A NEW ILL-CONDITIONED POWER FLOW ALGORITHM CONSIDERING THE ODDITIONAL NODAL POWERS

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Abstract: - The ill-conditioned load flow algorithm is important to power plan and analysis on security, stability in power system. The new ill-conditioned load flow algorithm based on node lopsided powers is proposed by transforming power flow equations to non-linear programming. It has the all-right astringency arose from the beginning value and no feasible solution in the power flow calculating progress and no larger calculating cost by compared to traditionary power flow calculation. The case study is made at IEEE-30 nodes system and the validity is proved.

Key-Words: - Ill-conditioned load flow; Optimal multiplier; Newdon power flow; Power system; Nonlinear programming;

1 Introduction

The Newton method is main one to solve the power flow equations, which is unable to judge the solution of power flow equations in existence with often appearing oscillations and divergences due to the ill-conditions arosed by strong nonlinear. Recently, more outage failures have occured at many countries and areas, and the facts are indicated that reliable operation of power system is based on the exact analysis. Therefore, the ill-conditioned power flow problem must be seriously considered[1], furthermore it is significant for programming, transmission capacity, security and stability of power network.

The reasons of ill condition at solving power flow problem can be summarized as followings[2]: 1) The unreasonable initialization of state variables led to difficult convergence of power flow calculation; 2) The power flow equations can be not solved, in other words, the solutions are not in existence; 3) The solutions can be not found due to localization in the arithmetic. At present, the optimal multiplier method and nonlinear programming method are main researches in ill-conditioned power flow. The optimal multiplier method under rectangular coordinates is recognized as a best method[3], which also is called by damp Newton method[4], but it is difficult under polar coordinate because of higher item in Taylor formula. The better effects are obtained by using orthogonal transformation damp multiplier method to solve the optimal multiplier under polar coordinate[5]. It is proved properly exact[6], that quadric functions are used to approach the power flow equations near solutions. The detailed calculating process of nonlinear programming method is expatiated at [7], it is uncommonly applied because that is more complicated to solve K-T conditions than power flow equations. More attentions are focused at voltage stability study with ill-conditioned power flow arithmetic, a new ill-conditioned arithmetic[8] is proposed to correct Jacob matrix near transmission limit point for analyzing voltage stability. At [9], an optimal multiplier method considering static voltage characteristic of load is used. The homotopy arithmetic[10] is also introduced in ill-conditioned power flow calculation.

In conclusion, the optimal multiplier method is better one, but the dependence on initialization can't be avoided, and while the solutions of power flow equations is inexistent, the information about nodal power imbalance can’t be provided by the optimal multiplier method. In this article, the nodal injected imbalance powers are introduced as the control variable and the power flow calculation is converted as a nonlinear programming problem, the calculating costs are not increased comparing with the general power flow calculation with better convergence while the ill-condition which is brought by unreasonable initialization or inexistent solutions occurs.

2 Mathematical Models

The key reasons of difficult convergence of power flow calculation are exhibited in nodal injected power imbalance as following:

$$P_{Gi} - P_{Li} - e_i \sum_{k \in i} (G_{ki} e_k - B_{ki} f_k) - f_i \sum_{k \in i} (G_{ki} f_k + B_{ki} e_k) = u_{pi} \quad (1)$$

$$Q_{Gj} - Q_{Lj} - f_j \sum_{k \in j} (G_{kj} e_k - B_{kj} f_k) + e_j \sum_{k \in j} (G_{kj} f_k + B_{kj} e_k) = u_{qj} \quad (2)$$

where, the $i = 1,2,\cdots N$ denotes number of active power flow equations; the $j = 1,2,\cdots M$ denotes reactive power flow equations; u_p and u_q respectively denote the imbalance of nodal injected active and reactive powers. The nonlinear programming form of power flow equations can be performed, objective function:

$$\min \quad \sum_{i=1}^{N} u_{pi}^2 + \sum_{j=1}^{M} u_{qj}^2 \quad (3)$$

s.t.

$$P_{Gi} - P_{Li} - e_i \sum_{k \in i} (G_{ki} e_k - B_{ki} f_k) - f_i \sum_{k \in i} (G_{ki} f_k + B_{ki} e_k) - u_{pi} = 0 \quad (4)$$

$$Q_{Gj} - Q_{Lj} - f_j \sum_{k \in j} (G_{kj} e_k - B_{kj} f_k) + e_j \sum_{k \in j} (G_{kj} f_k + B_{kj} e_k) - u_{qj} = 0 \quad (5)$$

While the convergence of power flow calculation is obtained, the $u_{pi} = 0$ and $u_{qj} = 0$; otherwise, the u_p and u_q respectively represent the nodal injected imbalance powers.

3 Calculating method

The expanded Lagrange function is performed by integrating formulas (3)~(5) as:

$$L(x,u) = \sum_{i=1}^{N} u_{pi}^2 + \sum_{j=1}^{M} u_{qj}^2 + \sum_{i=1}^{N} \lambda_{pi} (P_{Gi} - P_{Li} - e_i \sum_{k \in i} (G_{ki} e_k - B_{ki} f_k) - f_i \sum_{k \in i} (G_{ki} f_k + B_{ki} e_k) - u_{pi}) + \sum_{j=1}^{M} \lambda_{qj} (Q_{Gj} - Q_{Lj} - f_j \sum_{k \in j} (G_{kj} e_k - B_{kj} f_k) + e_j \sum_{k \in j} (G_{kj} f_k + B_{kj} e_k) - u_{qj}) \quad (6)$$

The optimal conditions are:

$$\frac{\partial L}{\partial u} = 2u - \lambda = 0 \quad (7)$$

$$\frac{\partial L}{\partial x} = f_x^T \lambda = 0 \quad (8)$$

$$\frac{\partial L}{\partial \lambda} = f(x,u) = 0 \quad (9)$$

where, $x = [V \quad \theta]^T$; $u = [u_p \quad u_q]^T$; $\lambda = [\lambda_p \quad \lambda_q]^T$; $f(x,u) = [f_p \quad f_q]^T$; f_p and f_q represented respectively active and reactive power balance equations. It is can be obtained from formula (7):

$$\lambda = 2u \quad (10)$$

To introduce formula (10) into (8):

$$g(x,u) = 2\left[\frac{\partial f(x,u)}{\partial x}\right]^T u = 0 \quad (11)$$

The solution of formula (11) and (9) is equivalent with ill-conditioned problem (3)~(5). It is important to perform Jacob matrix for solving problem (9) and (11) with Newton method. The Jacob matrix of equations (9) and (11) are as following:

$$J = \begin{bmatrix} f_x & f_u \\ g_x & g_u \end{bmatrix} \quad (12)$$

where, the f_x is Jacob matrix of conventional power flow equations. It can be obtained from formula (4) and (5) as $f_u = -E$, the E is unit diagonal matrix. The $g_u = f_x$ is gained from formula (11). The corrective functions are:

$$\begin{bmatrix} \Delta f \\ \Delta g \end{bmatrix} = \begin{bmatrix} f_x & -E \\ g_x & f_x^T \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta u \end{bmatrix} \quad (13)$$

where, the Δf corresponds active power error ΔP or reactive power error ΔQ. The above formula is extended as:

$$\Delta f = f_x \Delta x - \Delta u$$
$$\Delta g = g_x \Delta x + f_x^T \Delta u \quad (14)$$

viz.

$$(f_x^T f_x + g_x)\Delta x = f_x^T \Delta f + \Delta g \quad (15)$$

After Gauss iterations they are obtained:

$$\Delta x = (f_x^T f_x + g_x)^{-1} (f_x^T \Delta f + \Delta g) \quad (16)$$

and

$$\Delta u = f_x \Delta x - \Delta f \quad (17)$$

Therefore, the final solutions are completely obtained.

4 Arithmetic Evaluation

The calculation costs are increased compared with conventional power flow calculation at followings:

1) It is additional to perform matrix g_x;

2) The addition, subtraction and multiplication operations of matrix are added;

The added calculation costs are less, because the matrix dimensions are not changed in Gauss iteration process. It is worthy to pay attention for sparsity of the matrix g_x, which is as following under rectangular coordinates:

$$g(x,u)=\begin{bmatrix} u_{p1}H_{11}+u_{p2}H_{21}+\cdots+u_{pN}H_{N1} \\ +u_{q1}J_{11}+u_{q2}J_{21}+\cdots+u_{qM}J_{M1} \\ u_{p1}H_{12}+u_{p2}H_{22}+\cdots+u_{pN}H_{N2} \\ +u_{q1}J_{12}+u_{q2}J_{22}+\cdots+u_{qM}J_{M2} \\ \vdots \\ u_{p1}H_{1N}+u_{p2}H_{2N}+\cdots+u_{pN}H_{NN} \\ +u_{q1}J_{1N}+u_{q2}J_{2N}+\cdots+u_{qM}J_{MN} \\ u_{p1}N_{11}+u_{p2}N_{21}+\cdots+u_{pN}N_{N1} \\ +u_{q1}L_{11}+u_{q2}L_{21}+\cdots+u_{qM}L_{M1} \\ u_{p1}N_{12}+u_{p2}N_{22}+\cdots+u_{pN}N_{N2} \\ +u_{q1}L_{12}+u_{q2}L_{22}+\cdots+u_{qM}L_{M2} \\ \vdots \\ u_{p1}N_{1M}+u_{p2}N_{2M}+\cdots+u_{pN}N_{NM} \\ +u_{q1}L_{1M}+u_{q2}L_{2M}+\cdots+u_{qM}L_{MM} \end{bmatrix} \quad (18)$$

The $g(x,u)$ is consisted of $N+M$ equations, and their elements are:

$$\begin{cases} \dfrac{\partial g_i}{\partial e_i}=-u_{pi}2G_{ii}+u_{qi}2B_{ii} \\ \qquad or \quad -u_{pi}2G_{ii}+2u_{qi} \\ \dfrac{\partial g_i}{\partial e_j}=-u_{pi}G_{ij}-u_{pj}G_{ji}+u_{qi}B_{ij}+u_{qj}B_{ji} \\ \qquad or \quad -u_{pi}G_{ij}-u_{pj}G_{ji} \\ \dfrac{\partial g_i}{\partial f_i}=0 \\ \dfrac{\partial g_i}{\partial f_j}=u_{pi}B_{ij}-u_{pj}B_{ji}+u_{qi}G_{ij}-u_{qj}G_{ji} \\ \qquad or \quad u_{pi}B_{ij}-u_{pj}B_{ji} \\ \dfrac{\partial g_{N+i}}{\partial e_i}=0 \end{cases} \quad (19)$$

and

$$\begin{cases} \dfrac{\partial g_{N+i}}{\partial e_j}=-u_{pi}B_{ij}+u_{pj}B_{ji}-u_{qi}G_{ij}+u_{qj}G_{ji} \\ \qquad or \quad -u_{pi}B_{ij}+u_{pj}B_{ji} \\ \dfrac{\partial g_{N+i}}{\partial f_i}=-u_{pi}2G_{ii}+u_{qi}2B_{ii} \\ \qquad or \quad -u_{pi}2G_{ii}+2u_{qi} \\ \dfrac{\partial g_{N+i}}{\partial f_j}=-u_{pi}G_{ij}-u_{pj}G_{ji}+u_{qi}B_{ij}+u_{qj}B_{ji} \\ \qquad or \quad -u_{pi}G_{ij}+u_{pj}B_{ji} \end{cases} \quad (20)$$

It can be seen that matrix $g(x,u)$ is a function vector, there every element is corresponding to a node, which contains the information at this node and relative nodes and the elements of matrix $g(x,u)$ only exists in these row and arrange. So a conclusion can be got, that matrix $g(x,u)$ is same in structure with the node conductance matrix.

Moreover, the matrix $f_x^T f_x + g_x$ in formula (16) is greatly sparse due to the sparsity and symmetry of Jacob matrix, even if the matrix $f_x^T f_x$ is different from Jacob matrix in structure.

5 Case Study

The case study is made at IEEE-30 nodes system. At normal initial value, the calculating results are listed in table 1 using ill-conditioned power flow method in this article.

It can be seen that imbalance powers at every node are very small.

The active power is changed from 0.106 to 1.106 at node 30 for the sake of testing the effect of proposed ill-conditioned power flow method. The calculating results are listed in table 2.

It can be found, that imbalance powers and voltage drops are biggish at node 30 and 29, since the heavy loads are at node 30 and the node 29 is very close to it.

Table 1 The Calculating Results at Normal Case

node	Voltage magnitude	Voltage Angle	Injective Active Power	Injective Reactive Power	Active power Imbalance	Reactive Power Imbalance
1	1.05000	0	2.385440	0.971437	0	0
2	1.03380	-2.73577	0.358601	-0.107681	-7.64116e-007	-2.27344e-006
3	1.03136	4.68341	0.023998	-0.011998	-1.55274e-006	-1.55476e-006
4	1.02638	5.61	-0.075998	-0.015998	-1.76185e-006	-1.40946e-006
5	1.00580	8.99436	0.696399	0.031314	-7.39647e-007	-1.68147e-006
6	1.02167	6.47644	0	0	-7.43012e-006	5.90618e-007
7	1.00743	8.03754	0.227998	-0.108999	-2.23318e-006	-1.11143e-006
8	1.02300	-6.47435	0.049999	0.016944	5.79214e-007	-2.71078e-006
9	1.05830	8.09944	0	-0.002253	9.80911e-007	-1.94656e-006
10	1.05316	9.95912	0.057995	-0.019999	-4.86605e-006	-3.75756e-007
11	1.09130	-6.24895	0.179302	0.176036	-2.00428e-006	-1.12855e-006
12	1.06083	9.19311	0.111998	-0.007498	-2.35256e-006	-1.42298e-006
13	1.08830	-8.01813	0.169101	0.215281	-1.01293e-006	-1.01435e-006
14	1.04670	-10.0877	-0.061998	-0.015998	-1.52749e-006	-1.53449e-006
15	1.04268	10.202	-0.081998	-0.024998	-1.52402e-006	-1.53213e-006
16	1.05041	9.79032	-0.034998	-0.017998	-1.52948e-006	-1.53419e-006
17	1.04704	10.1145	-0.089998	-0.057998	-1.49275e-006	-1.55201e-006
18	1.03431	10.8031	-0.031998	-0.008998	-1.53106e-006	-1.53664e-006
19	1.03246	10.9707	-0.094998	-0.033998	-1.50563e-006	-1.54759e-006
20	1.03686	10.7748	-0.021998	-0.006998	-1.53102e-006	-1.54795e-006
21	1.04049	10.4137	-0.174991	-0.112004	-9.47263e-006	4.40902e-006
22	1.04090	10.4056	0	-0.012930	8.15912e-006	-8.44917e-006
23	1.03359	10.6536	-0.031998	-0.015998	-1.52796e-006	-1.53682e-006
24	1.02986	10.9151	-0.086997	-0.066999	-2.8651e-006	-3.04656e-007
25	1.02980	10.9043	0	-0.002340	-9.56503e-007	-2.05346e-006
26	1.01231	11.3125	-0.034998	-0.022998	-1.88466e-006	-1.18409e-006
27	1.03880	10.6539	0	0.008351	-1.58602e-006	-1.77005e-006
28	1.01770	6.87109	0	-0.010505	3.83777e-007	-1.84907e-006
29	1.01929	11.8465	-0.023998	-0.008998	-1.81952e-006	-1.31704e-006
30	1.00800	12.7018	-0.105998	-0.018998	-1.71298e-006	-1.38878e-006

Table 2 The Calculating Results at Heavy Loads Case

node	Voltage magnitude	Voltage Angle	Injective Active Power	Injective Reactive Power	Active power Imbalance	Reactive Power Imbalance
1	1.05000	0	2.689440	0.9553260	0	0
2	1.03098	3.37079	0.374602	0.1699930	-0.0080010	0.00581676

3	1.03055	5.69676	0.002007	0.0126617	-0.0130037	0.00033087
4	1.02531	6.86885	0.044437	-0.0173984	-0.0157811	0.00069920
5	1.00700	10.1332	0.668480	0.0602611	-0.0139600	-0.00240801
6	1.01992	8.28764	0.038603	-0.0010787	-0.0193017	0.00053937
7	1.00709	9.52897	0.193561	-0.1098010	-0.0172195	0.00040061
8	1.01941	-8.47083	0.091074	0.0514476	-0.0205370	0.00733658
9	1.05795	9.22968	0.047021	-0.0219539	-0.0235108	0.00074819
10	1.05522	10.9966	0.006622	-0.0192284	-0.0256886	-0.00038582
11	1.09130	7.13573	0.226321	0.1787000	-0.0235107	1.85164e-009
12	1.06415	9.75287	0.066553	-0.0063559	-0.0227233	-0.00057203
13	1.08922	8.42535	0.214520	0.1992420	-0.0227101	-0.00199649
14	1.05342	10.5358	0.013521	-0.0148196	-0.0242394	-0.00059020
15	1.04819	10.8627	0.030932	-0.0248151	-0.0255338	-9.2474e-005
16	1.05470	-10.3603	0.013131	-0.0171185	-0.0240657	-0.00044073
17	1.05030	10.9328	0.039448	-0.0570850	-0.0252760	-0.00045749
18	1.04251	-11.1699	0.019473	-0.0084978	-0.0257369	-0.00025107
19	1.04061	11.3393	0.043348	-0.0332623	-0.0258256	-0.00036886
20	1.04406	-11.2405	0.029568	-0.0062466	-0.0257842	-0.00037666
21	1.04224	11.6693	0.119622	-0.1117560	-0.0276892	-0.00012214
22	1.04221	-11.7662	0.056494	-0.0578696	-0.0282472	-0.00273711
23	1.03799	-12.1892	0.028674	-0.0171201	-0.0303375	0.00056003
24	1.03002	14.086	0.013545	-0.0686701	-0.0367270	0.00083504
25	1.02445	-20.1036	0.118573	-0.2348710	-0.0592864	0.01098000
26	1.02195	-19.3003	0.082218	-0.0216818	-0.0586091	-0.000659102
27	1.07413	-25.9898	0.144601	1.1146700	-0.0723005	-0.07465440
28	1.01594	-10.1912	0.051629	0.1609690	-0.0258145	0.00358317
29	0.92627	-33.6877	0.209292	0.0211749	-0.1166460	-0.01508740
30	0.77496	-48.7055	-0.766148	0.0476632	-0.1699260	-0.03333160

6 Conclusions

The ill condition of power flow calculation is caused by imbalanced powers, so the ill-conditioned power flow arithmetic is proposed in this article based on the nodal imbalance powers, and the better effects are obtained by test at IEEE-30 system.

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