Multi-criteria Decision Making with Incomplete Linguistic Preference Relations

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Abstract: - This study proposes some issues to solve the Incomplete Linguistic Preference Relations under Multi-Criteria Decision Making. The proposed method has simple calculation and can speed up the process of comparison and selection of alternative. Experts obtain the matrix by choosing a finite and fixed set of alternatives and perform a pairwise comparison based on their different preferences and knowledge. This method considers only $n-1$ judgments, whereas the traditional analytic hierarchy approach takes $n(n-1)/2$ judgments in a preference matrix with n elements. In evaluation, experts make pairwise comparisons objectively and definitely. This way not only presents a method of resolving inconsistent problem but also boosts the efficiency in computation. Furthermore, this study is developed by the preference relationship between complete and incomplete, then applying Multi-Criteria Incomplete Linguistic Preference Relation, Constituting Decision Matrix, and analyzing the result of the experiment via selecting hearing aids. Consequently, we draw the conclusion and verify the consistency linguistic preference relations.

Key-Words: - Incomplete linguistic preference relations, AHP, Decision analysis, Consistent fuzzy preference relations, Multi-criteria decision making

1. Introduction

Decision-making is the practice of finding the overall best option from the feasible alternatives. The chief difficulty is that almost decision issues have multiple, even conflicting criteria [1]. In many decision making processes, people often express their preferences by using pairwise comparisons. Based on using the concept of hierarchy structure to make evaluation by related decision makers, the Analytic Hierarchy Process (AHP) is a methodology proposed by Saaty [2, 3]. However, it often leads to uncertain predicting outcomes and inconsistency as it is applied to measure performance, some of the evaluators will not be able to obtain weights because of the experts' different opinions [4].

By using analytic hierarchy structure in linguistic preference relations, decision makers can obtain a matrix, $n(n-1)\ 2$, and then obtain a whole preference relation matrix. Recently, decision makers usually analyze based on linguistic preference relations in many research papers [5, 6, 7, 8, 9, 10, 11, 12]. As Herrera-Viedma et al. [13] proposed Fuzzy Preference Relation, people can obtain a weight value after making only $n-1$ pairwise comparisons.

Xu [14] brought a concept that every decision maker could choose a clear preference item for standard while pairwise comparison, and then compare with the adjacent items, so the evaluating process can be more flexible. It only focuses on comparing unique standard on study. This study bases on that theory structure, and extends multi-criteria decision making. It could spread experts in each domain by not affecting the evaluating results on participators' problems.

2. Complete and Incomplete Linguistic Preference Relations

In traditional AHP or Fuzzy AHP pairwise comparison, if evaluated matrix has n candidate item, we have to do $n(n-1)\ 2$ times pairwise comparisons. Therefore after Herrera-Viedma et al. [13] addressing the consistent fuzzy preference relations, we do $n-1$ times pairwise comparison only, then generate the other unknown data from formula. It is called "Complete Linguistic Preference Relations", if decision makers could do pairwise comparison for all properties of preference matrices. It is called "Incomplete Linguistic Preference Relations", when decision makers obtain results from the known preference values or calculate the other unknown data indirectly from formula. The detailed definitions of "Complete and Incomplete Linguistic Preference Relations" are as follows:

2.1 Complete Linguistic Preference Relations

Let S set be a finite set, $S=\{s_\alpha | \alpha=-t,...,t\}$, S_i is the linguistic variable, having the specifics as follows, if $\alpha>\beta$, $S_\alpha > S_\beta$, neg. $(S_\alpha)=S_{-\alpha}$ is reverse formula, neg. $(S_0)=S_0$. Let $X=\{x_1,x_2,...,x_n\}$, A set is preference values set of X proposal, matrix $A \subset X \times X$, preference relation matrix $A=(a_{ij})_{n\times n}$, a_{ij} is preference value of pairwise element x_i and x_j.

(a) Operational Laws of Complete Linguistic Relation

Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is a complete linguistic operation, then the operation is as follows:

$S_\alpha, S_\beta \in S$

$S_\alpha \oplus S_\beta = \max\{S_{-t}, \min\{S_{\alpha+\beta}, S_t\}\}$

$\lambda S_\alpha \oplus S_{\lambda\alpha}, \lambda \in [0,1]$

(b) Additive Preference Relation

For all i, j, decision makers can make comparison for each pair, a_{ij} and a_{ji} are equal to zero on contrary number respectively.

$$a_{ij} \in S,\ a_{ij} \oplus a_{ji} = S_0,\ a_{ii} = S_0 \quad (1)$$

(c) Complete Consistent Additive Preference Relation

We suppose consistent preference relation factor is $A=(a_{ij})_{n\times n}$, for all i, j, k, decision makers can make comparison for each pair, if $a_{ik} > S_0$ indicates x_i is preferred to x_k, $a_{kj} > S_0$ indicates x_k is preferred to x_j, then get $a_{ij} > S_0$, x_i is preferred to x_j, the formula is as follows:

$$a_{ij} = a_{ik} \oplus a_{kj} \quad (2)$$

If $a_{ij} = S_0$, a_{ij}=0, represents x_i as good as x_j, perform $a_{ik} = a_{kj} = a_{ij} = S_0$.

2.2 Incomplete Linguistic Preference Relation

Let $A=(a_{ij})_{n\times n}$ be linguistic relation, if A is an incomplete linguistic preference relation, decision makers couldn't compare properties for each pairs. × represents unknown variable while decision makers couldn't compare its attribute.

(a) Incomplete Linguistic Preference Additive Relation

Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is an incomplete linguistic preference relation, for elements of decision makers, it satisfies Eq.(1) $a_{ij} \in S,\ a_{ij} \oplus a_{ji} = S_0,\ a_{ii} = S_0$.

(b) Incomplete Linguistic Preference Adjoining Relation

Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is an incomplete linguistic preference relation, if $(i,j) \cap (k,l) \neq \varnothing$, the element a_{ij} and a_{kl} are called adjoining.

(c) Incomplete Linguistic Preference Indirect Relation

Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is an incomplete preference relation, we suppose $a_{i_0 j_0}$ be the unknown value in preference matrix A. The element $a_{i_0 j_0}$ is called "Indirect Relation" which is obtained from the adjoining elements $a_{i_0 k}$ and a_{kj_0}.

(d) Acceptable Project of Incomplete linguistic Preference

Let $A=(a_{ij})_{n\times n}$ be linguistic preference relation, if A is an incomplete linguistic preference relation, it is called "Acceptable Project" by obtaining all unknown variable × through adjoining known elements. Therefore if A is acceptable project of incomplete linguistic preference, it could be the known value in a column or row, and having $n-1$ contrasting values by pairs.

3. Constituting Decision Matrix Model by Applying Incomplete Linguistic Preference Relation

Multiple Criteria Decision-Making (MCDM) was developed by Zimmerman in 1985 [15]. Multiple Criteria Decision-Making is the optimal choice, with different type depended on decision makers' preference, sorted of Multiple Objective Decision Making (MODM) and Multiple Attribute Decision Making (MADM). Hwang and Yoon [16] provided that Multiple Criteria Decision-Making is a possible evaluation scale for many characters or quantities of decision-makers' evaluation. It could be determined be advantage or ranking.

There are two main parts of application of Multiple Criteria Decision-Making matrix. One is constructing a decision making matrix for preference relation, and the other is introducing three types of preference relation matrix models.

3.1 Constructing Decision Matrix of Preference Relation

We often have to choice in our real life. If expert e ($e=1,2,...,n$), criteria r ($r=1,2,...,k$), proposals $i,j=1,2,...,m$, then any expert's decision matrix will be $A^{(1)},A^{(2)},...,A^{(n)}$. The decision matrix ${}^{r}A^{(n)}=\left[{}^{r}a_{ij}^{(n)}\right]_{m\times m}$ is the rth criterion of the nth expert.

i.e. any e expert's rth criterion decision matrix ${}^{r}A^{(e)}$ is

$$
{}^{r}A^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & {}^{r}a_{12}^{(e)} & {}^{r}a_{13}^{(e)} & {}^{r}a_{14}^{(e)} & \cdots & {}^{r}a_{1m}^{(e)} \\ {}^{r}a_{21}^{(e)} & 0 & {}^{r}a_{23}^{(e)} & {}^{r}a_{24}^{(e)} & \cdots & {}^{r}a_{2m}^{(e)} \\ {}^{r}a_{31}^{(e)} & {}^{r}a_{32}^{(e)} & 0 & {}^{r}a_{34}^{(e)} & \cdots & {}^{r}a_{3m}^{(e)} \\ {}^{r}a_{41}^{(e)} & {}^{r}a_{42}^{(e)} & {}^{r}a_{43}^{(e)} & 0 & \cdots & {}^{r}a_{4m}^{(e)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ {}^{r}a_{m1}^{(e)} & {}^{r}a_{m2}^{(e)} & {}^{r}a_{m3}^{(e)} & {}^{r}a_{m4}^{(e)} & \cdots & 0 \end{bmatrix}_{m\times m}
$$

If expert's weight is $\omega^{(1)},\omega^{(2)},...,\omega^{(n)}$, $\sum_{e=1}^{n}\omega^{(e)}=1$

Every criterion's weight is ${}^{1}\omega,{}^{2}\omega,...,{}^{k}\omega$, $\sum_{r=1}^{k}{}^{r}\omega=1$

For the inconsistent problem of multi-criteria, multi-decision, and multi-proposals, the steps of acquiring complete preference decision making are as follows:

Step1: The unknown matrix of the first criterion of each decision-making expert.

The unknown matrix of the kth criterion of each decision-making expert

$$
{}^{k}A^{(n)}=\begin{bmatrix} 0 & {}^{k}a_{12}^{(n)} & \cdots & {}^{k}a_{1m}^{(n)} \\ {}^{k}a_{21}^{(n)} & 0 & \cdots & {}^{k}a_{2m}^{(n)} \\ \cdots & \cdots & \cdots & \cdots \\ {}^{k}a_{m1}^{(n)} & {}^{k}a_{m2}^{(n)} & \cdots & 0 \end{bmatrix}
$$

Step 2: For Eq.(1), it obtains the according value, and for Eq.(2), it obtains the other × unknown value, then acquires complete preference matrix.

Step 3: Integrate a complete matrix.

Step 4: Complete matrix multiply criterion's weight.

$$
{}^{r}A^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}\quad e=1,2,\cdots,n
$$

$$
\overline{A}^{(e)}=\frac{1}{k}\left[\sum_{i=1}^{k}{}^{i}A^{(e)}\otimes{}^{i}\omega\right]
$$

Step 5: Integrate the above steps, then multiply expert's

$$
\overline{\overline{A}}=\frac{1}{n}\left[\overline{A}^{(1)}\otimes\omega^{(1)}+\overline{A}^{(2)}\otimes\omega^{(2)}+...+\overline{A}^{(n)}\otimes\omega^{(n)}\right]
$$

$$
=\frac{1}{n}\left[\sum_{e=1}^{n}\overline{A}^{(e)}\otimes\omega^{(e)}\right]
$$

3.2 Incomplete Preference Relation's Decision-Making Matrix

Let X set be decision-making proposal, $X=\{x_1,x_2,...,x_n\}$, base on S_i linguistic relation value, every decision-making expert makes adjoining comparison for the known factor. It could generate $n-1$ preference value, and obtain acceptable incomplete linguistic relation matrix $A=(a_{ij})_{n\times n}$. Then based on equations, preference relation matrix is generated. For different known factors of decision-making expert's choice, it could obtain few matrices that are shown as follows:

Type 1: **Horizontal Comparison of Each Pairs**

$$
{}^{r}A^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & {}^{r}a_{12}^{(e)} & {}^{r}a_{13}^{(e)} & {}^{r}a_{14}^{(e)} & \cdots & {}^{r}a_{1m}^{(e)} \\ \times & 0 & \times & \times & \times & \times \\ \times & \times & 0 & \times & \times & \times \\ \times & \times & \times & 0 & \times & \times \\ \times & \times & \times & \times & 0 & \times \\ \times & \times & \times & \times & \times & 0 \end{bmatrix}_{m\times m}
$$

Type 2: Vertical Comparison of Each Pairs

$$
{}^{r}A^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & \times & {}^{r}a_{13}^{(e)} & \times & \times & \times \\ \times & 0 & {}^{r}a_{23}^{(e)} & \times & \times & \times \\ \times & \times & 0 & \times & \times & \times \\ \times & \times & {}^{r}a_{43}^{(e)} & 0 & \times & \times \\ \times & \times & \vdots & \times & 0 & \times \\ \times & \times & {}^{r}a_{m3}^{(e)} & \times & \times & 0 \end{bmatrix}_{m\times m}
$$

Type 3: **Oblique Comparison of Each Pairs**

$$
{}^{r}A^{(e)}=\left[{}^{r}a_{ij}^{(e)}\right]_{m\times m}=\begin{bmatrix} 0 & {}^{r}a_{12}^{(e)} & \times & \times & \times & \times \\ \times & 0 & {}^{r}a_{23}^{(e)} & \times & \times & \times \\ \times & \times & 0 & {}^{r}a_{34}^{(e)} & \times & \times \\ \times & \times & \times & 0 & \ddots & \times \\ \times & \times & \times & \times & 0 & {}^{r}a_{m-1\,m}^{(e)} \\ \times & \times & \times & \times & \times & 0 \end{bmatrix}_{m\times m}
$$

(a) When the above-mentioned incomplete reference relation matrices are applied to linguistic value, preference unknown matrix of example of Type 1 is as follows:

$$ {}^{r}\mathrm{A}^{(e)} = \left[{}^{r}a_{ij}^{(e)} \right]_{m\times m} = \begin{bmatrix} 0 & S_{-3} & S_1 & S_{-1} & S_{-2} & S_1 \\ \times & 0 & \times & \times & \times & \times \\ \times & \times & 0 & \times & \times & \times \\ \times & \times & \times & 0 & \times & \times \\ \times & \times & \times & \times & 0 & \times \\ \times & \times & \times & \times & \times & 0 \end{bmatrix}_{6\times6} $$

(b) Base on Eq.(1) to obtain the corresponding value

$$ {}^{r}\mathrm{A}^{(e)} = \left[{}^{r}a_{ij}^{(e)} \right]_{m\times m} = \begin{bmatrix} 0 & S_{-3} & S_1 & S_{-1} & S_{-2} & S_1 \\ S_3 & 0 & \times & \times & \times & \times \\ S_{-1} & \times & 0 & \times & \times & \times \\ S_1 & \times & \times & 0 & \times & \times \\ S_2 & \times & \times & \times & 0 & \times \\ S_{-1} & \times & \times & \times & \times & 0 \end{bmatrix}_{6\times6} $$

(c) Base on Eq.(2) to obtain the other known $\times$ of triangular first half

$$ {}^{r}a_{23}^{(e)} = S_1 + S_3 = S_4 \,,\, {}^{r}a_{24}^{(e)} = S_3 + S_{-1} = S_2 \,,\ldots, $$

$$ {}^{r}a_{56}^{(e)} = S_2 + S_1 = S_3 $$

(d) Then base on Eq.(1) to obtain the other known $\times$ of triangular second half.

$$ {}^{r}a_{32}^{(e)} = -{}^{r}a_{23}^{(e)} = S_{-4} \,,\, {}^{r}a_{42}^{(e)} = -{}^{r}a_{24}^{(e)} = S_{-2} \,,\ldots, $$

$$ {}^{r}a_{65}^{(e)} = -{}^{r}a_{56}^{(e)} = S_{-3} $$

(e) Finally, obtain the full preference matrix.

$$ {}^{r}\mathrm{A}^{(e)} = \left[{}^{r}a_{ij}^{(e)} \right]_{m\times m} = \begin{bmatrix} 0 & S_{-3} & S_1 & S_{-1} & S_{-2} & S_1 \\ S_3 & 0 & S_4 & S_2 & S_1 & S_4 \\ S_{-1} & S_{-4} & 0 & S_{-2} & S_{-3} & S_0 \\ S_1 & S_{-2} & S_2 & 0 & S_{-1} & S_2 \\ S_2 & S_{-1} & S_3 & S_1 & 0 & S_3 \\ S_{-1} & S_{-4} & S_0 & S_{-2} & S_{-3} & 0 \end{bmatrix}_{6\times6} $$

4. Illustrative Example

We have to make decision without acceptable method in real life, therefore making wrong decision will waste time in the evaluation process. In this paper, we give an example of selecting audiphones. Giving expert's survey to 3 kinds of decision-making experts (doctors, students, and parents), evaluating 4 criteria (hearing improvement, price, appearance of audiphones, and after-sales service), and evaluating 5 brands (A, B, C, D, and E). In the decision-making expert's weight, doctor's is $\omega^{(1)} = 0.5$, student's is $\omega^{(2)} = 0.2$, parents' is $\omega^{(3)} = 0.3$, then criteria weight is ${}^{k}\omega = {}^{1}\omega = 0.2, {}^{2}\omega = 0.3, {}^{3}\omega = 0.3, {}^{4}\omega = 0.2$, the weight of linguistic variable is shown in Table 1:

Table 1. Evaluation of linguistic variables

Item	Linguistic value	Item	Linguistic value
Extremely poor(EP)	S_{-4}	Extremely good(EG)	S_4
Very poor(VP)	S_{-3}	Very good(VG)	S_3
Poor(P)	S_{-2}	Good(G)	S_2
Slightly poor(SP)	S_{-1}	Slightly good(SG)	S_1
Fair(F)	S_0		

In expert's survey of 3 decision-making experts (doctor, student, parents), they choose a specific criterion of their preference from 4 criteria (hearing improvement, price, appearance of audiphones, after-sales service), then the decision makers compare each pair from 5 brands (A, B, C, D, E). We obtain 4 linguistic preference values, there are 12 matrices as follows:

(1) $\mathrm{A}^{(1)}$ is doctor's, $\mathrm{A}^{(2)}$ is student's, $\mathrm{A}^{(3)}$ is parents', the preference unknown matrix of criterion ${}^{1}\mathrm{A}^{(e)}$ for hearing improvement are as follows:

$$ {}^{1}\mathrm{A}^{(1)} = {}^{1}a_{ij}^{(1)} = \{a_{12}, a_{13}, a_{14}, a_{15}\} = \{S_{-3}, S_2, S_{-1}, S_2\} $$

$$ {}^{1}\mathrm{A}^{(2)} = {}^{1}a_{ij}^{(1)} = \{a_{12}, a_{23}, a_{34}, a_{45}\} = \{S_1, S_2, S_4, S_{-3}\} $$

$$ {}^{1}\mathrm{A}^{(3)} = {}^{1}a_{ij}^{(3)} = \{a_{14}, a_{23}, a_{34}, a_{54}\} = \{S_1, S_3, S_3, S_{-1}\} $$

(2) ${}^{2}\mathrm{A}^{(e)}$ for hearing improvement are as follows:

$$ {}^{2}\mathrm{A}^{(1)} = {}^{2}a_{ij}^{(1)} = \{a_{12}, a_{32}, a_{42}, a_{52}\} = \{S_2, S_1, S_1, S_3\} $$

$$ {}^{2}\mathrm{A}^{(2)} = {}^{2}a_{ij}^{(2)} = \{a_{15}, a_{25}, a_{35}, a_{45}\} = \{S_1, S_2, S_3, S_4\} $$

$$ {}^{2}\mathrm{A}^{(3)} = {}^{2}a_{ij}^{(3)} = \{a_{51}, a_{52}, a_{53}, a_{54}\} = \{S_2, S_{-2}, S_{-3}, S_1\} $$

(3) ${}^{3}\mathrm{A}^{(e)}$ for appearance of audiphones are as follows:

$$ {}^{3}\mathrm{A}^{(1)} = {}^{3}a_{ij}^{(1)} = \{a_{13}, a_{23}, a_{43}, a_{53}\} = \{S_2, S_{-1}, S_2, S_1\} $$

$$ {}^{3}\mathrm{A}^{(2)} = {}^{3}a_{ij}^{(2)} = \{a_{31}, a_{32}, a_{34}, a_{35}\} = \{S_{-3}, S_2, S_1, S_3\} $$

$$ {}^{3}\mathrm{A}^{(3)} = {}^{3}a_{ij}^{(3)} = \{a_{15}, a_{25}, a_{35}, a_{45}\} = \{S_1, S_2, S_3, S_4\} $$

(4) ${}^{4}\mathrm{A}^{(e)}$ for after-sales service are as follows:

$$^4\mathrm{A}^{(1)} = {}^4a_{ij}^{(1)} = \{a_{12}, a_{23}, a_{34}, a_{45}\} = \{S_3, S_{-1}, S_4, S_{-1}\}$$

$$^4\mathrm{A}^{(2)} = {}^4a_{ij}^{(2)} = \{a_{21}, a_{23}, a_{24}, a_{25}\} = \{S_2, S_1, S_4, S_{-1}\}$$

$$^4\mathrm{A}^{(3)} = {}^4a_{ij}^{(3)} = \{a_{14}, a_{23}, a_{34}, a_{54}\} = \{S_{-1}, S_2, S_3, S_4\}$$

Step 1: For the first criterion of all experts, the preference unknown matrix is determined by 5 proposals that compare each pair $^1\mathrm{A}^{(1)}$, for example:

$$^1\mathrm{A}^{(1)} = \left[{}^1a_{ij}^{(1)}\right]_{m\times m} = \begin{bmatrix} s_0 & s_{-3} & s_2 & s_{-1} & s_2 \\ \times & s_0 & \times & \times & \times \\ \times & \times & s_0 & \times & \times \\ \times & \times & \times & s_0 & \times \\ \times & \times & \times & \times & s_0 \end{bmatrix}$$

Step 2: Base on Eq.(1) to determine the unknown matrix $^1\mathrm{A}^{(1)}$, the example is as follow:

$$^1\mathrm{A}^{(1)} = \left[{}^1a_{ij}^{(1)}\right]_{m\times m} = \begin{bmatrix} s_0 & s_{-3} & s_2 & s_{-1} & s_2 \\ s_3 & s_0 & \times & \times & \times \\ s_{-2} & \times & s_0 & \times & \times \\ s_1 & \times & \times & s_0 & \times \\ s_{-2} & \times & \times & \times & s_0 \end{bmatrix}$$

Step 3: Integrate a complete matrix based on Eq.(2) , obtain unknown × variable of the first half matrix, $^1\mathrm{A}^{(1)}$, for example.

$$^1\mathrm{A}^{(1)} = \left[{}^1a_{ij}^{(1)}\right]_{m\times m} = \begin{bmatrix} s_0 & s_{-3} & s_2 & s_{-1} & s_2 \\ s_3 & s_0 & s_5 & s_2 & s_5 \\ s_{-2} & \times & s_0 & s_{-3} & s_0 \\ s_1 & \times & \times & s_0 & s_3 \\ s_{-2} & \times & \times & \times & s_0 \end{bmatrix}$$

Then apply Eq.(1), obtain unknown × variable of the second half matrix, generate a complete preference matrix as follows:

$$^1\mathrm{A}^{(1)} = \left[{}^1a_{ij}^{(1)}\right]_{m\times m} = \begin{bmatrix} s_0 & s_{-3} & s_2 & s_{-1} & s_2 \\ s_3 & s_0 & s_5 & s_2 & s_5 \\ s_{-2} & s_{-5} & s_0 & s_{-3} & s_0 \\ s_1 & s_{-2} & s_3 & s_0 & s_3 \\ s_{-2} & s_{-5} & s_0 & s_{-3} & s_0 \end{bmatrix}$$

Step 4: the complete matrix is multiplied by weight vector

$$\overline{\mathrm{A}}^{(1)} = \frac{1}{k}\left[{}^1\mathrm{A}^{(1)} \otimes {}^1\omega + {}^2\mathrm{A}^{(1)} \otimes {}^2\omega + \dots + {}^k\mathrm{A}^{(1)} \otimes {}^k\omega\right]$$
$$= \frac{1}{4}\left[{}^1\mathrm{A}^{(1)} \otimes 0.2 + {}^2\mathrm{A}^{(1)} \otimes 0.3 + {}^3\mathrm{A}^{(1)} \otimes 0.3 + {}^4\mathrm{A}^{(1)} \otimes 0.2\right]$$

$$\overline{\mathrm{A}}^{(1)} = \begin{bmatrix} s_0 & s_{0.375} & s_{0.425} & s_{0.325} & s_{0.350} \\ s_{-0.375} & s_0 & s_{0.050} & s_{-0.050} & s_{-0.225} \\ s_{-0.425} & s_{-0.050} & s_0 & s_{-0.100} & s_{-0.075} \\ s_{-0.325} & s_{0.050} & s_{0.100} & s_0 & s_{0.025} \\ s_{-0.350} & s_{0.225} & s_{0.075} & s_{-0.025} & s_0 \end{bmatrix}$$

used the same method

$$\overline{\mathrm{A}}^{(2)} = \begin{bmatrix} s_0 & s_{0.150} & s_{0.300} & s_{0.775} & s_{0.750} \\ s_{-0.150} & s_0 & s_0 & s_{0.475} & s_{0.450} \\ s_{-0.300} & s_0 & s_0 & s_{0.475} & s_{0.450} \\ s_{-0.775} & s_{-0.475} & s_{-0.475} & s_0 & s_{-0.025} \\ s_{-0.750} & s_{-0.450} & s_{-0.450} & s_{0.025} & s_0 \end{bmatrix}$$

$$\overline{\mathrm{A}}^{(3)} = \begin{bmatrix} s_0 & s_{-0.425} & s_{-0.625} & s_{-0.400} & s_{-0.425} \\ s_{0.425} & s_0 & s_{0.050} & s_{0.025} & s_0 \\ s_{0.625} & s_{-0.050} & s_0 & s_{0.225} & s_{0.200} \\ s_{0.400} & s_{0.050} & s_{-0.225} & s_0 & s_{-0.025} \\ s_{0.425} & s_{0.225} & s_{-0.200} & s_{0.025} & s_0 \end{bmatrix}$$

Step 5: Integrate the above steps, then multiply expert's weight vector

$$\overline{\overline{\mathrm{A}}} = \frac{1}{3}\left[\overline{\mathrm{A}}^{(1)} \otimes 0.5 + \overline{\mathrm{A}}^{(2)} \otimes 0.2 + \overline{\mathrm{A}}^{(3)} \otimes 0.3\right]$$

$$\overline{\overline{\mathrm{A}}} = \begin{bmatrix} s_0 & s_{0.030} & s_{0.028} & s_{0.066.} & s_{0.066} \\ s_{-0.030} & s_0 & s_{0.013} & s_{0.026} & s_{-0.008} \\ s_{-0.028} & s_{-0.013} & s_0 & s_{0.038} & s_{0.038} \\ s_{-0.066} & s_{-0.026} & s_{-0.038} & s_0 & s_0 \\ s_{-0.066} & s_{0.008} & s_{-0.038} & s_0 & s_0 \end{bmatrix}$$

Step 6: Rank each proposal and average the preference value of all proposals, we obtain the table of the preference values of each proposal.

Table 2. The avg. preference values of all proposals and rankings

	A	B	C	D	E	Avg.	Rank
A	0.000	0.030	0.028	0.066	0.066	0.0380	1
B	-0.030	0.000	0.013	0.026	-0.008	0.0003	3
C	-0.028	-0.013	0.000	0.038	0.038	0.0067	2
D	-0.066	-0.026	-0.038	0.000	0.000	-0.0258	5
E	-0.066	0.008	-0.038	0.000	0.000	-0.0192	4

Then we rank preference value of all proposals and obtain Brand A> Brand C> Brand B> Brand E> Brand D. Therefore, Brand A is the best decision proposal.

5. Conclusions

The problems of social economics and environmental change are more complicated and uncertain in the real world. We could make decision by unique criterion, but need to consider relative factors as well. Therefore, multi-criteria decision making could satisfy the need. It causes inconsistent of multi-criteria for many decision makers and many candidate proposals when carrying out the solving level-analysis. Thus Herrera-Viedma et al. [13] denoted fuzzy preference relation to solve those. It's hard to analyze each pair of specific choice for decision makers. Perhaps the issue is the pressure of time, lack of knowledge or information, or the realization. In this paper, we provide a method that decision-maker makes comparison for each pair by themselves in n factors. It needs n-1 comparisons, which is better than $\frac{n(n-1)}{2}$ comparisons of traditional AHP method. It's simple calculation and could spread experts of each domain.

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