Forchheimer's Sound Waves Propagation in a Cylindrical Tube Filled with a Porous Media

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Abstract: **-** A rigid frame, cylindrical capillary theory of sound propagation in porous media that includes the nonlinear effects of the Forchheimer type is laid out by using variational solutions. It is shown that the five main parameters governing the propagation of sound waves in a fluid contained in rigid cylindrical tubes filled with a saturated porous media are shear wave number, $s = R\sqrt{\bar{\rho}\omega / \mu}$, reduced frequency parameter, $k = wR/\bar{a}$, porosity, ε, Darcy number, $Da = R/K$, and Forchheimer number, $C_s^* = 2C_F$. The manner in which the flow influences the attenuation and the phase velocities of the forward and backward propagating isentropic acoustic waves is deduced. It is found that the inclusion of the solid matrix increases wave's attenuations and phase velocities for both forward and backward sound waves, while increasing the porosity and the reduced frequency number decreased attenuation and phase velocities. The effect of the steady flow is found to decrease the attenuation and phase velocities for forward sound waves and enhance them for the backward sound waves.

Key-Words: **-** sound waves, porous medium, fluid flow

1 Introduction

Acoustic problem covers a wide range of practical problems. If the acoustic improvements are restricted to interior spaces (buildings halls, theaters, dwellings, factories, vehicle cabins, etc.), usually mineral wools or open pore foams can be used to solve the problem. For outdoor problems, for instant, acoustic noise barriers against traffic noise, the absorption is provided by granular materials such as porous concrete or similar materials, as they behave better with bad weather and other atmospheric phenomena, as well as they can be cleaned (with a pressurized water) without loosing their acoustic properties.

In porous materials such as fibrous and granular, the absorption process of the acoustic wave takes place through viscosity and thermal losses of the acoustic energy inside the micro tubes forming the material. The problem of a propagation of sound waves in fluids contained in a plain medium is a classical one, to which famous names are connected like Helmholtz [1], Kirchhoff [2] and Rayleigh [3]. Since then many papers have been written on the subject; often in relation to the studied dealing with the dynamics response of pressure transmission lines. A variational treatment of the problem of sound transmission in narrow tubes is described by Cummings [4] as an alternative to the more usual analytical procedure which is limited to mathematically tractable geometries. A first approximation to the effects of mean flow on sound propagation through cylindrical capillary tubes is achieved by Peat [5]. A sound transmission in narrow pipes with superimposed uniform mean flow and acoustic modeling of automobile catalytic converters is done by Dokumaci [6]. A numerical study on the propagation of sound through capillary tubes with mean flow is achieved also by Jeong and Ih [7] and finally an approximate dispersion equation for sound waves in a narrow pipe with ambient gradients is done by Dokumaci [8].

The problem of sound waves propagation in a stationary or flowing fluid in a porous medium is not addressed yet. An attempt is made in this article to develop a simplified nonlinear theory that predicts the propagation characteristics of a stationary or flowing fluid in saturated porous media. This theory is an extension of the classical plain medium theory, using a modification to Darcy's law due to the Forchheimer effects. Analytical expressions for the propagation constant are obtained from variational solutions. Comparison with previous works in the limit of plain medium shows an excellent agreement.

2 Problem Formulation

Consider a rigid tube filled with a saturated porous material, the fluid is assumed to be a stationary or movable inside the tube. The x-coordinate is measured along the tube and the r-coordinate is measured normal to the axial direction.

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Under the boundary layer approximations the basic equations which govern acoustic wave propagation in a rigid tube filled with a porous media are the continuity and momentum equations:

$$\varepsilon\frac{\partial\rho^*}{\partial t^*}+u^*\frac{\partial\rho^*}{\partial x^*}+v^*\frac{\partial\rho^*}{\partial r^*}+\rho^*(\frac{\partial v^*}{\partial r^*}+\frac{v^*}{r^*}+\frac{\partial u^*}{\partial x^*})=0 \quad (1)$$

$$\rho^*\left[\varepsilon^{-1}\frac{\partial u^*}{\partial t^*}+\varepsilon^{-2}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial r^*}\right)\right]=-\frac{\partial p^*}{\partial x^*} \quad (2)$$

$$-\frac{\mu}{K}u^*-\frac{C_F\rho^*u^{*2}}{K^{1/2}}+\mu\varepsilon^{-1}\left(\frac{\partial^2u^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

$$-\frac{\partial p^*}{\partial r^*}=0 \quad (3)$$

Where u^*, v^* are the velocity components in the axial and normal directions; respectively. ρ^* and p^* are the fluid density and pressure, μ is the absolute viscosity and K is the permeability of the porous media and ε is the porosity of the porous medium. Further simplification of the governing equations results from the assumption that the radial velocity component, v, is zero. Since one is dealing only with capillary tubes the radial velocity might be expected to be negligible, if not identically zero. The effect of this is to decouple the continuity equation (4) from the momentum equation (5). Next, it is assumed that the flow through the capillary duct is a superposition of a fully developed laminar, incompressible, axial steady flow and a small harmonic acoustic disturbance of frequency ω. The steady flow is taken to have constant density $\bar{\rho}$ and a speed of sound $\bar{a}$ such that the fluid variables can be expanded in the form:

$$\rho^*=\bar{\rho}\left(1+\alpha\rho(\eta)e^{\Gamma\xi}e^{i\omega t^*}\right)\ u^*=\bar{a}\left(M_0(\eta)+\alpha u(\eta)e^{\Gamma\xi}e^{i\omega t^*}\right),$$

$$v^*=\bar{a}\alpha v(\eta)e^{\Gamma\xi}e^{i\omega t^*},\ p^*=(\bar{\rho}\bar{a}^2/\gamma)\left(p_0(\xi)+\alpha p(\eta)e^{\Gamma\xi}e^{i\omega t^*}\right) \quad (4\text{-}7)$$

Where $\alpha<<1$ and γ is the ratio of specific heats. It is seen that the steady flow variables p_0 and Mach number M_0 together with acoustic variables ρ, u, v and p are dimensionless. Now introduce the following variables in the transformations:

$$\xi=\omega x^*/\bar{a} \qquad \eta=r^*/R \qquad (8)$$

R is the radius of the capillary duct. The axial acoustic wave motion has been assumed to have complex propagation constant Γ which can be expanded as:

$$\Gamma=\Gamma'+i\Gamma'' \quad (9)$$

Where Γ' represents the wave attenuation per unit distance and Γ'' represents the phase shift over the same distance. The assumed forms of the variables, equations (4-7) are substituted into the governing equations (1) and (2)-(3) and terms of similar order in α equated. It is found that for zeroth order, the steady flow solution, the equations of continuity and radial momentum are identically satisfied, while the axial momentum equation (6) becomes:

$$\frac{s^2}{\gamma}\frac{dp_0}{d\xi}=\frac{1}{\varphi\eta}\frac{d}{d\eta}\left(\eta\frac{dM_0}{d\eta}\right)-Da^2M_0-\frac{C_Fs^2}{k}M_0^2 \quad (10)$$

Here $s=R\sqrt{\bar{\rho}\omega/\mu}$ is the shear wave number, $k=\omega R/\bar{a}$ is the reduced frequency parameter, $Da=R/K$ is the Darcy number and C_F is the Forchheimer number. This is the classical equation of Hagen-Poiseuilli flow, the solution of which, with no-slip boundary conditions, gives a parabolic velocity profile:

$$M_0=\frac{s^2}{\gamma}\frac{dp_0}{d\xi}\left(\frac{1-\eta^2}{4}\right)=2\bar{M}(1-\eta^2) \quad (11)$$

Where $\bar{M}$ is the mean Mach number of the steady flow. The linearized acoustic equations follow from equating terms of first order in α in the governing equations, and are:

$$\frac{i\rho}{\varepsilon}+\Gamma u+2\bar{M}\Gamma(1-\eta^2)\rho=0 \quad (12)$$

$$[\frac{iu}{\varepsilon}+\frac{2\bar{M}\Gamma}{\varepsilon^2}(1-\eta^2u]=(-\Gamma/\gamma)p+ \quad (13)$$

$$(1/s^2\varepsilon)\left[\frac{d^2u}{d\eta^2}+(1/\eta)du/d\eta\right]-(Da^2/s^2)u-(2C_FDa/k)M_0u$$

Where p is the pressure, ε is the porosity of the porous medium. Note that the ε, Da and C_F will reflect the effect of the porous matrix size on the acoustic problem under consideration. The case of $\varepsilon=1$ or $Da=0$ corresponds to the plain medium without the presence of the solid matrix and any values of $0<\varepsilon<1$ or $Da>0$ represent a porous medium with different pore spaces. For the case of $\varepsilon=1$ and $Da=0$, the governing equations (12) and (13) reduces to those obtained by Peat [5] for the case of a pure plain medium. In the limit of zero steady flow, $\bar{M}=0$, these equations are found to reduce to those for the reduced frequency solution of Tijdeman [9]. It will be assumed hat the tubes are rigid which implies the no-slip boundary condition of the fluid velocity at wall:

$$u=0 \text{ at } \eta=1 \quad (14)$$

The solution of equations (15)-(17) is greatly simplified if one assumes that the acoustic disturbances occur isentropically, since then:

$$p=\rho\gamma \quad (15)$$

3. Variational Solutions

Now the problem reduces to that of solving the continuity and axial momentum equations for the velocity component and the pressure p, which is constant over a radial cross section. A variational solution with the following form of axial acoustic velocity variation is sought:

$$u=C(1-\eta^2) \quad \text{where } C \text{ is constant} \quad (16)$$

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Equation (13) corresponds to the Euler-Lagrange equation:

$$\frac{\partial f}{\partial u}=\frac{d}{d\eta}\left(\frac{\partial u}{\partial(du/d\eta)}\right) \qquad (17)$$

$$f=\left[\begin{array}{l}(1/s^2\varepsilon)\eta\left(\frac{du}{d\eta}\right)^2+\frac{iu^2\eta}{\varepsilon}+(\frac{2\Gamma}{\gamma})pu\eta+\\ \frac{2\overline{M}\Gamma}{\varepsilon^2}(1-\eta^2)\eta u^2+\frac{Da^2}{s^2}u^2\eta+\frac{C_s^*Da}{k}2\overline{M}(1-\eta^2)\eta u^2\end{array}\right]d\eta \qquad (18)$$

Here $C_s^*=2C_F$, and thus for a given form of trial solution, the best approximation to equation (18) corresponds to the minimum of the functional:

$$F=\int_0^1\left[\begin{array}{l}(1/s^2\varepsilon)\eta\left(\frac{du}{d\eta}\right)^2+\frac{iu^2\eta}{\varepsilon}+(\frac{2\Gamma}{\gamma})pu\eta+\\ \frac{2\overline{M}\Gamma}{\varepsilon^2}(1-\eta^2)\eta u^2+\frac{Da^2}{s^2}u^2\eta+\frac{C_s^*Da}{k}2\overline{M}(1-\eta^2)\eta u^2\end{array}\right]d\eta \qquad (19)$$

Thus, the assumed form of trial solution for *u*, equation (16), is substituted into this expression and the minimum is found by setting $\partial F/\partial C=0$, so that

$$C=-\frac{p\Gamma}{2\gamma}\Bigg/\left(\frac{2}{s^2\varepsilon}+\frac{i}{3}+\frac{\overline{M}\Gamma}{2\varepsilon^2}+\frac{Da^2}{3s^2}+\frac{C_F^*Da\overline{M}}{2k}\right) \qquad (20)$$

The known from of trial solution is now inserted into the continuity equation (12) and integrated over the domain; this leads to:

$$(1-\frac{2\overline{M}^2}{\varepsilon^2})\Gamma^2-\left(\frac{8}{s^2\varepsilon}+\frac{4i}{3\varepsilon}+\frac{2i}{\varepsilon^3}+\frac{4Da^2}{3s^2}+\frac{C_s^*Da^2\overline{M}}{k}\right)\overline{M}\Gamma+\left(\frac{4}{3\varepsilon^2}-\frac{8i}{s^2\varepsilon^2}-\frac{4Da^2i}{3s^2\varepsilon}-\frac{2C_s^*Da\overline{M}i}{k\varepsilon}\right)=0 \qquad (21)$$

With a solution of the propagation constants of the form:

$$\Gamma=\frac{\left(\frac{8}{s^2\varepsilon}+\frac{4i}{3\varepsilon}+\frac{2i}{\varphi^3}-\frac{4Da^2}{3s^2}+\frac{C_s^*Da\overline{M}}{k}\right)\overline{M}}{2(1-\frac{2\overline{M}^2}{\varepsilon^2})}\pm\frac{\left\{\left(\frac{8}{s^2\varepsilon}+\frac{4i}{3\varepsilon}+\frac{2i}{\varepsilon^3}+\frac{4Da^2}{3s^2}+\frac{C_s^*Da\overline{M}}{k}\right)^2\overline{M}^2-4(1-\frac{2\overline{M}^2}{\varepsilon^2})\left(\frac{4}{3\varepsilon^2}-\frac{8i}{s^2\varepsilon^2}-\frac{4Da^2i}{3s^2\varepsilon}-\frac{2C_s^*Da\overline{M}}{k\varepsilon}\right)\right\}^{1/2}}{2(1-\frac{2\overline{M}^2}{\varepsilon^2})} \qquad (22)$$

4. Results and Discussion

In the limit of zero steady flow $\overline{M}=0$, comparison of variational solution with exact solution as given by Peat [5] in the limits of plain medium for $\varepsilon=1$ and $Da=0$ is shown in table 1. Figure 1 is a plot of the modulus of wave attenuation per unit distance, Γ', and phase shift, $|\Gamma''|$, for varying shear wave number and Mach numbers $\overline{M}=0,0.1,0.2,0.3$ and for selected values of $Da=10, C_s^*=0.1, \varepsilon=0.8$ and $k=0.15\pi$. It is clear that as the Mach number is increased the attenuation and phase velocities are decreased for the forward sound waves and increased for the backward sound waves propagated in a porous media; this is due unfavorable collision with the solid matrix for the forward sound waves and favorable fluid flow velocities on the propagation of the backward sound waves.

Figure .2 shows the effect of increasing Darcy numbers $Da=0,0.1,1,10,100$ for selected values of $\overline{M}=0.1, C_s^*=0.1, \varepsilon=0.8$ and $k=0.15\pi$, it is clear that as the Darcy number is increased the attenuation and phase velocities are increased for both the forward and backward sound waves; this is due to favorable solid matrix effects on damping the propagated sound waves. Figure .3 shows the effect of porosity $\varepsilon=0.5,0.6,0.7,0.8,0.9$ on attenuation and phase velocities for selected values of $Da=10, C_s^*=0.1$ $\overline{M}=0.1$ and $k=0.15\pi$, it is found that increasing of the porosity decreases the attenuation and phase velocities for both the forward and backward propagated sound waves; this is due to the small effect of the solid matrix as moving toward the plain media limit. Figure .4 shows the effect of Forchheiemr term $C_s^*=0.1,1,5,10,100$ on attenuation and phase velocities for selected values of $Da=10, \varepsilon=0.8, \overline{M}=0.1$ and $k=0.15\pi$, it is found that as the Forchheimer term is increased the attenuation and phase velocities are decreased for the forward and backward sound waves; this is due to favorable damping effects of the fluid inside the large used pores of the solid matrix.

Finally figure. 5 shows the effect of increasing the reduced frequency parameter $k=0.05\pi, 0.5\pi, 0.1\pi$ $, 0.2\pi, 0.3\pi, 0.5\pi$ on the attenuation and phase velocities for selected value of $Da=10, \varepsilon=0.8, \overline{M}=0.3$ and $C_s^*=10$, it is found that as the reduced frequency is increased both the attenuation and phase velocities for the forward and backward sound waves are decreased; this is due to higher frequency of the impacted sound waves on the solid matrix, it is important to note that the same effect is noticed for sound waves propagated in a plain medium.

5. Conclusion

1- It is found that the effect of increasing Darcy number or Forchheimer number is to increase the attenuation and phase velocities for both forward and backward sound waves; this is due to favorable role of solid matrix in damping sound waves.

2- It is found that the effect of increasing porosity or reduced frequency parameter is to decrease attenuation and phase velocities for both forward and backward sound waves; this is due to absence of

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favorable role of porous matrix and high incident sound waves strength respectively.

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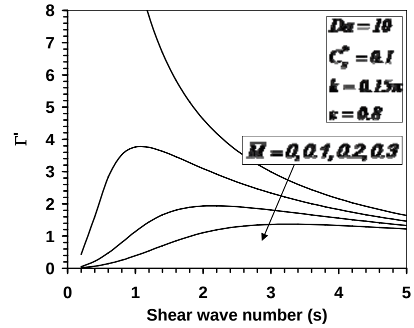
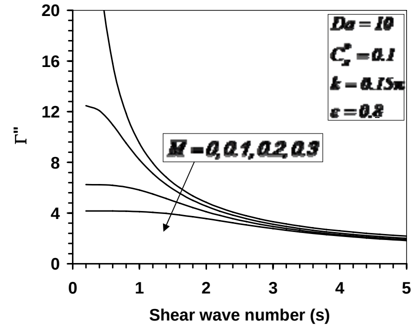
Table 1: Comparison of variational solution with exact

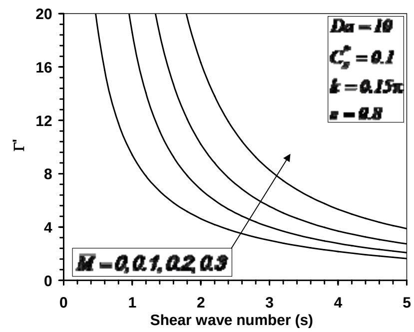
Shear wavenumber,s	Present	Peat [8]
0.2	9.967	9.975
0.4	4.934	4.950
1.0	1.841	1.879
2.0	0.732	0.786
3.0	0.367	0.411
4.0	0.213	0.243
5.0	0.138	0.158



Attenuation - Forward wave



Phase Shift – Forward wave



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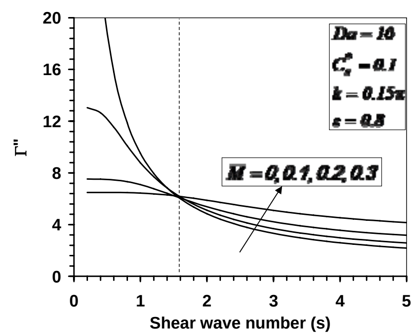

Phase Shift – Backward wave

Fig. 1 Effect of Mach Number

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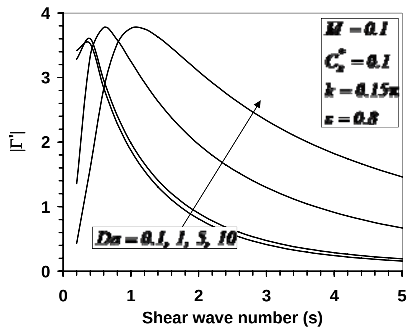

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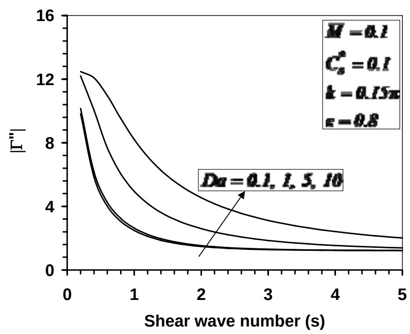
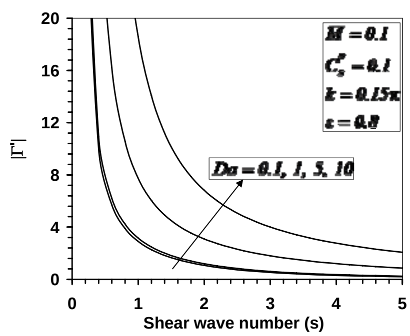
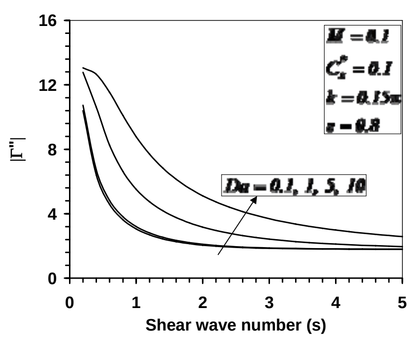


Phase Shift – Forward wave

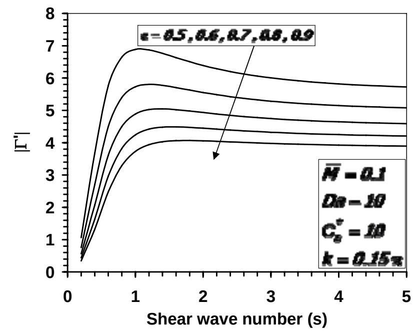


Attenuation – Backward wave

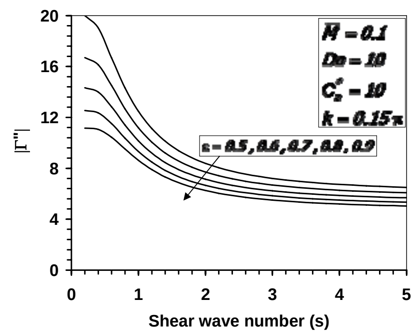


Phase Shift – Backward wave

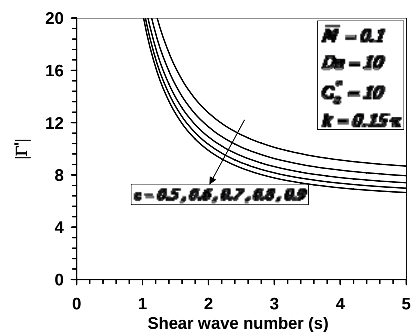
Fig. 2 Effect of Darcy number



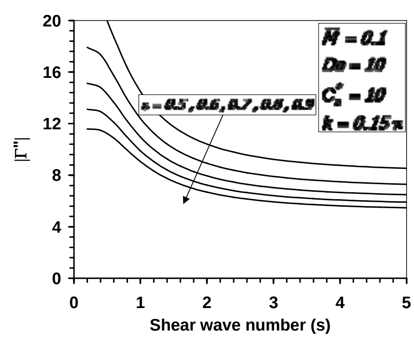
Attenuation - Forward wave



Phase Shift – Forward wave



Attenuation – Backward wave



Phase Shift – Backward wave

Fig. 3 Effect of Porosity

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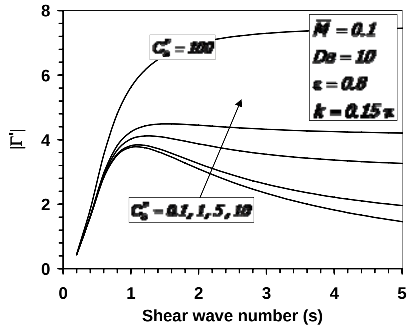
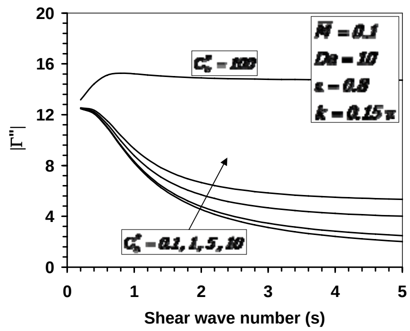
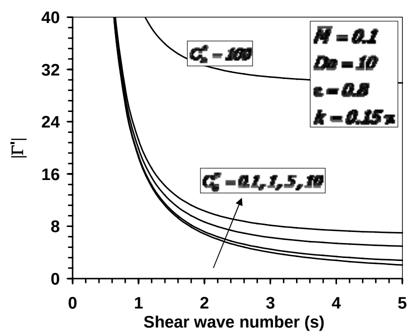


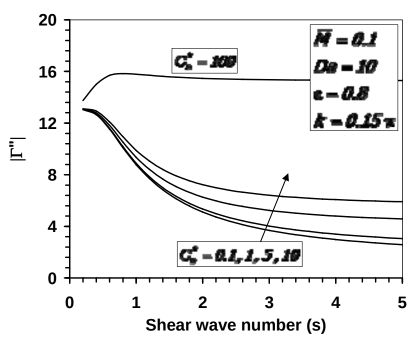
Attenuation - Forward wave



Phase Shift – Forward wave

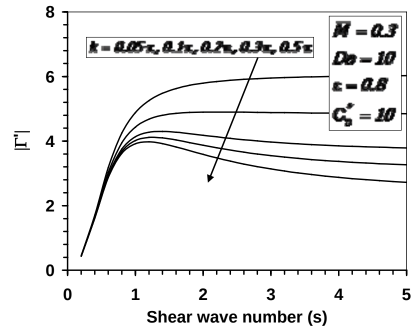


Attenuation – Backward wave

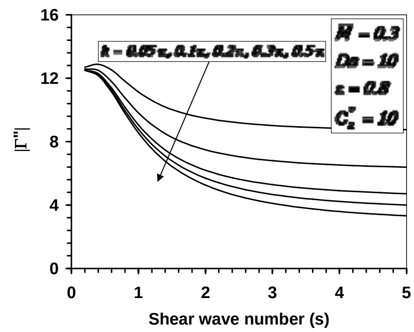


Phase Shift – Backward wave

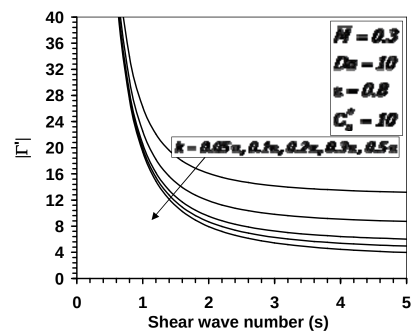
Fig. 4 Effect of Forchheimer number



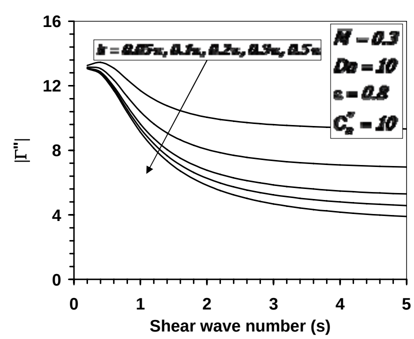
Attenuation - Forward wave



Phase Shift – Forward wave



Attenuation – Backward wave



Phase Shift – Backward wave

Fig. 5 Effect of reduced frequency number

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