A New estimation method of the poles for the generalized Orthonormal bases filters

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Abstract: - Modeling multivariable LTI systems using generalized Orthonormal basis functions requires the determination of optimal poles of these bases. This paper develops a new iterative method of poles optimization for the generalized Orthonormal Base using the Gauss-Newton algorithm and the state space representation of these bases.

Key-Words: - Optimization; Multivariable systems; LTI systems; Estimation; Generalized Orthonormal Base

1 Introduction

The generalized Orthonormal Basis functions [1], [2], [3] admit a variety of real or conjugate poles and regroup the common FIR, Laguerre and Kautz [4], [5] model structures which are restrictive special cases of this complete construction. As a consequence, these bases represent all type of linear, causal and stable systems.
Notice that in [6], [7], [8] the representation of MIMO (Multi-Input Multi-Output) linear systems on the generalized Orthonormal basis functions supposes that the poles are fixed. In this paper we surmounted these difficulties by using these bases with ordinary poles. Therefore, to facilitate the estimation problem of the poles we decomposed the MIMO system in MISO (Multi-Input Single-Output) subsystems represented on the generalized Orthonormal basis functions.

The poles of the generalized Orthonormal bases filters operate nonlinear (contrary to the Fourier coefficients) in their constitution and condition their performances. The choice of these poles is primordial to permit the truncating of the network filters as a minimal order without too alter in the quality of the approximation. However, to situate the dynamics of the system and to permit an optimal and adequate choice of the poles, some approaches [9], [10], [11], [12], [13], [14] for estimation of the poles exist in the literature. In contrast to these approaches of poles optimization that leads voluminous computation when determining the filters sensitivities, we introduced the state space representation of the generalized Orthonormal base to solve these problems.

This paper is organized as follows. In the second section we present the new formulation of the state space representation of the generalized Orthonormal base and the matrix form of the quadratic criteria to minimize. In the third section the problem of poles estimation, the matrix computations of the gradient and the Hessien as well as the determination of filter sensitivities by using the derivative of the state space representation are expressed. The fourth section summarizes some simulation results and finally, a conclusion is given in the last section.

2 Problem formulation

We consider a MIMO linear system having r inputs and m outputs described by its transfer matrix $G(q^{-1})$. Each elementary transfer function $G_{ij}(q^{-1})$ (i=1,2,…,m ; j=1,2,…,r) can be decomposed on the generalized Orthonormal bases filters as follows :

$$G_{ij}(z) = \sum_{n=0}^{N_{i,j}} g_n^{i,j} \mathcal{B}_n^{i,j}(z, \underline{\xi_{i,j}}) \quad (1)$$

where:

{ $g_n^{i,j}$ }, $N_{i,j}$ and $\underline{\xi_{i,j}}$ are respectively the set of Fourier coefficients, the truncating order and the poles vector of the network (i,j).

with:

$\left\{\mathcal{B}_n^{i,j}(z, \underline{\xi_{i,j}})\right\}$ represent the generalized Orthonormal basis functions [1] defined by:

$$\mathcal{B}_n^{i,j}(z) = \frac{\sqrt{1-\left|\xi_n^{i,j}\right|^2}}{z-\xi_n^{i,j}} \prod_{k=0}^{n-1}\left(\frac{1-\bar{\xi}_k^{i,j} z}{z-\xi_k^{i,j}}\right) \quad (2)$$

($\xi_k^{i,j}$ and its conjugated $\bar{\xi}_k^{i,j}$ are the poles of the filter *k*).
Every network (*i,j*) of the generalized Orthonormal bases filters can be described by a state space representation which is reformulated in a simplified version and rewritten in matrix form as follows:

$$\begin{cases} X_{i,j}(k+1) = A_{i,j} X_{i,j}(k) + B_{i,j} u_j(k) \\ \hat{y}_{i,j}(k) = \theta_{i,j}^T X_{i,j}(k) \end{cases} \tag{3}$$

with $X_{i,j}(k)$ is a state vector of dimension $(N_{i,j}+1)$:

$$X_{i,j}(k) = \left[x_0^{i,j}(k)\ x_1^{i,j}(k) \cdots x_{N_{i,j}}^{i,j}(k)\right]^T \tag{4}$$

where:

$$x_n^{i,j}(k) = \mathcal{Z}^{-1}\left\{\mathcal{B}_n^{i,j}(z, \underline{\xi_{i,j}})\right\} u_j(k) \tag{5}$$

$A_{i,j}$ is a matrix of dimension $(1+N_{i,j}) \times (1+N_{i,j})$ defined by:

$$A_{i,j}(p,q) = \begin{cases} \xi_{p-1}^{i,j} & if \quad p = q \\ a_{i,j}(p,q) & if \quad p \succ q \\ 0 & if \quad p \prec q \end{cases} \tag{6}$$

where:

$$a_{i,j}(p,q) = (-1)^{p+q+1} \alpha_{p-1}^{i,j} (1 - \xi_{q-1}^{i,j} \bar{\xi}_{q-1}^{i,j}) \prod_{l=q+1}^{p-1} \alpha_{l-1}^{i,j} \bar{\xi}_{l-1}^{i,j} \tag{7}$$

$B_{i,j}$ and $\theta_{i,j}$ are vectors of dimensions $(1+N_{i,j})$:

$$B_{i,j}(p) = (-1)^{p+1} \alpha_{p-1}^{i,j} \prod_{l=1}^{p-1} \alpha_{l-1}^{i,j} \bar{\xi}_{l-1}^{i,j} \quad (p = 1, 2, \cdots, N_{i,j}+1) \tag{8}$$

$$\theta_{i,j} = \left[g_0^{i,j}\ \ g_1^{i,j} \cdots g_{N_{i,j}}^{i,j}\right]^T \tag{9}$$

where:

$$\alpha_l^{i,j} = \frac{\sqrt{1 - \left|\xi_l^{i,j}\right|^2}}{\sqrt{1 - \left|\xi_{l-1}^{i,j}\right|^2}} \quad (l \succ 0) \quad \text{and} \quad \alpha_0^{i,j} = \sqrt{1 - \left|\xi_0^{i,j}\right|^2} \tag{10}$$

The model output of a MISO subsystem can be written, as follows:

$$\hat{y}_i(k) = \sum_{j=1}^{r} \theta_{i,j}^T X_{i,j}(k) = \sum_{j=1}^{r} \sum_{n=0}^{N_{i,j}} g_n^{i,j} \mathcal{B}_n^{i,j}(k, \underline{\xi_{i,j}}) u_j(k) \tag{11}$$

We define the quadratic criterion in a finished horizon time for each subsystem as:

$$J_i = \sum_{k=1}^{H} \left[y_{im}(k) - \hat{y}_i(k)\right]^2 = \sum_{k=1}^{H} \left[e_i(k)\right]^2 \tag{12}$$

where:

y_{im}, $e_i(k)$ are respectively the measurement output of the system and the prediction error.

$$e_i(k) = \left(y_{im}(k) - \sum_{j=1}^{r} \sum_{n=0}^{N_{i,j}} g_n^{i,j} x_n^{i,j}(k) \right) \tag{13}$$

This criterion can be written in matrix form as:

$$J_i = E_i E_i^T = \left(Y_{im} - \Psi_i \Theta_i\right)^T \left(Y_{im} - \Psi_i \Theta_i\right) \tag{14}$$

with:

$$E_i = \left[e_i(1)\ e_i(2) \cdots e_i(H)\right]^T \tag{15}$$

$$Y_{im} = \left[y_{im}(1)\ y_{im}(2) \cdots y_{im}(H)\right]^T \tag{16}$$

$$\Psi_i = \left[\Psi_{i,1}\ \Psi_{i,2} \cdots \Psi_{i,r}\right] \tag{17}$$

$$\Psi_{i,j} = \left[\Psi_0^{i,j}\ \Psi_1^{i,j} \cdots \Psi_{N_{i,j}}^{i,j}\right] \tag{18}$$

$$\Psi_n^{i,j} = \left[x_n^{i,j}(1)\ x_n^{i,j}(2) \cdots x_n^{i,j}(H)\right]^T \tag{19}$$

$$\Theta_i = \left[\theta_{i,1}^T\ \theta_{i,2}^T \cdots \theta_{i,r}^T\right]^T \tag{20}$$

Since the model is linear in parameters, the optimal Fourier coefficients can be obtained by solving directly the following normal equation, as explained in [6], [7]:

$$\hat{\Theta}_i = \left(\Psi_i^T \Psi_i\right)^{-1} \Psi_i^T Y_{im} \tag{21}$$

3 Poles Estimation

To calculate the poles of the generalized Orthonormal basis functions, an iterative algorithm is necessary minimizing the quadratic criterion expressed nonlinear to these poles. This iterative optimization method of the second order uses Newton type algorithms, characterized by adaptive steps according to the poles emplacement in each iteration time. These algorithms are based on the development of the criteria *J* in order two.
The poles vector of the MISO subsystem at the iteration (*s+1*) can be written in function of the poles vector at the iteration (*s*), as follows:

$$\underline{\xi_i}^{(s+1)} = \underline{\xi_i}^{(s)} - \mu_i \left(\frac{\partial^2 J_i^{(s)}\left(\underline{\xi_i}^{(s)}\right)}{\partial \underline{\xi_i}^T \partial \underline{\xi_i}} \right)^{-1} \frac{\partial J_i^{(s)}\left(\underline{\xi_i}^{(s)}\right)}{\partial \underline{\xi_i}^T} \qquad (22)$$

where the step μ_i of the Newton algorithm has for role to attenuate or to amplify the effect of the Hessien inverse.

3.1 Determination of the gradient and the Hessien

The poles of each subsystem regrouped in the vector $\underline{\xi_i}$, will be optimized by using the iterative method (22).

To calculate the gradient we differentiating (14) in relation to the poles vector $\underline{\xi_i}$:

$$P_i = \frac{\partial J_i}{\partial \underline{\xi_i}} = 2 \frac{\partial E_i^T}{\partial \underline{\xi_i}} E_i = 2R_i E_i \qquad (23)$$

with:

$$R_i = \frac{\partial E_i^T}{\partial \underline{\xi_i}} = -\Theta_i^T \frac{\partial \Psi_i^T}{\partial \underline{\xi_i}} = \left[R_{i,1}^T \; R_{i,2}^T \cdots R_{i,r}^T \right]^T \qquad (24)$$

where $R_{i,j}$ is a matrix defined by:

$$R_{i,j} = -\begin{bmatrix} \Theta_i^T \dfrac{\partial \Psi_i^T}{\partial \xi_0^{i,j}} \\ \vdots \\ \Theta_i^T \dfrac{\partial \Psi_i^T}{\partial \xi_{N_{i,j}}^{i,j}} \end{bmatrix} = -\begin{bmatrix} \theta_{i,j}^T \dfrac{\partial \Psi_{i,j}^T}{\partial \xi_0^{i,j}} \\ \vdots \\ \theta_{i,j}^T \dfrac{\partial \Psi_{i,j}^T}{\partial \xi_{N_{i,j}}^{i,j}} \end{bmatrix} \qquad (25)$$

Determining the second derivative of the error on the poles required computing all filter sensitivities of order two. Therefore, we determine the approximate Hessien taking place from the only knowledge of function sensitivities of the first order, used in gradient calculation. The Hessien is calculated by differentiating the gradient (23) on the poles vector:

$$Q_i = \frac{\partial^2 J_i}{\partial \underline{\xi_i}^T \partial \underline{\xi_i}} \approx 2 \frac{\partial E_i^T}{\partial \underline{\xi_i}} \frac{\partial E_i}{\partial \underline{\xi_i}^T} = 2R_i S_i \qquad (26)$$

with:

$$S_i = \frac{\partial E_i}{\partial \underline{\xi_i}^T} = -\frac{\partial \Psi_i}{\partial \underline{\xi_i}^T} \Theta_i = \left[S_{i,1} \; S_{i,2} \cdots S_{i,r} \right] \qquad (27)$$

where $S_{i,j}$ is a matrix defined by:

$$S_{i,j} = -\left[\frac{\partial \Psi_i}{\partial \xi_0^{i,j}} \Theta_i \cdots \frac{\partial \Psi_i}{\partial \xi_{N_{i,j}}^{i,j}} \Theta_i \right] \qquad (28)$$

We define the filter sensitivities in relation to the poles for each subsystem. Either for j=1,2,…,r and p=0,1,…, $N_{i,j}$:

$$\frac{\partial \Psi_{i,j}}{\partial \xi_p^{i,j}} = \left[O_{H\times p} \; \frac{\partial \Psi_0^{i,j}}{\partial \xi_p^{i,j}} \frac{\partial \Psi_1^{i,j}}{\partial \xi_p^{i,j}} \cdots \frac{\partial \Psi_{N_{i,j}}^{i,j}}{\partial \xi_p^{i,j}} \right] \qquad (29)$$

By using (29), the matrixes $R_{i,j}$ and $S_{i,j}$ can be written as follow:

$$R_{i,j} = -\begin{bmatrix} \sum_{n=0}^{N_{i,j}} \beta_{n,0}^{i,j}(1) & \cdots & \sum_{n=0}^{N_{i,j}} \beta_{n,0}^{i,j}(H) \\ \vdots & \ddots & \vdots \\ \beta_{N_{i,j},N_{i,j}}^{i,j}(1) & \cdots & \beta_{N_{i,j},N_{i,j}}^{i,j}(H) \end{bmatrix} \qquad (30)$$

$$S_{i,j} = -\begin{bmatrix} \sum_{n=0}^{N_{i,j}} \beta_{n,0}^{i,j}(1) & \cdots & \beta_{N_{i,j},N_{i,j}}^{i,j}(1) \\ \vdots & \ddots & \vdots \\ \sum_{n=0}^{N_{i,j}} \beta_{n,0}^{i,j}(H) & \cdots & \beta_{N_{i,j},N_{i,j}}^{i,j}(H) \end{bmatrix} \qquad (31)$$

where:

$$\beta_{n,p}^{i,j}(k) = g_n^{i,j} \frac{\partial x_n^{i,j}(k)}{\partial \xi_p^{i,j}} \qquad (p = 0,1,\cdots,N_{i,j}) \qquad (32)$$

$$\dim R_i = \left(\sum_{j=1}^{r} (1 + N_{i,j}) \right) \times H \qquad (33)$$

$$\dim S_i = H \times \left(\sum_{j=1}^{r} (1 + N_{i,j}) \right) \qquad (34)$$

According to (30) and (31) we deduce that:

$$R_i = S_i^T \qquad (i = 1,2,\cdots,m) \qquad (35)$$

By using (24), (27) and (15), we can determine directly the gradient (23) and the approximate Hessien (26) as follow:

$$P_i = 2R_i E_i = \left[P_{i,1}^T P_{i,2}^T \cdots P_{i,r}^T \right]^T \qquad (36)$$

$$Q_i = 2R_i S_i = 2R_i R_i^T \qquad (37)$$

with $P_{i,j}$ $(j = 1,2\cdots,r)$ are vectors defined by:

$$P_{i,j} = -2\begin{bmatrix} \sum_{k=1}^{H}\left(\sum_{n=0}^{N_{i,j}}\beta_{n,0}^{i,j}(k)\right)e_i(k) \\ \vdots \\ \sum_{k=1}^{H}\beta_{N_{i,j},N_{i,j}}^{i,j}(k)e_i(k) \end{bmatrix} \quad (38)$$

where:

$$\dim P_i = \left(\sum_{j=1}^{r}(1+N_{i,j})\right)\times 1 \quad (39)$$

$$\dim Q_i = \left(\sum_{j=1}^{r}(1+N_{i,j})\right)\times\left(\sum_{j=1}^{r}(1+N_{ij})\right) \quad (40)$$

We notice that we have a total of $\sum_{j=1}^{r}(N_{i,j}+1)$ nonlinear parameters to estimate in each subsystem.

3.2 Determination of the filter output sensitivities

The filter sensitivities in relation to the poles at each instant *k* can be written as:

$$\frac{\partial x_n^{i,j}(k)}{\partial \xi_n^{i,j}} \qquad (n = 0,1,\cdots,N_{i,j};\quad j = 1,2,\cdots,r) \quad (41)$$

To determine the sensitivities, we derivate the state space representation of the general Orthonormal base (3) as follows:

$$Z_{i,j}(k+1) = A_{i,j}Z_{i,j}(k) + F_{i,j}X_{i,j}(k) + G_{i,j}u_j(k) \quad (42)$$

Where:

$$Z_{i,j}(k) = \frac{\partial X_{i,j}(k)}{\partial \xi_n^{i,j}},\quad F_{i,j} = \frac{\partial A_{i,j}}{\partial \xi_n^{i,j}} \text{ and } G_{i,j} = \frac{\partial B_{i,j}}{\partial \xi_n^{i,j}} \quad (43)$$

The elements of the matrix $F_{i,j}$ (p=1,2,…, $N_{i,j}+1$ q=1,2,…, $N_{i,j}+1$) are given by:

$$F_{i,j}(p,q) = \begin{cases} 0 & if\ \ p \prec q \\ U_{i,j}(p,q) & if\ \ p = q \\ V_{i,j}(p,q) & if\ \ p \succ q \end{cases} \quad (44)$$

with:

$$U_{i,j}(p,q) = \begin{cases} 1 & if\ \ p = n+1 \\ 0 & if\ \ p \neq n+1 \end{cases} \quad (45)$$

$$V_{i,j}(p,q) = \begin{cases} 0 & if\ \ p \prec n+1 \\ \overline{V}_{i,j}(p,q) & if\ \ p = n+1 \\ \overline{\overline{V}}_{i,j}(p,q) & if\ \ p \succ n+1 \end{cases} \quad (46)$$

where:

$$\overline{V}_{i,j}(p,q) = (-1)^{p-q+2}\frac{\left(1-\xi_{q-1}^{i,j\,2}\right)}{\left(1-\xi_{p-1}^{i,j\,2}\right)}\prod_{l=q+1}^{p-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j} \quad (47)$$

$$\overline{\overline{V}}_{i,j}(p,q) = \begin{cases} 0 & if\ \ q \succ n+1 \\ \overline{W}_{i,j}(p,q) & if\ \ q = n+1 \\ \overline{\overline{W}}_{i,j}(p,q) & if\ \ q \prec n+1 \end{cases} \quad (48)$$

where again:

$$\overline{W}_{i,j}(p,q) = (-1)^{p-q+2}\xi_{q-1}^{i,j}\alpha_{q-1}^{i,j}\prod_{l=q+1}^{p-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j} \quad (49)$$

$$\overline{\overline{W}}_{i,j}(p,q) = (-1)^{p-q+1}\left(1-\xi_{q-1}^{i,j\,2}\right)\alpha_{p-1}^{i,j}\alpha_{n-1}^{i,j}\prod_{l=q+1}^{n-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j}\prod_{l=n+1}^{p-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j} \quad (50)$$

The elements of the vector $G_{i,j}$ (p=1,2,…, $N_{i,j}+1$) are given by:

$$G_{i,j}(p) = \begin{cases} 0 & if\ \ p \succ n+1 \\ \overline{G}_{i,j}(p) & if\ \ p = n+1 \\ \overline{\overline{G}}_{i,j}(p) & if\ \ p \prec n+1 \end{cases} \quad (51)$$

where:

$$\overline{G}_{i,j}(p) = (-1)^{p+2}\frac{\prod_{l=1}^{p}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j}}{\left(1-\xi_{p-1}^{i,j\,2}\right)} \quad (52)$$

$$\overline{\overline{G}}_{i,j}(p) = (-1)^{p+1}\alpha_{p-1}^{i,j}\alpha_{n-1}^{i,j}\prod_{l=1}^{n-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j}\prod_{l=n+1}^{p-1}\xi_{l-1}^{i,j}\alpha_{l-1}^{i,j} \quad (53)$$

Determining the optimal poles for each MISO subsystem can be summarized as follows:

1) *Off line computation:*
- Acquisition of necessary input-output signals,
- Determination of optimal truncating orders.

2) Computation at each iteration:

- Determination of the Fourier coefficients vector $\hat{\Theta}_i$ from (21),
- Determination of the matrix $F_{i,j}$ and the vector $G_{i,j}$ from (44) and (51),
- Determination of the filter sensitivities from (42),
- Determination of the vector E_i and the matrix R_i from (13) and (30),
- Determination of the gradient and the Hessien from (36) and (37),
- Determination of optimal poles from (22).

4 Simulation Results

The utility of optimization method of the poles for the generalized Orthonormal basis functions is illustrated with a brief simulation study. Therefore, we consider a MIMO linear system with r=2 inputs and m=2 outputs described by the following transfer matrix with real poles.

$$H(z^{-1}) = \begin{bmatrix} \dfrac{z^{-1}(1-0.525z^{-1})}{1-0.825z^{-1}} & \dfrac{z^{-1}(-0.515+0.895z^{-1})}{(1-0.315z^{-1})(1+0.575z^{-1})} \\ \dfrac{-z^{-1}(0.712+0.810z^{-1})}{1+0.715z^{-1}} & \dfrac{z^{-1}(0.510+0.475z^{-1})}{(1+0.225z^{-1})(1-0.695z^{-1})} \end{bmatrix} \quad (54)$$

The input signals selected for simulation are of amplitudes and random periods. We consider the situation where to estimate the poles of the system from the input-output sequences of H=200 points data record.

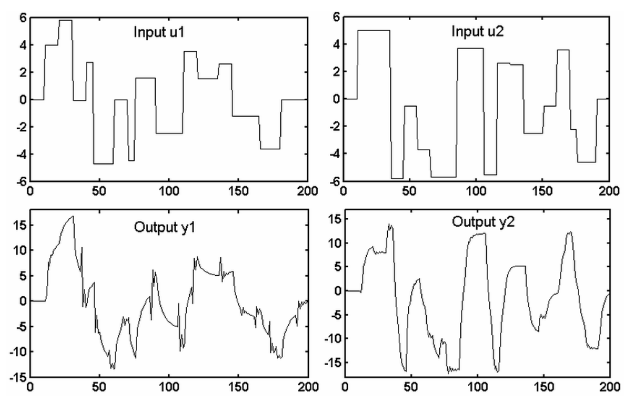

Fig1. Input-Output signals of the system

The quadratic error J_i (i=1,2) is evaluated for various truncating orders varying for example from 0 to 5. It becomes minimal from $N_{i,j} = 1$ (i=1,2; j=1,2). We will keep these optimal truncating orders in the rest of simulations.

The vector of the poles $\underline{\xi_i}$ $(i=1,2)$ is initialized at zeros. By applying the optimization algorithm of the poles, we obtained the results shown in figures 2 and 3.

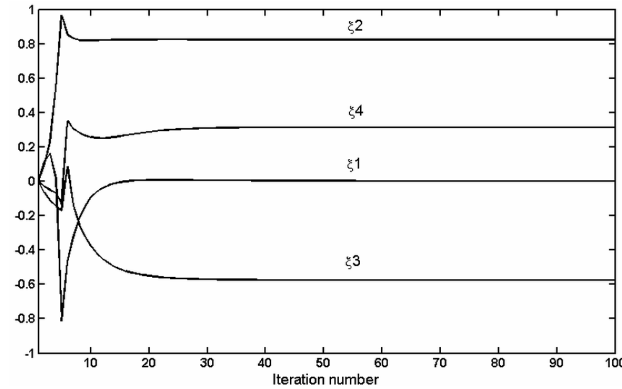

Fig.2. Estimation of the poles for the first subsystem.

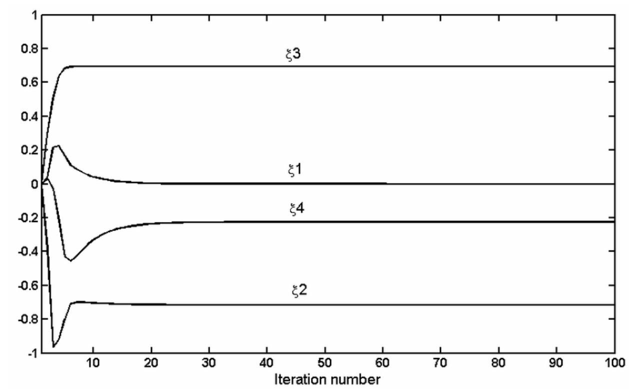

Fig.3. Estimation of the poles for the second subsystem

According to figures 2 and 3, we note that the optimal poles converge quickly in a reduce number of iterations, either from 20 iterations for first subsystem case and 15 iterations for the second.

In the follows table we summarize the optimal poles estimated by the optimization method and the Fourier coefficients estimated by the simple mean square method.

Table 1: Optimal poles and Fourier coefficients

Optimal poles	Fourier coefficients
$\underline{\xi_{1opt}} = [0.825\ \ 0\ \ 0.315\ \ -0.575]^T$	$\hat{\Theta}_1 = [1.0031\ \ -0.5250\ \ -0.2079\ \ 1.2326]^T$
$\underline{\xi_{2opt}} = [-0.715\ \ 0\ \ 0.695\ \ -0.225]^T$	$\hat{\Theta}_2 = [-0.1900\ \ -0.8100\ \ 1.0104\ \ 0.3197]^T$

To validate the optimization method after estimation of the poles, we draw in figures 4 and 5 the model and estimation outputs. The prediction errors (error between the model output and its estimation) are given in the same figures. We note that this error is negligible of 4×10^{-14} and 2×10^{-14} orders.

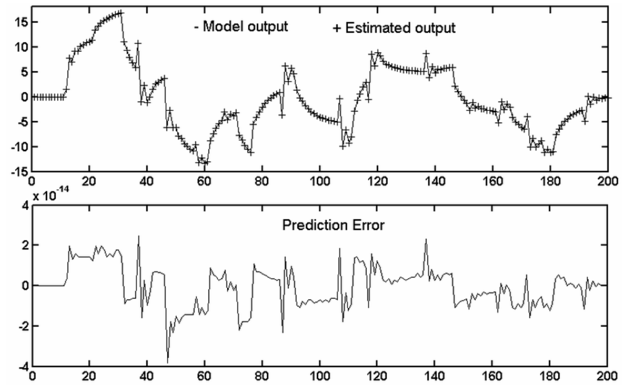

Fig.4. Model and Estimation outputs of the first subsystem

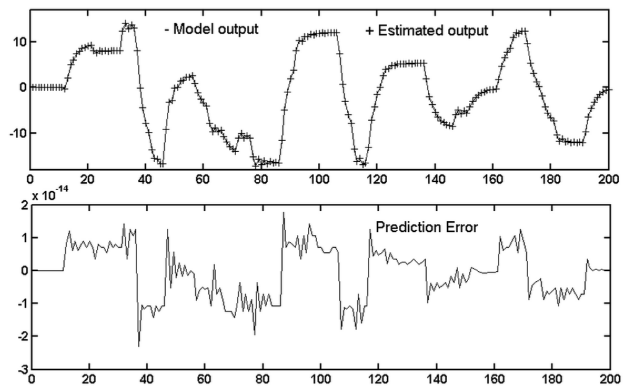

Fig.5. Model and Estimation outputs of the second subsystem

5 Conclusion

In this paper, a new optimization method of ordinary real poles for the generalized Orthonormal basis functions has been proposed. We estimated the poles of a MIMO linear system represented on the generalized Orthonormal basis functions from an ordinary data record. The efficiency of this method mainly resides in the initialization at zeros of the poles, what avoids the research of initial conditions different to zero. As a consequence, the models obtained in simulations prove the successfully of this new method which will be studied in nonlinear systems represented on these bases.

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