The Irreversible Power Cycles Preliminary Design

GHEORGHE DUMITRASCU, BOGDAN HORBANIUC
Engineering Thermodynamics, Thermal Machines, and Refrigeration Department,
"Gh. Asachi" Technical University of Iasi,
Bd. D. Mangeron 59-61, 700050-Iasi,
ROMANIA,
http://www.tuiasi.ro

Abstract: In designing power cycles, the approach of Novikov–Curzon–Albhorn, maximum power criterion, cannot be applied since it has some unclearness regarding the real link between heat exchange temperature differences, and the overall heat transfer coefficients and the heat transfer areas. The industrial practice demonstrated that it is possible to obtain more power by using advanced heat exchangers with higher effectiveness. In building the Irreversible Power Cycles Design, they were identified firstly the second law effectiveness of the cycle heat exchangers by defining the concept of NTUS (number of transfer units per entropy variation rate of the working fluid). The Irreversible Power Cycles Design highlights the main way in designing new advanced power systems. The delivered power cannot be optimized mathematically, but can be increased systematically by using new heat exchangers with the second law effectiveness closer and closer to the unity. The second law effectiveness can make the difference between the heat transfers, at the hot and the cold reservoirs. Since the second law effectiveness also adds up the internal irreversibility by the intermediary of the NTUS and $NTUS_0$, the Irreversible Power Cycles Design might in fact judge the overall irreversibility, internal and external. The entropy generation caused by the internal irreversibility is put in storage in the entropy variation of the working fluid at the hot and the cold sinks.

Key-Words: - Number of transfer units per entropy variation, Second law effectiveness for preliminary power design.

1 Introduction

The thermodynamic analysis and optimization is made on the basis of ideal cycles. An ideal cycle is characterized by no entropy generation, $\dot{S}_{gen} = 0$. By this very concise condition, one can originate a lot of ideal cycles. In classical Thermodynamics the ideal cycles come in contact with two external heat sources, see Figure 1.

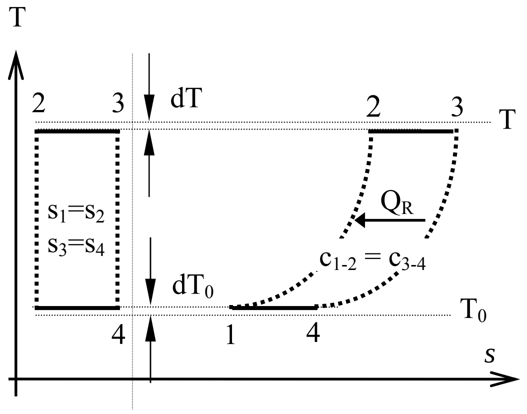

Fig. 1. The possible ideal engine cycles in temperature – entropy diagram

An ideal completely reversible cycle exchanges heat with at least two external heat reservoirs (having infinite heat capacity respectively constant temperatures) at infinitesimal temperature differences. There are infinite variants possible to analyze, all of them complying with the same single rule previously considered in defining the operational thermal frame. In accord to Figure 1, for all these ideal possible engine cycles the heat transfers are made at infinitesimal temperature differences dT and dT_0. With the exception of the Carnot cycle (left), for the other ones (right), the two isothermal processes are linked by two non-adiabatic processes, and they must exchange each other the internal heat Q_R at infinitesimal temperature difference. The non-adiabatic processes 1 – 2 and 3 – 4 might be two any politropic ones. The internal heat exchange must fulfill the following constraint:

$$s_2 - s_1 = s_3 - s_4 = c_n \ln(T/T_0) \quad (1)$$

where c_n is the heat capacity
(e.g. at v = ct or p = ct)

It is very easy to demonstrate for all above considered ideal cycle, that the first law and second law efficiencies are identical, alike the Carnot cycle.

Any irreversible (thermally – ΔT and ΔT_0 and ΔT_R finite, and frictionally – internal irreversible flow) engine cycle has both efficiencies smaller.

Development of the second law analysis or of the so called finite time thermodynamics (FTT) method leans basically on the complete ideal cycles, having the maximum possible second law efficiency. The precursors of F.T.T. were CHAMBADAL (1957) [1] and NOVIKOV [2] (1958), which studied the scheme of nuclear cycles considering the internal and external irreversibility. Curzon et Ahlborn [3] introduced the time-based thermodynamic analysis in view of the real heat transfer made at finite temperature difference. F.T.T. was employed to analyze known cycles (Carnot, Brayton, Stirling, Ericson, Otto, Diesel), and further to reverse cycles (refrigeration machines, heat pumps). Worth mentioning YAN et al. [4], GROSU et al. [5], CHEN et al. [6,7] which analyzed the cycles with three external heat sinks. Meanwhile, numerous works were performed in FTT [8-26]. The finite time thermodynamics considers the ideal reversible cycles, previously presented, as useless because they might supply the maximum engines work but it is supposed that either the needed time to perform the cyclic path or the needed heat transfer areas are tending to infinite since the reversible heat transfer at infinitesimal temperature difference requests it. The real heat transfer can be made only at finite temperature difference and as a result it was defined a new finite time ideal cycle (on the basis of Carnot) called Novikov–1958, Curzon&Albhorn–1975. The Novikov– Curzon–Albhorn cycle is in fact the endoreversible Carnot cycle that it is exchanging heat with external heat reservoirs at the finite temperature differences ΔT and ΔT_0, and it can supply the maximum power. Therefore, the finite thermodynamic introduced a new second law-optimizing criterion, respectively the Maximum Power Generation that might be used in optimizing thermal systems with a Minimum Time of Life. For imposed T, T_0, and supposing any overall heat transfer coefficients and the correspondent heat transfer areas for the heat exchange with hot and cold thermal reservoirs, it results that the output power can be mathematically maximized. The first law efficiency, in this case, becomes:

$$\eta_I = \frac{P}{\dot{Q}} = 1 - \sqrt{\frac{1}{\tau}} \quad \text{where } \tau = \frac{T}{T_0} \tag{2}$$

Unfortunately, Novikov-Curzon-Albhorn variant of endoreversible Carnot cycle cannot be considered for cycles having an internal heat exchange, e.g. Stirling, or all possible alternatives, see Figure 2.

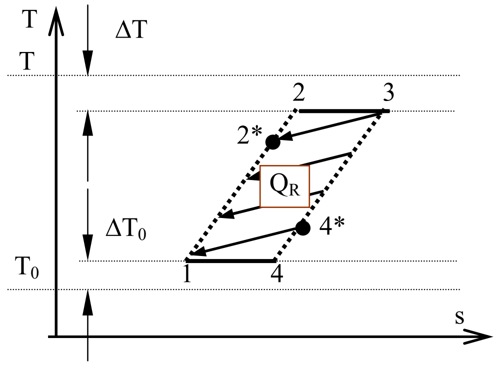

Fig. 2. The finite time ideal cycle with real internal heat exchange, temperature – entropy diagram

The same grounds might be re-used in order to find the new ideal maximum power/minimum operation time but taking into account that even the internal heat transfer must be completed in finite time by considering also the necessary finite temperature difference to endorse this heat transfer. A very simple calculus is cooperative, so that we re-get the same relation for output maximum power, and consequently the same optimum parameters, but the first law efficiency is lower and obviously depends on overall heat transfer coefficients and mass flow rate and nature of fluid:

$$\eta_I = \frac{P}{\dot{Q}_{2-3} + Q_S} = \frac{\eta_{Curzon-Albhorn}}{1 + \frac{\dot{Q}_S}{\dot{Q}_{2-3}}} = \frac{1 - \sqrt{1/\tau}}{1 + \frac{\dot{m}c(1-\varepsilon_I)(\overline{K}\sqrt{\tau}+1)}{\overline{K}\sqrt{\tau}}} \tag{3}$$

where $\overline{K} = \frac{U \cdot A}{U_0 \cdot A_0}$, and $\varepsilon_I = \frac{NTU_I}{NTU_I + 1}$ is the effectiveness of the internal heat exchanger – countercurrent balanced.

Remark: The real time internal heat exchange set up a direct gateway between hot and cold heat sinks by

$$\dot{Q}_{2^*-2} = \dot{Q}_{4^*-4} = \dot{m}c_n(T - \Delta T - T_0 - \Delta T_0)(1 - \varepsilon_I) = \dot{Q}_S$$

2 The new maximum power criterion

In building the new maximum power criterion, they were identified firstly the second law effectiveness of the endoreversible CARNOT cycle heat exchangers, see Figure 3.

2.1 The second law effectiveness of the heat exchange at the hot and cold reservoir

It was introduced the concept of NTUS (number of transfer units per entropy variation rate of the working fluid) in defining the second law effectiveness of this very distinctive heat exchangers (they might be considered acting like a balanced heat exchangers that satisfies simultaneously the constraints of constant temperatures and constant heat flux on both sides):

$$NTUS = \frac{UA}{\dot{m}\Delta s} \text{ and } NTUS_0 = \frac{U_0 A_0}{\dot{m}\Delta s} \quad (4)$$

They were adopted the following notations: U, U_0, are the overall heat transfer coefficients at the hot and cold thermal reservoirs, A, A_0, are the correspondent heat transfer areas, ΔT, ΔT_0, are the heat transfer temperature differences.

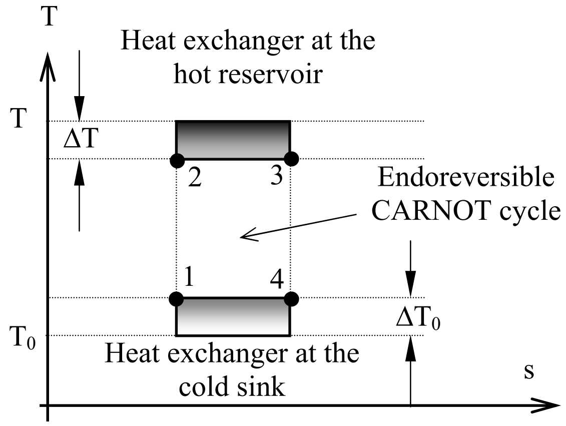

Fig. 3. Scheme of an endoreversible Carnot cycle.

2.1.1 The second law effectiveness at the hot source

The heat transfer balance gives:

$$\dot{Q} = UA\Delta T = \dot{m}(T - \Delta T)\Delta s \quad (5)$$

From equation (5) it results:

$$\Delta T = T\frac{1}{NTUS + 1} \text{ and}$$
$$\dot{Q} = \dot{m}(T - \Delta T)\Delta s = \dot{m}T\Delta s\frac{NTUS}{NTUS + 1} \quad (6)$$

The effectiveness at the hot source can be set as:

$$\varepsilon = \frac{\dot{Q}}{(\dot{Q})_{NTUS\to\infty}} = \frac{NTUS}{NTUS + 1} < 1 \quad (7)$$

for Δs = const.

In obtaining this effectiveness we had to impose one restrictive condition regarding the heat transfers, $\dot{Q}$ and $(\dot{Q})_{NTUS\to\infty}$. For the sake of simplicity we supposed constant Δs for both heat transfers.

In this way, the heat rate at the hot source becomes:

$$\dot{Q} = \varepsilon\dot{Q}_{NTUS\to\infty} = \varepsilon\dot{Q}_{max} = \varepsilon\dot{m}T\Delta s \quad (8)$$

The meaning of $\dot{Q}_{max} = \dot{m}T\Delta s$ is the maximum reversible heat rate that can be received from the hot source and so the effectiveness, $\varepsilon < 1$, in this case reflects the irreversibility caused by the real heat transfer at finite temperature difference ΔT, i.e. $\dot{Q} < \dot{Q}_{max}$.

2.1.2 The second law effectiveness at the cold source

The heat transfer balance gives:

$$\dot{Q}_0 = U_0 A_0 \Delta T_0 = \dot{m}(T_0 + \Delta T_0)\Delta s \quad (9)$$

From equation (9) it results:

$$\Delta T_0 = T_0\frac{1}{NTUS_0 - 1} \text{ and}$$
$$\dot{Q}_0 = \dot{m}(T_0 - \Delta T_0)\Delta s = \dot{m}T_0\Delta s\frac{NTUS_0}{NTUS_0 - 1} \quad (10)$$

The effectiveness at the cold source is setting as:

$$\varepsilon_0 = \frac{\dot{Q}_0}{(\dot{Q}_0)_{NTUS_0\to\infty}} = \frac{NTUS_0}{NTUS_0 - 1} > 1 \quad (11)$$

for Δs = const.

In obtaining this effectiveness we had to impose one restrictive condition regarding the heat transfers, $\dot{Q}_0$ and $(\dot{Q}_0)_{NTUS\to\infty}$. We supposed constant Δs for both heat transfers, because the cycle is endo-reversible, $s_3 - s_2 = s_4 - s_1$.

In this way, the heat rate at the cold source becomes:

$$\dot{Q}_0 = \varepsilon_0(\dot{Q}_0)_{NTUS_0\to\infty} = \varepsilon_0\dot{Q}_{min} = \varepsilon_0\dot{m}T_0\Delta s \quad (12)$$

The meaning of $\dot{Q}_{min} = \dot{m}T_0\Delta s$ is the minimum reversible heat rate that can be rejected to the cold sink and so the effectiveness, $\varepsilon_0 > 1$, in this case reflects the irreversibility caused by the real heat transfer at finite temperature difference ΔT_0, respectively $\dot{Q}_0 > \dot{Q}_{min}$

Figure 4 shows the dependences between the second law effectiveness, NTUS and $NTUS_0$ and the irreversibility of heat transfers.

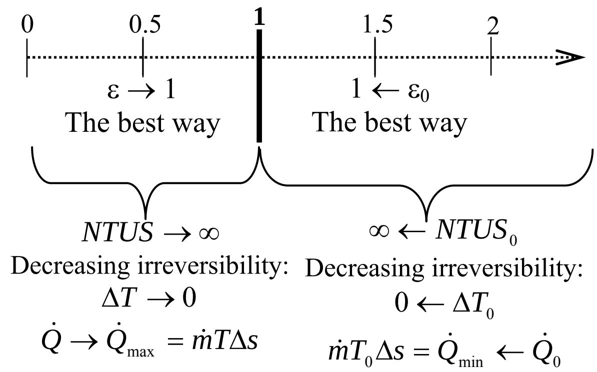

Fig. 4. The dependence second law effectiveness – NTUS

This method to describe the second law effectiveness can be applied at any other heat exchanger by the intermediary of the mean thermodynamic temperature of non-adiabatic processes, Baehr – 1973, and the heat transfer temperature differences related to the heat transfer processes. By combining the previous equations (5) and (12) it is obtaining:

$$P = \dot{Q} - \dot{Q}_0 = \dot{Q}\left(1 - \frac{\dot{Q}_0}{\dot{Q}}\right) = Q_{\max}\varepsilon\left(1 - \frac{\dot{Q}_{\min}}{\dot{Q}_{\max}}\frac{\varepsilon_0}{\varepsilon}\right)$$
$$= \dot{m}T\Delta s\varepsilon\left(1 - \frac{1}{\tau}\frac{\varepsilon_0}{\varepsilon}\right) \quad (13)$$

Analyzing this expression, it is yielding that the delivered power cannot be optimized mathematically, but can be increased step by step by using new heat exchangers with the second law efficiencies closer and closer to the unity. The first and second law efficiencies for *The New Maximum Power Criterion* analysis are:

$$\text{first law efficiency: } \eta_I = 1 - \frac{1}{\tau}\frac{\varepsilon_0}{\varepsilon},$$
$$\text{second law efficiency } \eta_{II} = \frac{1 - \frac{1}{\tau}\frac{\varepsilon_0}{\varepsilon}}{1 - \frac{1}{\tau}} \quad (14)$$

2.2 Some considerations regarding the real power cycles

The real power cycles are also internally irreversible, respectively:

– the heat transfer processes, 2–3 and 4–1, are non-isothermal;

– the adiabatic processes, 2–3 and 4–1, are non-isentropic;

– the external heat sources have finite heat capacities, respectively are non-isothermal;

– the mean log temperature difference pertaining to the heat transfers has a value that it is not equalizing the difference of the mean thermodynamic temperatures.

This dissimilarity can be solved step by step by introducing the necessary adjustment coefficients in order to take into account also the internal irreversibility. We rework below, for instance, the real heat transfers, see Figure 5.

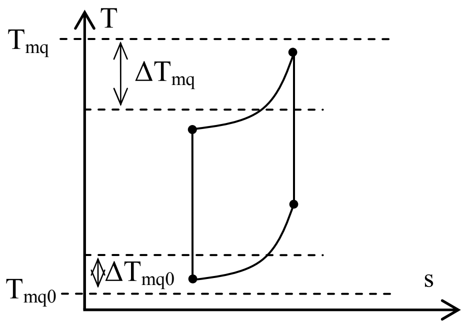

Fig. 5. Scheme of a power cycle with two non-isothermal processes in contact to external heat sources

2.2.1 The second law effectiveness at the hot source

The heat transfer balance gives:

$$\dot{Q} = UA\Delta T_{mq}C_{\Delta T} = \dot{m}\left(T_{mq} - \Delta T_{mq}\right)\Delta s \quad (15)$$

From equation (15) it results:

$$\Delta T_{mq} = T_{mq}\frac{1}{NTUS \cdot C_{\Delta T} + 1} \text{ and}$$
$$\dot{Q} = \dot{m}\left(T_{mq} - \Delta T_{mq}\right)\Delta s = \dot{m}T_{mq}\Delta s\frac{NTUS \cdot C_{\Delta T}}{NTUS \cdot C_{\Delta T} + 1} \quad (16)$$

The effectiveness at the hot source can be set as:

$$\varepsilon = \frac{\dot{Q}}{\left(\dot{Q}\right)_{NTUS\to\infty}} = \frac{NTUS \cdot C_{\Delta T}}{NTUS \cdot C_{\Delta T} + 1} < 1 \quad (17)$$

for Δs = const.

In obtaining this effectiveness we adopted $C_{\Delta T}$ as the ratio of the mean log temperature difference to the difference of mean thermodynamic temperatures of the hot source and of the working fluid. In defining the second law effectiveness we related to the reversible path for the $\left(\dot{Q}\right)_{NTUS\to\infty}$,. For the sake of simplicity we supposed constant Δs for both heat transfers. In this way, it is obtained a similar relation:

$$\dot{Q} = \varepsilon\dot{Q}_{NTUS\to\infty} = \varepsilon\dot{Q}_{\max} = \varepsilon\dot{m}T_{mq}\Delta s \quad (18)$$

The meaning of $\dot{Q}_{\max} = \dot{m}T_{mq}\Delta s$ is the same, the maximum reversible heat rate that can be received from the hot source.

2.2.2 The second law effectiveness at the cold source

The heat transfer balance gives:

$$\dot{Q}_0 = U_0 A_0 \Delta T_{mq0} C_{\Delta T0} = \dot{m}\left(T_{mq0} + \Delta T_{mq0}\right)\Delta s \quad (19)$$

From equation (19) it results:

$$\Delta T_{mq0} = T_{mq0}\frac{1}{NTUS_0 \cdot C_{\Delta T0} - 1} \text{ and}$$
$$\dot{Q}_0 = \dot{m}\left(T_{mq0} - \Delta T_{mq0}\right)\Delta s = \dot{m}T_{mq0}\Delta s\frac{NTUS_0 \cdot C_{\Delta T0}}{NTUS_0 \cdot C_{\Delta T0} - 1} \quad (20)$$

The effectiveness at the cold source is setting as:

$$\varepsilon_0 = \frac{\dot{Q}_0}{\left(\dot{Q}_0\right)_{NTUS_0\to\infty}} = \frac{NTUS_0 \cdot C_{\Delta T0}}{NTUS_0 \cdot C_{\Delta T0} - 1} > 1 \quad (21)$$

for Δs = const.

$C_{\Delta T0}$ is the ratio of the mean log temperature difference to the difference of mean thermodynamic temperatures of the cold source and of the working fluid. We related also to the reversible path for the $\left(\dot{Q}_0\right)_{NTUS\to\infty}$,. For the sake of simplicity we supposed

constant Δs for both heat transfers, because the adiabatic processes are supposed to be isentropic, $s_3 - s_2 = s_4 - s_1$.
The heat rate at the cold source becomes:

$$\dot{Q}_0 = \varepsilon_0 \left(\dot{Q}_0\right)_{NTUS_0 \to \infty} = \varepsilon_0 \dot{Q}_{\min} = \varepsilon_0 \dot{m} T_{mq0} \Delta s \qquad (22)$$

The meaning of $\dot{Q}_{\min} = \dot{m} T_{mq0} \Delta s$ is the same, the minimum reversible heat rate that can be rejected to the cold sink.
The first and second law efficiencies remain unaffected.

Remark: In defining the second law effectiveness it was considered the real fact that the entropy variation of the operating fluid is limited, likewise the flow heat capacity of a mono-phase working fluid, either by geometrical constraints (e.g. the entropy variation during the isothermal expansion or compression in a Stirling engine is depending on the limited admissible volumetric ratio) or by physical ones (e.g. the entropy variation for an isothermal phase change is limited and dependent on the nature of the working fluid), and so the heat rate during the heat transfer process might be mainly influenced by the overall thermal conductance UA, U_0A_0.
Hence, the first law efficiency of the New Maximum Power Criterion is non-restrictive, but it shows the ways to come in finite time closer and closer to Carnot machine, see annexed Figures that show the dependence of first law efficiency on τ, ε and ε_0.

3 Some Conclusions

The new design maximum power criterion highlights the main way in building new advanced power systems. The delivered power cannot be optimized mathematically, but can be increased systematically by using new heat exchangers with the second law effectiveness closer and closer to the unity.

The second law effectiveness can make the difference between the heat transfers, at the hot and the cold reservoirs. Therefore, the effectiveness at the hot source, ε < 1, reflects the irreversibility caused by the real heat transfer at finite temperature difference, i.e. $\dot{Q} < \dot{Q}_{\max}$, and the effectiveness at cold sink, $\varepsilon_0 > 1$, returns the irreversibility caused by the real heat transfer at finite temperature difference, respectively $\dot{Q}_0 > \dot{Q}_{\min}$.

Since the second law effectiveness also adds up the internal irreversibility by the intermediary of the NTUS and $NTUS_0$, the new maximum power criterion might in fact judge the overall irreversibility, internal and external. The entropy generation caused by the internal irreversibility is put in storage in the entropy variation of the working fluid at the hot and the cold sinks, by $\dot{S} = \dot{m} \cdot \Delta s$ and $\dot{S}_0 = \dot{m} \cdot \Delta s_0$.

The real time internal heat exchange set up a direct gateway between hot and cold heat sinks

References

[1] P. Chambadal, Les centrales nucléaires, Paris, *Armand Colin*, (1957).

[2] I.I. Novikov, The efficiency of atomic stations, *Journal Nuclear Energy*, vol. 7 (1958), 125-128.

[3] F.L. Curzon, B. Ahlborn, Efficiency of a Carnot engine at maximum power conditions, *Am. J. Phys.*, vol. 53, (1975), 570-573.

[4] Z. Yan, J. Chen, An optimal endoreversible three heat reservoir refrigerator, *J. Appl. Phys.*, 65 (1) (1989), 1-4.

[5] L. Grosu, M. Feidt, R. Bénelmir, Study of the improvement in the performance coefficient of machines operating with three heat reservoirs, *Int. J. Exergy*, 1 (1) (2004), 147-162.

[6] J. Chen, J.A. Schouton, Optimum performance characteristics of an irreversible absorption refrigeration system, *Energy Convers. Mgmt*, 39 (10) (1997), 582-587.

[7] J. Chen, The optimum performance characteristics of a four temperature levels irreversible absorption refrigeration at maximum specific cooling load, *J. Phys. – D : Appl. Phys.*, 32 (24) (1999), 3085-3091.

[8] A. Bejan, Advanced Engineering Thermodynamics, New York, *Wiley interscience*, (1988).

[9] A. Bejan, Entropy generation minimization : the new thermodynamics of finite-size device and finite-time processes, *J. Appl. Phys.*, vol. 79, n° 3 (1996), 1191-1218.

[10] A. Haj Taleb, M. Feidt, Analyse paramétrique de la performance optimale d'une machine frigorifique quadritherme, *Proceedings COFRET'04*, 22-24 Avril 2004, Nancy, France.

[11] Bejan A. "The Equivalence of Maximum Power and Minimum Entropy Generation Rate in the Optimization of Power Plants." *JERT 118*: (Jun) 1996, 98 - 101..

[12] Bejan, A. "Maximum power from fluid flow." *INTERNATIONAL JOURNAL OF HEAT AND MASS TRANSFER* 39: 1175, 1996..

[13] Bejan, A. "Models of power plants that generate minimum entropy while operating at maximum power." *AMERICAN JOURNAL OF PHYSICS* 64: 1054, 1996.

[14] Bejan, A; Errera, MR. "Maximum power from a hot stream." *INTERNATIONAL JOURNAL OF HEAT AND MASS TRANSFER* 41: 2025, 1998..

[15] C. Wu et al., General performance characteristics of finite speed Carnot refrigerator, *Appl. Therm. Eng.*, vol. 16 (1996), 299-303.

[16] E. Vasilescu, M. Feidt, R. Boussehain, L'optimisation des cycles idéaux exo-irréversibles des systèmes frigorifiques quadrithermes, *Proceedings COFRET'04*, 22-24 Avril 2004, Nancy, France.

[17] F. Sun et al., Optimal performance of an endoreversible Carnot heat pump, *Energy Convers. Mgmt*, vol. 38 (1997) 1439-1443.

[18] J.M. Gordon et al., Cool Thermodynamics, Cambridge Int. Sciences Publishers (2000).

[19] J.M. Gordon et al., Optimizing chiller operation based on finite time thermodynamics : universal modeling experimental confirmation, *Int. J. Refrig.*, vol. 20 (1997), 191-200.

[20] L. Chen, C. Wu, F. Sun, Finite time thermodynamic optimization or entropy generation minimization of energy systems, *J. Non-Equilib. Thermodyn.*, vol. 24, n° 4 (1999) 327-359.

[21] M. Feidt, Thermodynamique et optimisation énergétique des systèmes et procédés, Technique et Documentation, *Paris*, (1987).

[22] R.S. Berry, V.A. Kazakov, S. Sieniutycz, Z. Szwast, A.M. Tsirlin, Thermodynamic Optimization of Finite Time Processes, Chischester, *Wiley*, (1999).

[23] S. Petrescu et al., The study for optimization of Carnot cycle which develop with finite speed, *Proc. of the Inter. Sympos. ECOS'93*, Krakow, Poland (1993).

[24] T. Zheng, L. Chen, F. Sun, C. Wu, "Performance of a four heat reservoirs absorption refrigerator with heat resistance and heat leak, *Int. J. Ambient Energy*, 24 (3) (2003), 157-168.

[25] V.T. Radcenco, Generalized Thermodynamics, Bucharest, *Technica* (1994)

[26] Y. Goth, M. Feidt, Optimum COP for endoreversible heat pump or refrigerating machine, *C.R. Acad. Sci.* Paris, 303 (1) (1986) 19-24.

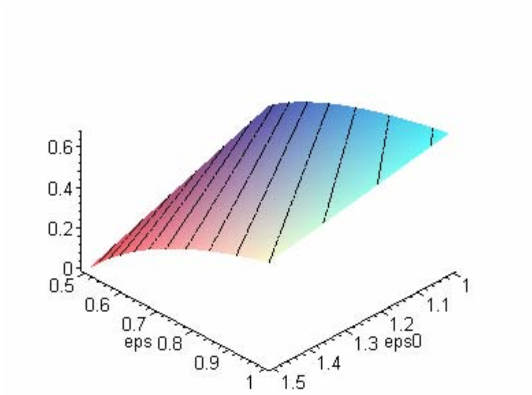

Fig. 6. First law efficiency function of second law effectiveness, τ = 3

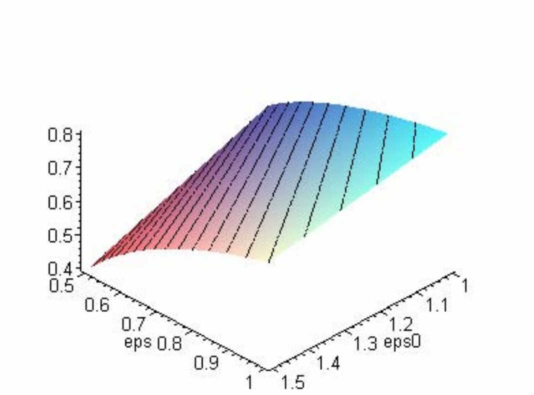

Fig. 7. First law efficiency function of second law effectiveness, τ = 5