spherical architectures, in which only revolute joints are used and their axes intersect at a common center point; as a result, the moving links constitute paths located on concentric spherical surfaces [1-6]. Others use dissimilar architectures, in which the links do not need to move along any spherical surfaces. They only require that the constraints imposed by the legs on the platform provide a fixed center of rotation. This kind of design can achieve the benefit of being able to choose leg topologies in a large range [7-12]. Another possible scheme uses an additional spherical pair which physically connects the platform to the center of rotation, so the legs do not have to provide any constraints which can give the platform a fixed center. As a result, the legs can be freely designed [13-16]. However, most of these configurations have coupled motion among the orientations of the moving platform. In fact, little research has been reported on the study of rotational decoupling [17-20], while much more research concentrate on the translational motion decoupling [21-31]. Besides, the workspaces of the above mentioned parallel spherical mechanisms are usually smaller than a hemisphere, which may limit the application area.

This paper focuses on the decoupled synthesis of 2-DOF spherical mechanism. The interest for this case is justified by the fact that, in many applications, a 2-DOF orientation device is sufficient. A typical application is the positioning of solar panel. In order to facilitate the control (tracing the sun), a decoupled mechanism is highly desirable.

The paper presents a fully decoupled parallel 2-DOF spherical mechanism, named HEMISPHERE.

A model of prototype is also presented to show one of the realizable types of structure. The rest of the paper is organized as follows. Section 2 studies the general idea of the decoupled 2-DOF spherical mechanisms. Section 3 presents the improved architectures. Section 4 analyzes the kinematics of the improved mechanism. Finally, Section 5 draws the conclusions.

2 A Study on The General Idea of A 2-DOF Spherical Mechanism

Figure 1 illustrates the general geometry of a decoupled 2-DOF spherical mechanism. The moving platform is anchored to the base by two legs. A leg consists of two revolute joints, R_1 and R_2, whose axes, z_1 and z_2, intersect at point o and connect to each other perpendicularly; so the value of α is $\pi/2$. The other leg consists of a prismatic pair P, a spherical joint S and a revolute joint R_3. The prismatic pair P and the revolute joint R_1 are mounted on the base, and the moving direction of the prismatic pair P is parallel to the axis of R_1. The spherical joint S is mounted on the prismatic pair P, and the rotation center of S is just on the axis of R_1. The two revolute joints, R_2 and R_3, are mounted on the moving platform and their axes are parallel.

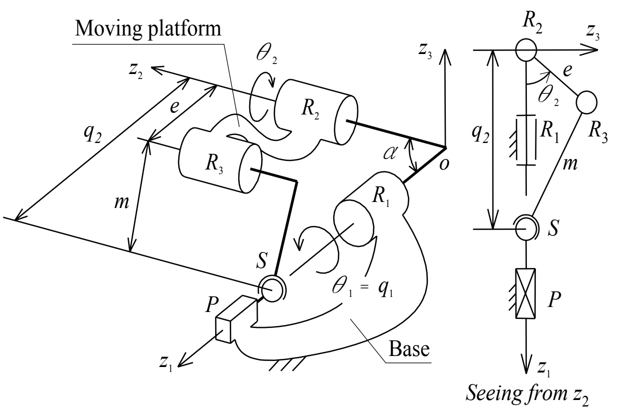

Fig.1: Illustration of a general spherical mechanism

Suppose that the input parameters, q_1 and q_2, represent the angular displacement of the revolute joint R_1 and the distance from S to the axis of R_2 separately. They are driven by a rotary actuator and a linear actuator. The pose of the moving platform is defined by the Euler angles θ_1 and θ_2 of the platform. When the value of q_1 changes and q_2 holds the line, only θ_1 alters. On the other hand, when the value of q_2 changes, only θ_2 changes. So, θ_1 and θ_2 are independently determined by q_1 and q_2 respectively, i.e., one output parameter only relates to one input parameter. In other words, the platform rotations around two axes are decoupled.

Let e be the distance between the axes of R_2 and R_3, m be the distance from center of S to the axis of R_3. Also suppose that, axis z_3 is through the point o and always perpendicular to the plane of z_1-z_2 and moreover, define the value of θ_2 is zero whenever the axis of R_3 is on the plane of z_1-z_2. Then the coordinates of S and R_3 for the axes z_1 and z_3 are

$$\begin{cases} S(z_1, z_3) = S(q_2, 0) \\ R_3(z_1, z_3) = R_3(e\cos\theta_2, e\sin\theta_2) \end{cases} \tag{1}$$

The displacement relationship between input and output is:

$$\begin{cases} q_1 = \theta_1 \\ (q_2 - e\cos\theta_2)^2 + e^2 \sin^2\theta_2 = m^2 \end{cases} \tag{2}$$

Taking the derivative of equation (2), it follows that

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J^{-1} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{3}$$

Where,

$$J = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{eq_2 \sin\theta_2}{e\cdot\cos\theta_2 - q_2} \end{bmatrix} \tag{4}$$

Furthermore, applying the principle of virtual work to equation (2), we get

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = J^T \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{5}$$

Where, T_j is the output torque of the platform, f_j is the input torque for R_1 and the input force for P, and $j = 1, 2$.

3 Improved Architecture To Get Real Hemisphere Workspace

There exist some workspace limitations in the mechanism shown in Figure 1. One of the limitations is that the workspace of θ_1 can never reach 2π, which is caused by the movement confines of the base. The link of SR_3 may be blocked by the base while the rotation of R_1. To overcome this shortcoming, we introduce a new architecture as shown in Figure 2.

The main modification of the new architecture is that a through hole is added to the center of the revolute joint R_1, so the prismatic pair P can be set in the center of the hole and rotates with R_1. As a result, none interference exists while the rotation of R_1. It is clearly that the workspace of θ_1 can reach 2π. Also, as the prismatic pair P rotates with R_1, the spherical joint S can be replaced by a revolute joint R_4.

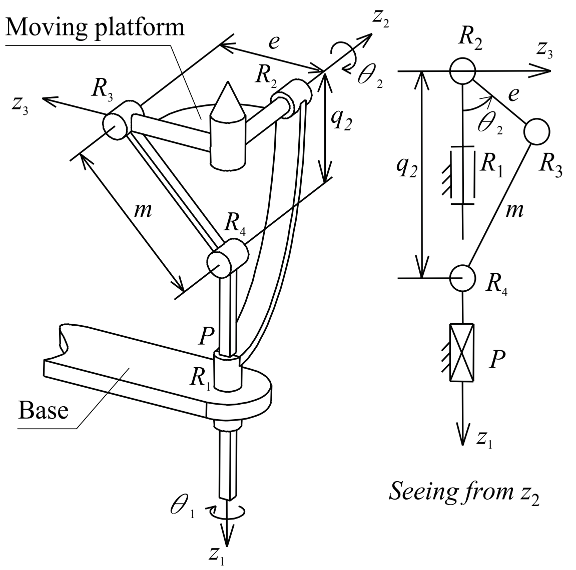

Fig.2: New architecture

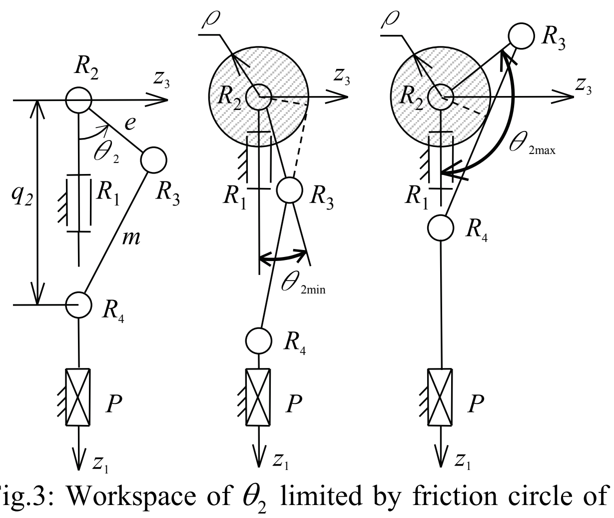

Fig.3: Workspace of θ_2 limited by friction circle of R_2

The other limitation of the mechanism is caused by the friction circle of R_2. This limitation can be described by Figure 3, from which we can see that the work space of θ_2 satisfies

$$\theta_{2\min} < \theta_2 < \theta_{2\max} \tag{6}$$

Where $\theta_{2\min}$ and $\theta_{2\max}$ are the minimum and the maximum boundaries, which can be simply calculated based on Figure 3 as follows

$$\theta_{2\min} = \arcsin \frac{m\rho}{e\sqrt{\rho^2 + (\sqrt{e^2-\rho^2}+m)^2}} > 0 \tag{7}$$

$$\begin{aligned}\theta_{2\max} &= \arctan \frac{m-\sqrt{e^2-\rho^2}}{\rho} \\ &+ \arctan \frac{\sqrt{e^2-\rho^2}}{\rho} < 2\pi\end{aligned} \tag{8}$$

It means that the workspace of the mechanism can not reach a hemisphere. Clearly, this is not desirable.

In fact, because the workspace of θ_1 is $[0, 2\pi]$, the mechanism workspace can reach a hemisphere only if the workspace of θ_2 is chosen $[0, \pi/2]$ or $[\pi/2, \pi]$. So there exist two methods to get a hemisphere workspace.

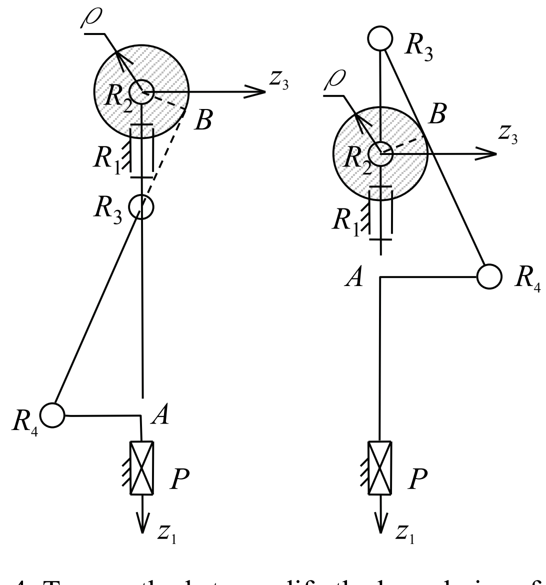

Fig.4: Two methods to modify the boundaries of θ_2: (a) $\theta_{2\min} = 0$, (b) $\theta_{2\max} = 2\pi$

Figure 4 shows the critical instances for both of them; each one uses the similar technique to offset the axis of R_4 from the axis z_1. Let n denotes the axis offset of R_4 (or the length of AR_4), and n_c is the special value of n for the critical configurations as shown in Figure 4, then n should be chosen as

$$n > n_c = \frac{m\rho}{e} \tag{9}$$

Using this technique, a hemisphere work space can be obtained.

The improved architectures are shown in Figure 5 and Figure 6, in which the workspace of θ_2 includes the area of [0, π/2] or [π/2, π] separately.

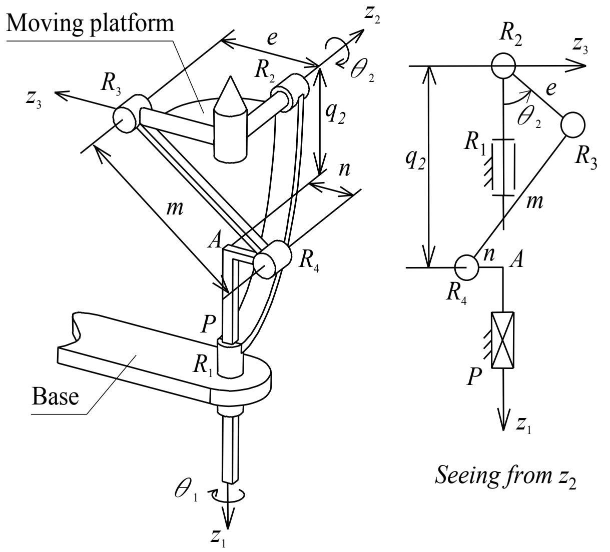

Fig.5: The improved mechanism for $\theta_2 \in [0, \pi/2]$

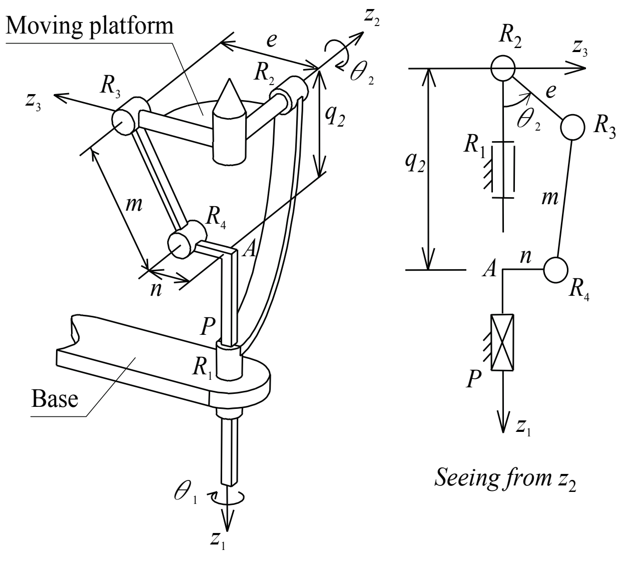

Fig.6: The improved mechanism for $\theta_2 \in [\pi/2, \pi]$

A prototype model of the mechanism for the condition of $\theta_2 \in [0, \pi/2]$ is designed. Figure 7 shows the outline picture of this model. In this design, one leg is actuated by a servo motor through a tooth belt; while the other leg is actuated by the other servo motor through a ball screw, which converts the rotational movement into the translational one. Both motors are fixed on the base. Besides, another revolute joint R_5 is added to connect the prismatic pair with the nut of the ball screw.

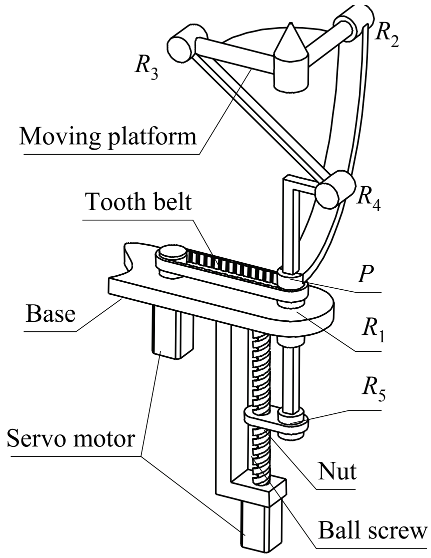

Fig.7: The prototype model for $\theta_2 \in [0, \pi/2]$

4 Kinematics analysis

The coordinates of R_3 are the same in both mechanisms in Figure 5 and 6, which are

$$R_3(z_1, z_3) = R_3(e\cos\theta_2, e\sin\theta_2) \tag{10}$$

While the coordinates of R_4 are different, but can be expressed in a uniform type as the following equation.

$$R_4(z_1, z_3) = R_4(q_2, \pm n) \tag{11}$$

Where, "+" is chosen for the design in Figure 6, while "-" for the design in Figure 5. Then the displacement relationships between input and output parameters are:

$$\begin{cases} q_1 = \theta_1 \\ (q_2 - e\cos\theta_2)^2 + (\pm n - e\sin\theta_2)^2 = m^2 \end{cases} \tag{12}$$

Taking the derivative of equation (12) and applying the principle of virtual work, we can find that equation (3) and (5) still hold, but J becomes

$$J = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{\pm ne\cos\theta_2 - eq_2\sin\theta_2}{q_2 - e\cdot\cos\theta_2} \end{bmatrix} \quad (13)$$

5 Conclusions

In this paper, a fully decoupled parallel 2-DOF spherical mechanism, named HEMISPHERE, is presented. This mechanism has a number of advantages including (a) the ability to position the payload at the geometric center of rotation thereby reducing inertia; (b) high stiffness enabling the use of high angular velocity and acceleration for orientating large payloads; (c) decoupled geometry making the kinematic calculation very simple; and (d) relatively large outward workspace (a hemisphere without singularity) and large internal free space for payload orientation. It is expected the new mechanism would find many applications.

Acknowledgments

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References:

[1] C.M. Gosselin, and J. Angeles, "The Optimum Kinematic Design of a Spherical Three-Degree-of-Freedom Parallel Manipulator," *Journal of Mechanisms, Transmissions, and Automation in Design*, 111(2), 202–207, 1989.

[2] W.M. Craver and D. Tesar, "Force and Deflection Determination for a Parallel 3-DOF Robotic Shoulder Module," *1990 ASME Design Technical Conferences*, DE-Vol. 24, pp. 473–479, 1990, Chicago, IL.

[3] C.M. Gosselin, and E. Lavoie, "On the Kinematic Design of Spherical Three-Degree-of-Freedom Parallel Manipulators," *International Journal of Robotics Research*, 12(4), 394–402, 1993.

[4] S. Leguay-Durand and C. Reboulet, "Optimal Design of a Redundant Spherical Parallel Manipulator," *Robotica*, 15(4), 399–405, 1997.

[5] C.W. Wampler, "Displacement Analysis of Spherical Mechanisms Having Three or Fewer Loops," *2002 ASME Design Engineering Technical Conferences*, Paper No. MECH-34326, 2002, Montreal, Canada.

[6] C.M. Gosselin, C. Cloutier and D. Rancourt, "Kinematic analysis of spherical two-degree-of-freedom parallel manipulators", *Proc. ASME Mechanisms*, Minneapolis, Vol. DE-72, September, pp. 255-262, 1994.

[7] R. Di Gregorio, "A New Parallel Wrist Using Only Revolute Pairs: the 3-RUU Wrist," *Robotica*, 19(3), 305–309, 2001.

[8] R. Di Gregorio, "Kinematics of a New Spherical Parallel Manipulator with Three Equal Legs: the 3-URC Wrist," *Journal of Robotic Systems*, 18(5), 213–219, 2001.

[9] M. Karouia, and J.M. Hervé, "A Family of Novel Orientational 3-DOF Parallel Robots," *The 14th CISM-IFToMM Symposium*, pp. 359–368, 2002, Udine, Italy.

[10] X. Kong, and C.M. Gosselin, "Type Synthesis of 3-DOF Spherical Parallel Manipulators Based on Screw Theory," *2002 ASME Design Engineering Technical Conferences*, Paper No. MECH-34259, 2002, Montreal, Canada.

[11] Z. Huang, and Q.C. Li, "Type Synthesis of Symmetrical Lower-Mobility Parallel Mechanisms Using Constraint Synthesis Method," *International Journal of Robotics Research*, 22(1), 59–79, 2003.

[12] P. Vischer, and R. Clavel, "Argos: a Novel 3-DOF Parallel Wrist Mechanism," *International Journal of Robotics Research*, 19(1), 5-11, 2000.

[13] V. Hayward, and R. Kurtz, "Modeling of a Parallel Wrist Mechanism with Actuator Redundancy," *Advances in Robot Kinematics*, S. Stifter and J. Lenarčič, ed., pp. 444–456, Springer-Verlag, Wien, 1991.

[14] C. Innocenti and V. Parenti-Castelli, "Echelon form Solution of Direct Kinematics for the General Fully Parallel Spherical Wrist," *Mechanism and Machine Theory*, 28(4), 553–561, 1993.

[15] K. Wohlhart, "Displacement Analysis of the General Spherical Stewart Platform," *Mechanism and Machine Theory* 29(4), 581–589, 1994.

[16] Y. Zhang, C. D. III Crane, and J. Duffy, "Determination of the Unique Orientation of Two Bodies Connected by a Ball-and-Socket Joint from Four Measured Displacements," *Journal of Robotic Systems*, 15(5), 299–308, 1998.

[17] M. Carricato, and V. Parenti-Castelli, "A Two-Decoupled-DOF Spherical Parallel Mechanism for Replication of Human Joints," *Servicerob 2001 – European Workshop on*

Service and Humanoid Robots, 2001, pp. 5–12, Santorini, Greece.

[18] M. Carricato, and V. Parenti-Castelli, "A Novel Fully Decoupled Two -Degrees-of-Freedom Parallel Wrist," *The International Journal of Robotics Research*, 23(6), 661-667, 2004.

[19] G. Gogu, "Fully-Isotropic Over-Constrained Parallel Wrists with Two Degrees of Freedom", *in Proceedings of 2005 IEEE International Conference on Robotics and Automation* (ICRA 2005), Barcelona, Spain , 18-22 April 2005, ISBN 0-7803-8915-8 , pp. 4025-4030.

[20] W.M. Li, J.G. Sun, J.J. Zhang, K. He and R. Du, "A Novel Parallel 2-DOF Spherical Mechanism with One-to-One Input-Output Mapping", *WSEAS Transactions on Systems* 5(6), 2006.

[21] L.W. Tsai, G.C. Walsh, and R. Stamper, "Kinematics of a Novel Three DOF Translational Platform," *Proceedings of the 1996 IEEE International Conference on Robotics and Automation*, pp. 3446–3451, 1996, Minneapolis, MM.

[22] L.W. Tsai, "Kinematics of a Three-DOF Platform Manipulator with Three Extensible Limbs," *Recent Advances in Robot Kinematics*, J. Lenarcic, and V. Parenti-Castelli, ed., pp.401–410, Kluwer Academic Publishers, London, 1996.

[23] T.S. Zhao and Z. Huang, "A Novel Three-DOF Translational Platform Mechanism and its Kinematics," *2000 ASME Design Engineering Technical Conferences*, Paper No. MECH-14101, 2000, Baltimore, MD.

[24] L. Baron and G. Bernier, "The Design of Parallel Manipulators of Star Topology under Isotropic Constraint," 2001 ASME Design Engineering Technical Conferences, Paper No. DAC-21025, 2001, Pittsburgh, PA.

[25] J.M. Hervé, and F. Sparacino, "STAR: a New Concept in Robotics," *International Conference of 3K-ARK*, pp. 176–183, 1992.

[26] M. Carricato, V. Parenti-Castelli, A family of 3-DOF translational parallel manipulators, in: Proceedings of the 2001 ASME Design Engineering Technical Conferences, Pittsburgh, PA, DAC-21035, 2001.

[27] X.W. Kong and C. Gosselin, "Kinematics and Singularity Analysis of a Novel Type of 3-CRR 3-DOF Translational Parallel Manipulator," *International Journal of Robotic Research*, 21(9), 791–798, 2002.

[28] H.S. Kim, and L.W. Tsai, "Design Optimization of a Cartesian Parallel manipulator," *Journal of Mechanical Design*, 125(1), 43–51, 2003.

[29] W.M. Li, F. Gao, and J.J. Zhang, "R-CUBE, a Decoupled Parallel Manipulator Only with Revolute Joints," *Mechanism and Machine Theory*, 40(4), 467-473, 2005.

[30] W.M. Li, J.J. Zhang and F. Gao, "P-CUBE, A Decoupled Parallel Robot Only with Prismatic Pairs", *The 2nd IEEE/ASME International Conference on Mechatronic and Embedded Systems and Applications*, Beijing, China on August 13-16, 2006

[31] F. Gao, Y. Zhang, and W.M. Li, "Type Synthesis of 3-DOF Reducible Translational Mechanisms," *Robotica*, 23(2) (2005) 239-245.