Single Input Fuzzy Control of Nonlinear and Time-Varying Plant

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Abstract: - The aim of this paper is to design a single input PI fuzzy controller for a nonlinear and time-varying plant – the biogas production rate in the anaerobic digestion of organic waste in waste water treatment. The main contributions are the design of the SI FC, the study of the closed loop control system performance and the estimation of the advantages of the SI FC system in comparison to other systems with advanced fuzzy controllers. The single input fuzzy controller (SI FC) reduces the complexity in the design and the tuning of fuzzy controllers and ensures stability and high performance of the closed loop system. The investigation is based on Matlab, Simulink and Fuzzy Logic Toolbox.

Key-Words: - Single input PI fuzzy controller, simulation, anaerobic digestion of organic waste, stability, Matlab

1 Introduction

The model-free fuzzy logic approach has been recently gaining popularity as an efficient and appealing solution to the control problem of complex nonlinear and time-varying plants under uncertainties [1-3]. Despite its well-developed theoretical framework the industrial applications are still limited to few areas. The reason is the practical completion and tuning of the fuzzy logic controller (FLC). Though the FLC design seams at first sight rather simple and straightforward there exist a lot of uncertainties, concerning the real world plant, to be tackled often by empirical tuning of quite a number of parameters – scaling coefficients, rules, membership functions, etc. First, to escape the confusion in the definition of the universes of discourse for the fuzzy sets that describe the terms of the linguistic variables, normalization and renormalization coefficients are added to allow tuning, which in most cases is carried out heuristically. Second, to facilitate the correct mapping of the available expert knowledge, expressed in words, onto the rule base PI fuzzy controllers are mainly developed, for which there are clear prescriptions on how to derive the rule base easily though not uniquely. The inclusion of the derivative of error as a controller's input, however, arouses new tuning problems in order to prevent noise interference and to ensure good system performance. Supervisory autotuning is suggested in the two-level fuzzy control algorithms, which is, however, accompanied by an increase of the controller complexity [4].

To cope with the two basic problems - "the curse of dimensionality" with the increase of linguistic variables and terms and the tuning of the rule base and the membership functions, fuzzy logic is complemented with artificial neural networks (ANN), which enhances the FLC adaptability, simplifying at the same time its structure [5,6]. An ANN, trained to fit the designed FLC control surface and to generalize well, can be easily embedded in controllers [6]. Also a Sugeno backpropagation ANN can be trained on rich in frequency and amplitude collection of input-output data [7] that reflect all possible disturbances, nonlinearities, model uncertainties, etc. to yield a Sugeno fuzzy controller with tuned membership functions and reduced number of rules rules. The curse of dimensionality is avoided, the tuning is automatic, but the real-world noisy data often leads to bad training or generalization thus deteriorating the system performance.

Various efforts have been made to derive a unique rule base out of stability and performance requirements, which are, however, restricted either to a particular plant, or need a plant model [1, 3, 7, 8].

So far neither solution to the complexity problems, related to redundancy of parameters to be tuned – scaling coefficients, derivative coefficients, rules, membership functions, etc. nor practical prescriptions for unique design of FLC have been found.

Therefore the suggested in [9] single input PI-like fuzzy controller (SI FC) is appealing for the unique construction and the reduction by times of the rule base that ensures both closed loop system stability for easily checked restrictions on the plant and control curve nonlinearity and also a simplified tuning because of the reduced number of tuning parameters.

The aim of the present paper is to design a single input PI fuzzy controller for a nonlinear and time-varying plant - the anaerobic digestion (AD) of organic waste in wastewater treatment processes, and to compare the system performance with the results, obtained from alternative fuzzy approaches for the control of the same plant using Simulink and Fuzzy Logic Toolbox of Matlab [10].

2 Problem Formulation

The standard PI-like fuzzy controller has inherited its structure from the incremental classic PI controller - its inputs are the system error e and the change-of-error (derivative) $\dot{e}$ and the output is the rate of the control action Δu. The rules are in the form:

R_k: IF e is LE_i AND $\dot{e}$ is LDE_j THEN Δu is LU_l ,

where $i=1\div M$, $j=1\div N$, LE_i, LDE_j and LU_l denote the corresponding linguistic values for e, $\dot{e}$ and Δu in the k[th] rule. The total number of rules is $M\times N$. The rules are located in the 2-D space and can be represented as a fuzzy associative matrix FAM in Table 1. , where the norm or zero term has a subscript “0”, the subscripts “-1” and “1” denote asymmetric terms, i.e., if LU_{-2} is NG (negative great) then LU_2 is PG (positive great).

Table 1. FAM of standard PI fuzzy controller

Δu: $\dot{e}$ \ e	LE_{-2}	LE_{-1}	LE_0	LE_1	LE_{-2}
LDE_{-2}	LU_0	$\Delta u_{12}=LU_{-1}$	LU_{-1}	LU_{-2}	LU_{-2}
LDE_{-1}	$\Delta u_{21}=LU_1$	LU_0	LU_{-1}	LU_{-1}	LU_{-2}
LDE_0	LU_1	LU_1	LU_0	LU_{-1}	LU_{-1}
LDE_1	LU_2	LU_1	LU_1	LU_0	LU_{-1}
LDE_2	LU_2	LU_2	LU_1	LU_1	LU_0

The standard FAMs exhibit skew symmetric properties - $\Delta u_{21}=-\Delta u_{12}$. The sign of Δu depends on the location of the point $(e,\dot{e})$ above or below the diagonal line, determined by the terms “Zero” LU_0. The magnitude of Δu is a function of the distance from the point to this line. For infinite number of terms (discretization levels) the FAM becomes continuous as shown in Fig.1. with diagonal switching line $s_l : \dot{e}+\lambda.e=0$.

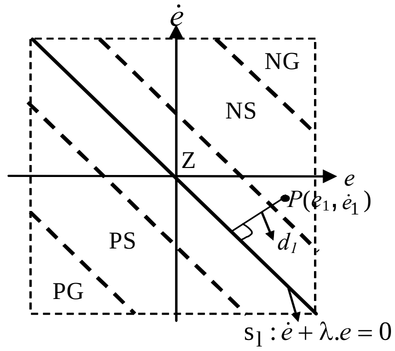
A generalized single input (SI) to the controller, called signed distance, can be introduced:

$$d_s = \mathrm{sign}(s_l)\frac{|\dot{e}+\lambda.e|}{\sqrt{1+\lambda^2}} = \frac{\dot{e}+\lambda.e}{\sqrt{1+\lambda^2}}, \mathrm{sign}(s_l) = \begin{cases} 1, & s_l > 0 \\ -1, & s_l < 0 \end{cases} .(1)$$

The output of the single input fuzzy controller (SI FC) is uniquely determined from $u=-d_s$ and has rules such as:

R_k: IF d_s is LDS_k THEN Δu is LU_l,

where LDS_k is the linguistic value of the signed distance d_s in the k[th] rule. The number of the rules is reduced five



NG –Negative Great, NS –Negative Small
Z–Zero, PG–Positive Great, PS–Positive Small

Fig.1. FAM with infinite number of discretization levels

times and the rules are located in the 1-D space. The new FAM is shown in Table 2.

Table 2 FAM of a single input PI fuzzy controller

d_s	LDS_{-2}	LDS_{-1}	LDS_0	LDS_1	LDS_{-2}
u	LU_2	LU_1	LU_0	LU_{-1}	LU_{-2}

The SI FC reduces the number of the tuning parameters to only derivative, normalization and denormalization coefficients. It has a control curve $u=\Psi(d_s)$ instead of a control surface and ensures system stability for sector bounded control and bounded plant nonlinearity [9].

This new approach to the design of fuzzy controllers is applied to the anaerobic digestion of organic waste in waters [4, 7].

The anaerobic digestion (methane fermentation) of organic waste is the last stage of water depollution technology, in which organic matter (animal litters, plant sludge, industrial and domestic waste) is mineralized by microorgamisms in the absence of oxygen to safely disposable in the environment substances. The anaerobic digestion treats higher organic load, produces less sludge and leads to energy recovery through the biogas produced. It is a multistage process that involves microorganisms of several diverse groups and can be viewed upon as a three-phase conversion of organic waste into biogas – hydrolysis, acidogenesis and methanogenesis. The process takes place under prescribed temperature and pH since the acidogenic bacteria are sensitive to temperature changes while the methanogenic bacteria cannot tolerate pH fluctuations. Microorganisms’ development undergoes four phases – lag phase (adaptation), exponential growth, stationary and dying phase. The models used are nonlinear both in terms of parameters and variables and time-varying, making the classical control theory inapplicable. The parameters cannot be precisely determined because of the specific features of the microorganisms, the low reproducibility of the experiments, the limited number of time-consuming and expensive measurements and complex laboratory analyses, the noisy experimental data, the great number of model parameters, etc.

The only continuously measurable and controlled plant output is the specific biogas production rate Q, l/l.d, $y=[Q]$, $Q(0)=0.94$. The plant inputs are the dilution rate as control variable D, d^{-1} ($D \in [0,0.3]$, $D(0)=0.04$) and the influent organic concentration as disturbance S_{oi}, g/l, ($S_{oi} \in [30,70]$). For the sake of simulations a fifth order Barth-Hill nonlinear model with specific growth rates of the acidogenic and the methanogenic bacteria in the form of Monod is used.

3 Design of Single Input Fuzzy Controller

The fuzzy sets for $d_s=e+\dot{e}$ and Δu of the SI FC are normalized in the range [-1,1] and shown in Fig.2.

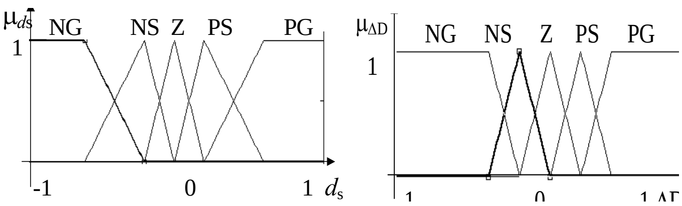


Fig.2. Membership functions for SI FC input and output

The rules are:

R$_1$: IF d_s is NG THEN ΔD is NG,
R$_2$: IF d_s is NS THEN ΔD is NS,
R$_3$: IF d_s is Z THEN ΔD is Z,
R$_4$: IF d_s is PS THEN ΔD is PS,
R$_5$: IF d_s is PG THEN ΔD is PG,

The calculated control curve is shown in Fig.3 and is sector bounded.

The designed SI FC configuration is depicted in Fig.4. It consists of a fuzzy, a pre- and a post- processing units. The change-of-error de is obtained at the output of a differentiator with tuning gain k_d and timeconstant T_d.

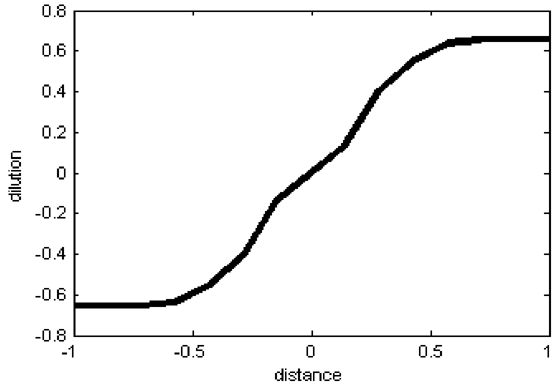


Fig.3. Control curve of SI FC

Other tuning parameters are the normalization K_{ds} and denormalization $K_{\Delta D}$ scaling coefficients for the input and the output of the fuzzy unit. Integrating actuator with timeconstant T_a controls the position of the valve for the pure water to insure the desired dilution rate D.

4 Closed Loop System Performance

In order to study the closed loop system performance simulations are carried out using Simulink. Seven different control algorithms are applied:

- classical PI, tuned using empirical formulae for overshoot of 0% for linearised in the operating point plant model to work with $K_p=0.019$, $T_i=1.723$;
- fuzzy two-level with standard Mamdani main controller [4];
- fuzzy Sugeno [7];
- fuzzy Sugeno with Sugeno plant predictor (S&pr) [7];
- fuzzy two-level with SI FC as main fuzzy controller;
- SI FC;
- SI FC with Sugeno plant predictor.

The algorithms are applied for the control an approximate Simulink plant model to keep the biogas production rate at a desired reference D_r at step reference

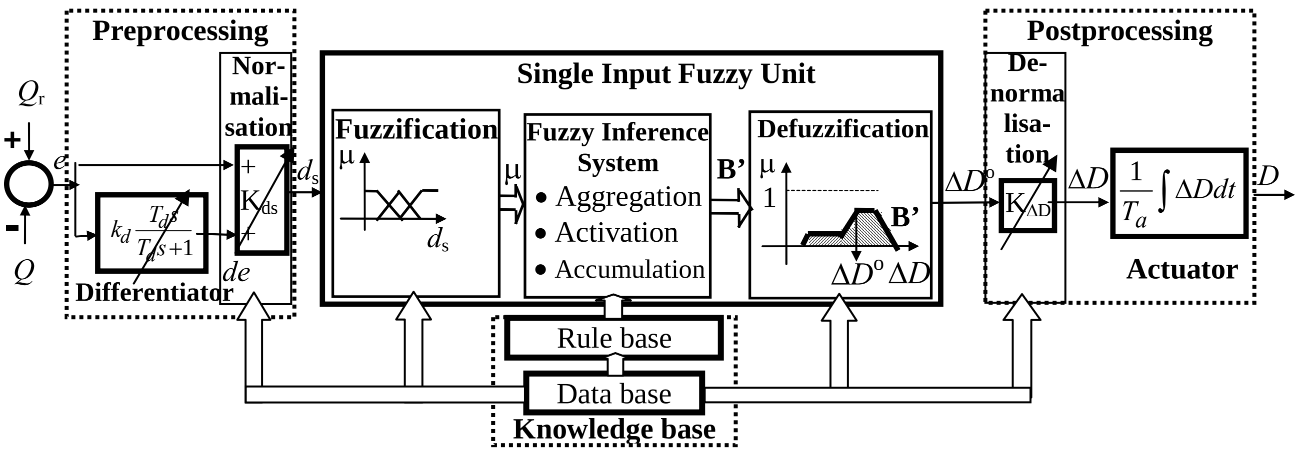


Fig.4. Single input PI fuzzy controller

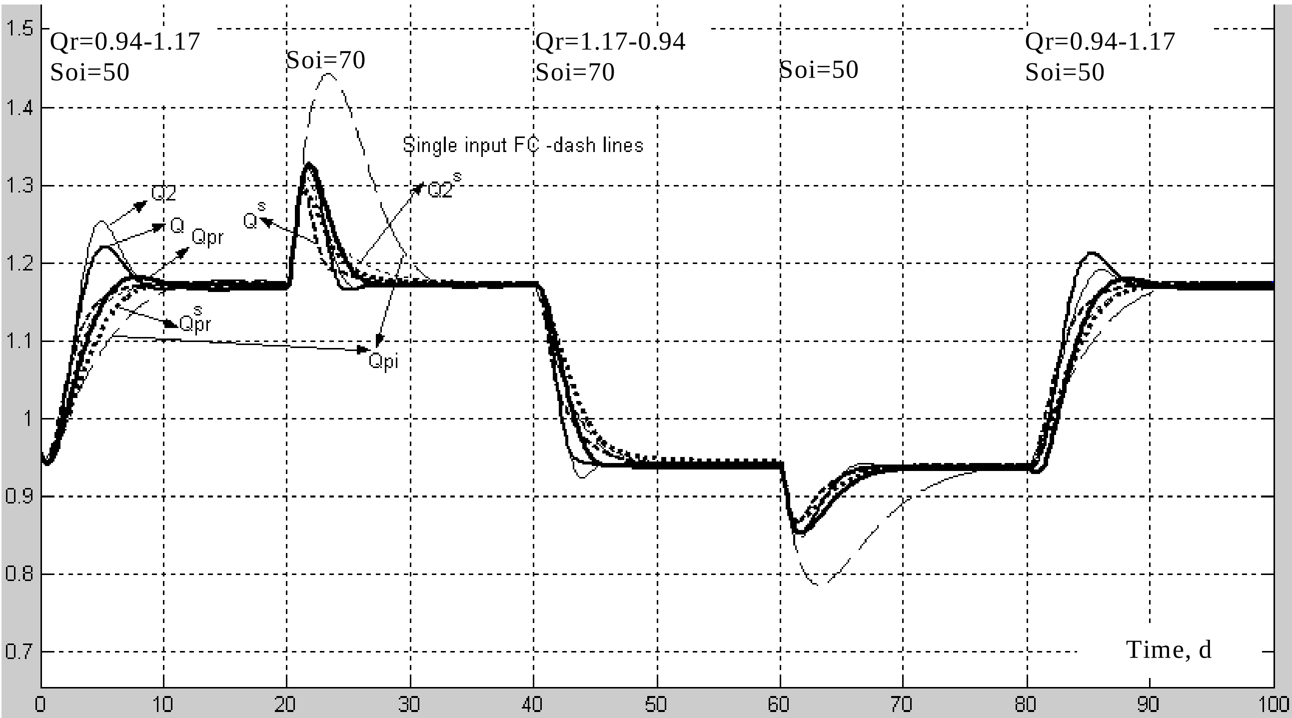


Fig.5. Step responses of closed loop systems with different controllers

Table 3. Plan of experiments and performance indices

Plan of simulation experiments			Performance measures			
t, d	ΔQ_r	S_{oi}	Maximal deviation		Settling time t_s , d	
			SI FC	S&pr	SI FC	S&pr
0	0.94-1.17	50	0	0.023	7	12
20		70	0.13	0.15	6	6
40	1.17-0.94	70	0	0	8	8
60		50	0.06	0.07	6	10
80	0.94-1.17	50	0	0.023	7	12

changes and disturbances in the initial organic load treated S_{oi}. The plan of the experiments is shown in Table 3, where also are given the estimated from the step responses performance indices of the system with SI FC as the best and with Sugeno with predictor (S&pr) as the second best.

The step responses are shown in Fig.5. The output variables of the systems with the different controllers are denoted as follows:

- *Q*pi - for classical PI controller;
- *Q*2 - for two-level fuzzy controller;
- *Q* – for Sugeno controller;
- *Q*pr – for Sugeno controller and plant predictor.

In dash lines are shown the denoted with the superscript "s" step responses of the systems with SI FC based controller.

The comparison of the performance indices shows that the SI FC is not only simple in structure and tuning but also ensures best system performance without autotuning and plant predictor. The two-level FC system to retunes its scaling coefficients for the error and the change-of-error at the end of the first step response. That is why the system performance of this first step response is worse that the last step response, which is provoked by the same step change in D_r. All Sugeno based controllers and plant predictors though simple and leading to high system performance are very sensitive to noisy real-world data used for training and in generalization.

The system stability can be qualitatively estimated using the phase plane method [7, 8]. The phase trajectory of the SI FC system, shown in Fig.6, keeps closer to the phase plane origin compared to the S&pr system. This is a confirmation for fast and overdamped step responses, high dynamic accuracy in tracing of the reference and in rejection of the disturbances.

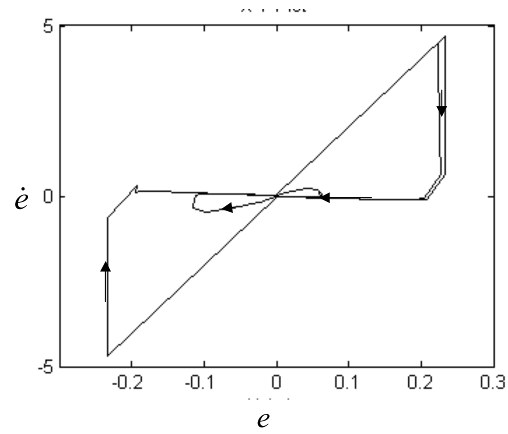


Fig.6. Phase plane SI FC system trajectories

5 Analysis and Conclusion

The main contributions presented in this paper are summarized as follows.

A single input PI fuzzy controller is designed using Fuzzy Logic Toolbox of Matlab.

It is applied for the control of a nonlinear and time-varying plant – the biogas production rate in anaerobic degradation of organic waste in wastewater treatment and the step responses are studied via simulation in Matlab and Simulink environment.

The designed SI FC is embedded as a main controller also in various control configurations – in a two level scheme with supervisory autotuning and in a scheme with Sugeno plant predictor.

The performances of the closed loop systems using standard and SI fuzzy controllers are compared.

A stability analysis is carried out by studying via simulation the phase trajectories of the SI FC system.

The SI FC is simple in structure and easy to design and tune. It has uniquely determined rule base, reduced number of tuning parameters and ensures stability and good performance indices – fast and overdamped step responses at different operating points of the nonlinear plant, high robustness to noise and disturbances.

Future research is foreseen in testing of discrete SI FC in conditions more close to industrial environment for the control of pilot plant using Matlab real time facilities.

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