THE DEA METHOD IN ECONOMICAL EFFICIENCY ANALYSIS (MICRO-LEVEL).

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Abstract: - In this work we will present an analysis of the economic situation of some branches of Moldova. This was possible using the technique DEA (data envelopment analysis) based on mathematical programmer approach. We're provided an analysis of some certain data concerning the efficiency or inefficiency of the branches, that use the Efficiency Measurement Programs (EMS).
Key-Words: - **:** **e**fficiency score, Malmquist productivity index, efficiency measurement, technical efficiency.

1. The method for economic efficiency measurement

The economic theory of production is based on efficient subsets of production sets, on dual value such as minimum cost functions and maximum revenue or profit functions. We know two approach methods for the construction of the production frontiers and for the measurement of efficiency: the econometric and mathematical programming approach. The econometric approach differs from the mathematical programming approach, because the econometric approach is stochastic, it attempts to distinguish the effects of noise from the effects of inefficiency and the programming approach being non-stochastic, it lumps noise and inefficiency together and calls the computations

1.1.The production frontiers and measurement of efficiency.

By the productivity of production unit we mean the ratio of its output to its inputs. The technical efficiency of a productive unit is a comparison between observed and optimal values (defined in terms of productions possibilities) of its outputs and inputs. This comparison can take the form of a ratio of observed to maximum potential output attainable from the given input, or the ratio of minimum potential to observed input required to produce the given output, or some combination of these two.

Suppose producers use input vector $x = (x_1, \ldots, x_n) \in R_+^N$ to produce output vector $y = (y_1, \ldots, y_m) \in R_+^M$. We refer to affine displacements of the input and output vectors by means of $\bar{x}_i = x_i + \alpha, \alpha \geq 0,$ and $\bar{y}_i = \bar{y}_i + \alpha, \beta \geq 0, i = 1, \ldots, I,$ so as to eliminate zero or negative values that may exist in x_I and y_I. Thus $\bar{x}_i \in R_{++}^N,\ \bar{y}_i \in R_{++}^M,\quad i = 1, \ldots, I.$ Further we consider $x_i = \bar{x}_i$ and $y_i = \bar{y}_i,\ \ i = 1, \ldots, I.$

We can represent the production technology with an input set $L(y) = \{x : (y, x) \textit{is feasible}\}$, and also $P(x) = \{y : (y, x) \textit{is feasible}\}$. The input distance function is $D_i(y, x) = \max\{\lambda : (x/\lambda) \in L(y)\}$, and the output distance function is $D_o(y, x) = \min\{\theta : (y/\theta) \in P(x)\}$, where D_i means D_{input}, D_o means D_{output}.

The corresponding Debrue- Farrell [1] input-oriented and output- oriented measures of technical efficiency can be defined as:

$$DF_i(y, x) = 1/D_i(y, x) = \min\{\lambda : \lambda x \in L(y)\},$$

$$\boldsymbol{DF}_o(\boldsymbol{y}, \boldsymbol{x}) = 1/\boldsymbol{D}_o(\boldsymbol{y}, \boldsymbol{x}) = max\{\theta : \theta \boldsymbol{y} \in \boldsymbol{P}(\ \boldsymbol{x}\ )\},$$

where DF_I means DF_{input} and DF_o means DF_{output}. The most general definition of efficient production is provided by Koopmans [1]: a producer is technically efficient if an increase in any output requires a reduction in at least one other output or an increase in at least one input, and if a reduction in any input an increase in at least one other input or a reduction in at least one output. Debrue and Farrell [1] introduced a measure of technical efficiency defined as one minus the maximum equiproportionate reduction in all inputs that still allows continued production of given outputs.

The mathematical programming approach to the setting up measurement of technical efficiency also economic efficiency frequently goes by the descriptive title of data envelopment analysis (DEA).

Since its foundation in 1978, with the study of Charnes, Cooper and Rhodes [2], the DEA methodology has been developed from a single linear programming model into a vast and still growing family of mathematical programming models. Consider a set of I producers using vector $x \in R^N_{++}$ to produce output vector $y \in R^M_{++}$. Let (x_0, y_0) be the input- output vector of the producer being evaluated and (x_i, y_i) the input- output vector of the i-th producer. The objectives analyze the performance of each producer comparative to the best-observed practice in the sample. In order to do that, we search for a set of nonnegative weights which, when applied to each producer's inputs and outputs, minimizes the ratio of weighted input to weighted output for the producer under evaluation, subjects to the normalizing constraint that no producer in the sample.

$$\min_{\mu,\nu} \frac{\nu^T x_0}{\mu^T y_0}, \frac{\nu^T x_i}{\mu^T y_i} \geq 1, i = 1,\ldots,0,\ldots I,\ \mu,\nu \geq 0,$$

where T means a transpose operation. The previous nonlinear ratio model can be converted into a linear programming problem via the change of variables

$$u = t\mu, v = t\nu, t = \left(\mu^T y_0\right)^{-1}.$$

The model becomes:

$$\min_{u,v} v^T x_0\ , \begin{cases} u^T y_0 = 1 \\ v^T x_i > u^T y_i, i = 1,\ldots,0,\ldots I, \\ u, v \geq 0 \end{cases}$$

And its dual is the linear programming "envelopment". $\max_{\theta,\lambda} \theta, \begin{cases} X\lambda \leq x_0, \\ \theta y_0 \leq Y\lambda, \\ \lambda \geq 0. \end{cases}$

Where X is an $N * I$ input matrix with columns x_i, Y is an $M*I$ output matrix with columns y_i and λ is an I*1 intensity vector.

If radial expansion is possible for a producer, its optimal $\theta > 1$, while if radial expansion is not possible, its optimal $\theta = 1$. We may now observe that optimal $\theta = 1$ is necessary but not sufficient for a producer to be technically efficient in the sense of Koopmans [5], since (θy°, x°) may contain slack in any of its $(N + M - 1)$ dimensions.

The output oriented version of the DEA problem can be handled analogously.

1.2 The Malmquist productivity index

The Malmquist productivity index [4] can be expressed as products of an index of technical change and an index of technical efficiency change.

Let $x^t = \left(x_1^t, \ldots, x_N^t\right) \in R^N_+$ and $y^t = \left(y_1^t, \ldots, y_M^t\right) \in R^M_+$

denote respectively an input vector and an output vector in period t, t = 1, …., T, where T means the time during the practice estimations.

The output oriented Malmquist productivity index can be defined using three approaches for the same orientation:

- a backward-looking approach which evaluates the performance of the data from periods t and $t+1$ relative to technology (production possibilities) from period t

$$M_o^t\left(x^t, y^t, x^{t+1}, y^{t+!}\right) = \frac{D_o^t\left(x^{t+1}, y^{t+1}\right)}{D_o^t\left(x^t, y^t\right)},$$

- a forward – looking approach which evaluates the performances of the data from periods t and $t+1$ relative to technology (production possibilities) from period $t+1$:

$$M_o^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_o^t\left(x^{t+1}, y^{t+1}\right)}{D_o^{t+1}\left(x^t, y^t\right)},$$

A value larger than 1 for $M_o^{t+1}\left(x^{t+1}, y^{t+1}, x^t, y^t\right)$ indicates positive productivity growth from period t observation to the period $t+1$ technology, while a value less than 1 indicates a productivity decline.

In the same manner can be defined an input-oriented Malmquist productivity index. Improvement in productivity occur whenever $M_i^{t+1}\left(x^{t+1}, y^{t+!}, x^t, y^t\right) < 1$.

The Malmquist productivity index can be decomposed into an index of technical change and an index of technical efficiency change. For the Malmquist index we obtain:

Forward:

$$M_o^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left[\frac{D_o^t\left(x^t, y^t\right)}{D_o^{t+1}\left(x^t, y^t\right)}\right]\left[\frac{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^t\left(x^t, y^t\right)}\right]$$

$$\Delta T\left(x^t, y^t\right)\Delta TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right)$$

Backward:

$$\boldsymbol{M}_o^{t+1}\left(\boldsymbol{x}^t, \boldsymbol{y}^t, \boldsymbol{x}^{t+1}, \boldsymbol{y}^{t+1}\right) =$$

$$\left[\frac{D_o^t\left(x^{t+1}, y^{t+1}\right)}{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}\right]\left[\frac{D_o^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^t\left(x^t, y^t\right)}\right] =$$

$$\Delta T\left(x^t, y^t\right)\Delta TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right).$$

- $\Delta T\left(x^t, y^t\right)$ is the index of technical change between periods t and $t+1$ with respect to the data from period t, i.e. the shift in frontier technology between periods;
- $\Delta TE\left(x^t, y^t, x^{t+1}, y^{t+1}\right)$ is an index of technical efficiency change between period's t and $t+1$.

2. The efficiency analysis.

In order to study the technical efficiency of industrial branches we used a DEA model with one input initially and one output, assuming constant returns to scale and free disposability of input and output [6]. We performed EMS for input and output orientations in order to construct the scores of efficiency and the Malmquist productivity index.

2.1 The Data

The entrepreneurial activity in Republic of Moldova is developed according with the Law about the free enterprise adopted on January 3-rd 1992 with the seldom yearly changes. This Law sets rights and obligations, the judicial and organization principles of entrepreneurial activity. It protects the producer but imposes the needs about the quality of final products. The goal of this Law is to protect the modern interior market in condition of the intense concurrence in the market economy.

We study three economical branches: the industry of wines, the milk industry, the canning industry. We collected the data for 22 firms of the first, 17 for the second and 15 for the third. The data source covers the period of 1995-1999 and consists of statistical reports set up by the Department of Statistics, Department of Economy, Bank Reports for the indicated period [7]. (We want to mention that according to the Law of Protection the Entrepreneur about keeping the confidence of information, the names of firms aren't registered.)

In this analysis we considered one input and one output: The yearly Volume of Production and the Total yearly Spending for every firm. The Volume of Production represents the sum value of delivered works (service) with industrial, semi-manufactured, manufactured character, including intermediate activity (allowed by the firm's statute), product stocks and unfinished production. The Total Spending represent the sum value, used for the development of all types of activities included in the statute of entrepreneurial firm's activity.

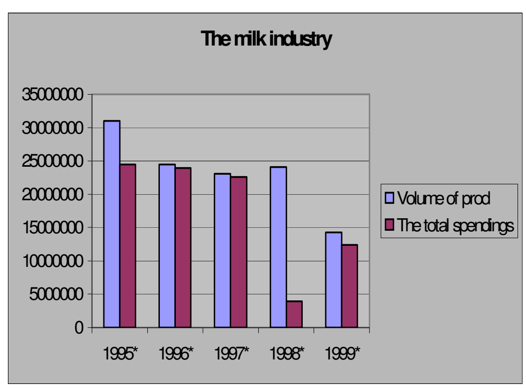
For performing the data analysis, consequently for comparing the firms among them, we have transformed the data in dollars, then we have converted the data according to 1995 price, using the following formula:

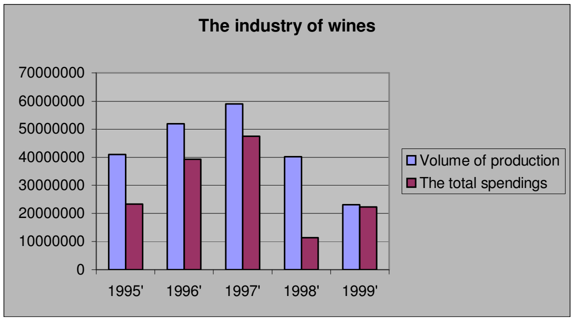
$$\sum_i \boldsymbol{q}_i \boldsymbol{p}_i^b = \frac{\sum \boldsymbol{q}_i \boldsymbol{p}_i^t}{\left(1+\frac{inf^{b+1}}{100}\right)\cdots\left(1+\frac{inf^{t}}{100}\right)}$$

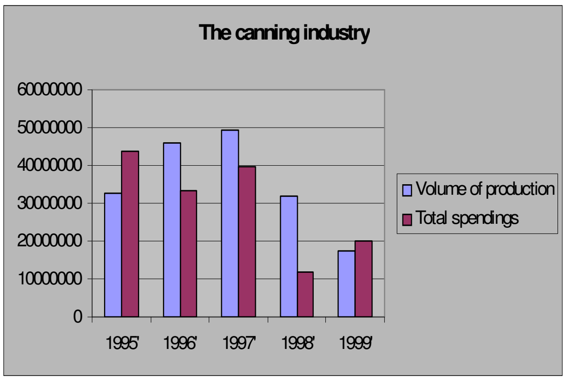
Here: q_I- -denotes the investments, industrial productions and export, respectively, expressed in physical units, corresponding to t year, p_i^b – denotes the price of the reference year, p_i^t – denotes the price of t year, inf^t – denotes the level of inflation in the t year, relatively to $(t-1)$ year, inf^{b+1} – denotes the level of inflation in the $b+1$ year, relatively to b year (reference year).

The data concerning the value rate of exchange and yearly inflation level have been collected from yearly Reports of the National Bank of Moldova.

The summary values of input and output of all firms for every year from 1995 to 1999 (for every branch)

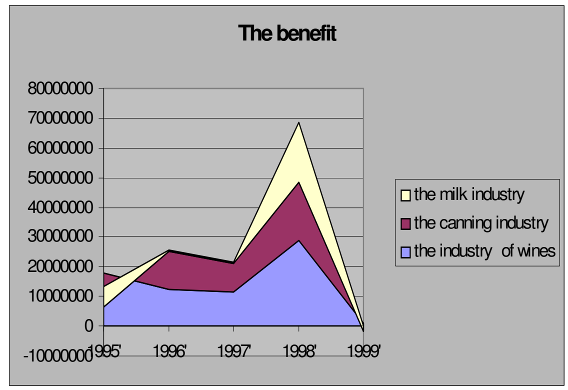






Analyzing the above placed diagrams we can state:
- for the chosen firms of milk industry the total volume of production is bigger than the total spending in every year from 1995-1999 period, consequently it is an profitable activity; a great profit has been registered in 1995 and more considerable in 1998, but we mention that the difference between the volume of production and the total spending in the other years is not considerable. - the general situation for all chosen firms of canning industry, accorded to the summary input and output was not always profitable; for example in 1995 and 1999 the total spending are bigger than the volume of production; the most profitable situation for these is registered in 1998. - the firms with the biggest profit are included in industry of wines, because they registered the biggest differences between the volume of production and the total spending, less on 1999; the highest profit was registered in 1998.

From the above placed diagrams of annual benefit for every branch, we conclude, that the most profitable firms activities were recorded by the industry of wines, except 1999, when the milk industry had bigger profit. If the firms of milk industry recorded always the benefit, but not all the time big, then the firms of the canning industry marked the non-benefit in 1995 and 1999.
We calculated the benefit using the following



formula:

$$B=\sum_i B_i=\sum_i VP_i-\sum_i S_i$$

where: $\boldsymbol{B}$ is the value of benefit; $\boldsymbol{B}_i$ is the firm's benefit; $\boldsymbol{VP}_i$ is the firm's benefit value of production; $\boldsymbol{S}_i$ is firm's spending; $\boldsymbol{i}$- represent number of firms.

Analyzing the above placed diagrams of firms profitableness we can conclude that the profitableness bigger than 1 always was recorded by the firms of the milk industry and the industry of wines; the firms of milk industry marked the top of profitableness in 1998.

We have used the following formula to estimate the profitableness: $\boldsymbol{P} = \frac{\sum_i \boldsymbol{VP}_i}{\sum_i \boldsymbol{S}_i}$, where ***P*** represents the profitableness. We keep the notation of benefit's formula.

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