A Genetic Algorithm for Optimization Design of Thermoacoustic Refrigerators

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Abstract: - In the recent years extensive research efforts have been made to develop alternative refrigeration technologies that are environmentally safe. Among of these are thermoacoustic refrigeration and pulse-tube technologies. The thermoacoustic refrigerators are devices which use acoustic power to pump heat. However, the efficiency of the thermoacoustic refrigerators currently is not competitive the conventional refrigerators. In this paper we present an alternative optimization design approach based on the multi-population genetic algorithms to maximize the efficiency of the thermoacoustic refrigerators. Since the governing equation of the thermoacoustic refrigerators is a system of two-point boundary-value differential equations, we impose a two-point boundary-value problem solver into the genetic algorithm such that the algorithm can solve the governing equation and do optimization at the same time. The results show that the modified genetic algorithm is able to obtain the design variables that maximize.

Key-Words: - Thermoacoustic Refrigerator, Genetic Algorithm, Optimization

1 Introduction

During the past several decades, environmentally safe refrigeration technologies have been interested widely in research communities. Among of these are thermoacoustic refrigeration and pulse-tube technologies [1], [2], [3]. Thermoacoustic refrigerators are devices which use acoustic power to pump heat. They consist mainly of an acoustic resonator filled with a non-hazardous gas. In the resonator a stack consisting of a number of parallel plates and two heat exchangers are installed. Whereas conventional refrigerators have crankshaft-coupled pistons, the thermoacoustic refrigerators only have a loudspeaker for generating a standing wave in the resonant tube. The thermal interaction of the oscillating gas with the surface of the stack generates a heat transfer from one end of the stack to the other end. The stack is an important element which determines the performance of the refrigerator.

Although the thermoacoustic refrigeration is attractive from an environment perspective, its efficiency is still very low compared to the conventional refrigeration. Thus, tools for optimizing the design variables to improve the performance of the thermoacoustic refrigerators must be explored. However, because the mathematical theory of thermoacoustics is complicated, most of optimization attempts rely on a more or less intuitive approach.

Recently, Wetzel and Herman [4] introduce a systematic design and optimization algorithm for the design of thermoacoustic refrigerators. They employ a simplified linear model of thermoacoustics to provide fast engineering estimates for initial design calculations. By excluding energy losses in resonator, heat exchangers, and loudspeaker, their calculations indicate that the thermoacoustic refrigeration can achieve the efficiency competitive to conventional refrigerators. Miner et al. [5] make use of the simplex method to design the thermoacoustic refrigerators to maximize the efficiency. They show that the improvement in efficiency achieved by the optimized design ranged between 100% and 200%.

In this paper we present an alternative optimization design for the thermoacoustic refrigerators to improve the cooling performance. The optimization is mainly based on the multi-population genetic algorithm. However, since the governing equation of the refrigerators is a system of two-point boundary-value differential equations, we impose a soothing method for solving two-point boundary-value problems into the genetic algorithm in order to numerically solve the governing equation and do optimization simultaneously.

2 Thermoacoustic Refrigeration

The thermoacoustic refrigeration system and its refrigeration thermodynamic cycle are briefly presented in this section. As shown in Fig. 1, the main parts of the thermoacoustic refrigerators comprise a loudspeaker, a stack, two heat exchangers and a resonant tube filled with a non-hazardous gas. The loudspeaker generates an acoustic standing wave in the working gas at the fundamental resonance frequency of the resonant tube. The length of the resonant tube corresponds to haft the wavelength of the standing wave. The heat exchangers exchange heat with the surroundings, at the cold and hot sides of the stack. The stack, which is the heart of the thermoacoustic refrigerator, determines the upper limit of the refrigerator's performance.

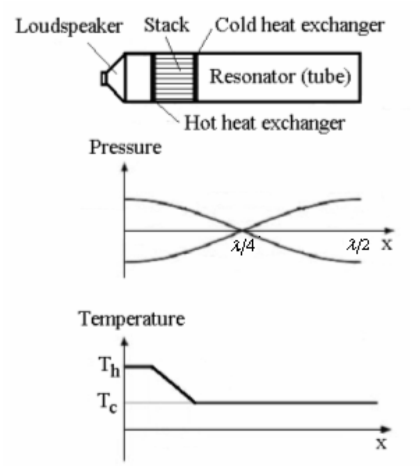
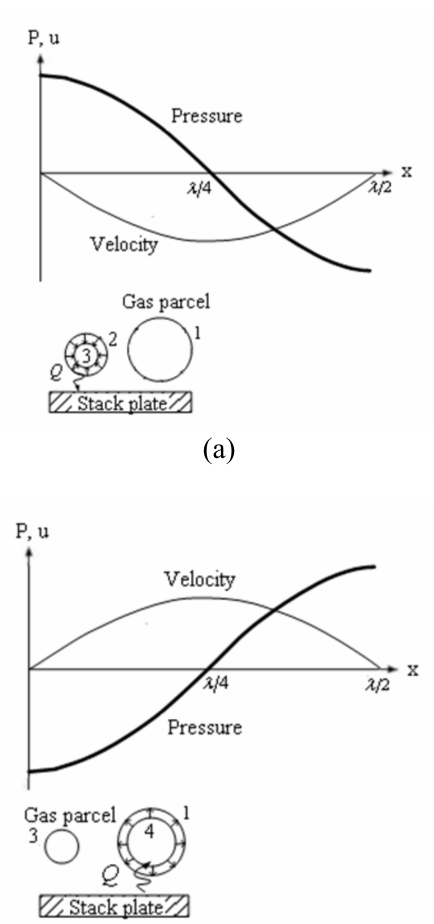


Fig. 1 Thermoacoustic refrigeration system and its pressure and temperature distributions

Fig. 2 explains the refrigeration thermodynamic cycle by considering the oscillation of a single gas parcel of the working gas along a stack plate. In the first haft cycle (see Fig. 2a), when the pressure of the acoustic waves at the stack location is positive, the gas parcel is compressed as it slightly moves to the left towards the loudspeaker. The compression process causes the temperature of the gas parcel to rise. In this state the temperature of the gas parcel is higher than that of the neighboring stack plate. Heat then transfers from the gas parcel to the stack plate. During the second half cycle (see Fig. 2b), when the pressure of the acoustic wave at the stack location is negative, the gas parcel is expanded and moved to the right towards its initial position. In this state the temperature of the gas parcel is lower than that of the neighboring stack plate. Heat is then absorbed from the stack plate to the gas parcel. After this state the gas parcel has completed one refrigeration thermodynamic cycle. It also has arrived back at its initial position and temperature. Since there are many gas parcels subjected to this cycle and the heat that is dropped by one gas parcel is transported further by the adjacent parcel, a temperature gradient develops along the stack plate as shown in Fig. 1. Therefore, a refrigeration system is obtained by installing the heat exchangers at the cold and hot sides of the stack to exchange heat with the surroundings.



(b)

Fig. 2 Illustration of the heat pumping cycle in the magnified region of the stack: (a) the gas parcel is compressed (1 to 2) and heat transfers from the gas parcel to the stack plate (2 to 3), (b) the gas parcel is expanded (3 to 4) and heat transfers from the stack plate to the gas parcel (4 to 1).

The governing equations of the thermoacoustic refrigerators are mainly derived from the continuity, momentum, and energy equations. At the steady state, the pressure $p(x)$, volume flow rate $U(x)$, and temperature $T(x)$ of the gas can be approximated as the sum of some mean value and an oscillating quantity:

$$p = p_m + p_1 e^{j\omega t}$$
$$U = U_m + U_1 e^{j\omega t}$$
$$T = T_m + T_1 e^{j\omega t},$$

where ω is the acoustic angular frequency, p_m, U_m, and T_m are the mean values, $p_1 e^{j\omega t}$, $U_1 e^{j\omega t}$, and $T_1 e^{j\omega t}$ are the oscillating quantities, and p_1, U_1, and T_1 are complex. To solve the governing equations, we need to solve a complex boundary-value problem. Readers are referred to [1] and [2] for details and discussions of the governing equations.

To solve the boundary-value problem, we utilize a shooting method [10]. The shooting method starts with guessing the unknown system's parameters such as acoustic frequency and volume flow rate. Then, we numerically integrate the equations from the loudspeaker end to the closed end of the resonator. After that the boundary conditions at the end of the resonator, which are zero acoustic power and zero energy conditions, are verified. If not, the value of the guessed parameters will be adjusted, and we can iteratively try again and again, until the boundary conditions are satisfied. Newton's method, for example, can be used to adjust the parameters. However, because of the complexity of the governing equations the convergence of this iterative process can not be guaranteed.

The coefficient of performance, COP, of the refrigerators is defined as

$$COP = \frac{Q_c}{W_{ac}}$$

where Q_c is the cooling load (the rate of the heat transfer at the cold heat exchanger) and W_{ac} is the input acoustic power provided by the loudspeaker. The second law of thermodynamics limits the value of COP as COP $\leq$ COP_C , where COP_C is the Carnot's coefficient of performance defined as

$$COP_C = \frac{T_c}{T_h - T_c}$$

where T_h and T_c are the temperatures of the hot and cold heat exchangers, respectively.

3 Multi-Population GAs

Genetic algorithms (GAs) are powerful search techniques inspired by Darwin's theory of "survival of the fittest." Nowadays, GAs have been applied successfully to solve optimization problems in many different disciplines [6],[7]. One of the most promising solutions to make GAs faster is to use parallel implementations. Multi-population GAs are the most parallel method among the parallel GAs with numerous publications. The multi-population GA splits the population into a finite number of subpopulations. Every subpopulation evolves over a number of generations isolated before some sort of communication between the subpopulations occurs. The most chosen communication operator is migration whereas the most universal communication topology is unrestricted migration topology. A detailed discussion of parallel GAs, including the multi-population GAs, can be found in [9].

4 Optimization Design Problems

In this paper we choose to design a thermoacoustic refrigerator for a temperature T_h of 300.7 K (27 °C) and a cooling load Q_c of 3 Watts. The refrigerator has the configuration of Hofler's prototype [1], [2], [5]. The geometry of the refrigerator is set corresponding to the desired cooling load of 3 Watts and the acoustic frequency around 500 Hz. The working gas is helium. The optimization problem is formulated to maximize the cost function

$$J = COPR(d) \tag{4}$$

where $COPR = COP / COP_C$ is the relative coefficient of performance defined as the ratio of the coefficient of performance to the Carnot's coefficient of performance and d is a vector of decision variables

The decision variables, d, comprise the half-plate spacing of the stack (h) and the stack length (Δx). The maximization is subject to three sets of the constraints. First, it require the refrigerator to provide the specified cooling load and temperature T_h. Second, the design parameters have to fall within specified bounds. Third, the system's parameters must satisfy the thermoacoustic nature (e.g., the system operates at the acoustic resonance, the zero acoustic power loss and zero energy boundary conditions at the end of the resonator, and $T_c < T_h$). The first constraint can be easily achieved by setting the cooling load and temperature T_h to be constant parameters of the system. The second constraint is also obvious for the GAs. The most difficult constraint to deal with is the third one. We need to solve a boundary-value problem using the shooting method to determine the system's unknown parameters. The parameters comprise the acoustic angular frequency (ω) and the volume flow rate (U_1). In theory the optimization process can easily follow the flow chat in Fig. 3.

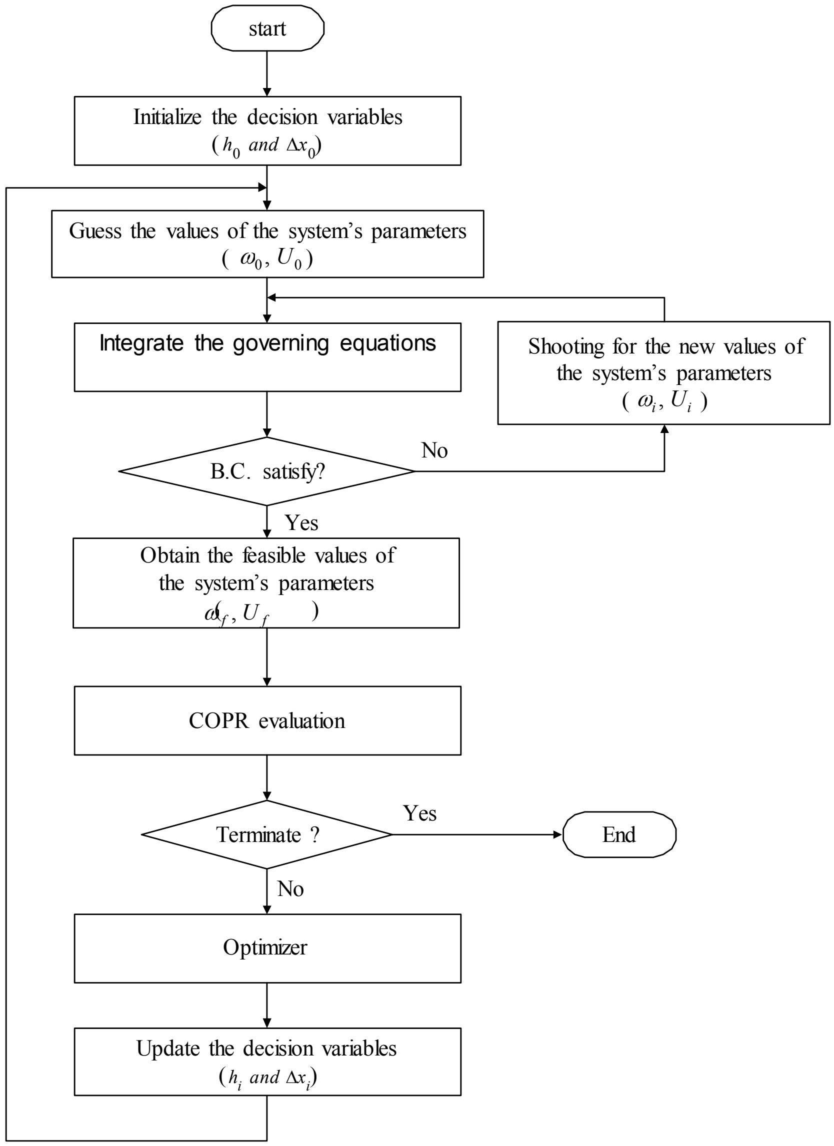


Fig. 3 Flow chart of an optimization process

However, for traditional optimization techniques such as the steepest descent technique, the derivative of the cost function must be available or can be estimated. This is not easy to do in our problem because of the complicatedness of the mathematic model of the system. The techniques that do not require the derivative information such as the simplex method and the GA-based approach are our candidates.

In this paper we are interested in the GA-based optimizer. By considering the flow chat in Fig. 3 carefully, we found that it is not suited for GA-based approach because it requires the iterative procedure to solve the governing equations for every chromosomes and generations. This consumes a lot of the computational time. Therefore, we proposes a modified GA that combines the soothing method into the genetic algorithm in order to numerically solve the governing equations and do optimization at the same time.

5 Modified Genetic Algorithm

The flow chat of the proposed genetic algorithm is shown in Fig. 4. The chromosomes do not contain only the decision variables, but also the system's unknown parameters:

$$chromosome = [\,h, \Delta x, \omega, U_1\,].$$

This implies that the GA will solve the boundary-value problem and do the optimization at the same time. In each generation the chromosomes must be evaluated and fixed prior the normal GA steps. The flow chat of the chromosome fixing is shown in Fig. 5. As shown in the figure, the number of iteration in the shooting loop is now limited to some small number and the satisfaction level of the boundary condition is relaxed to "roughly satisfy". When the shooting loop converges to the pre-specified region (in the neighborhood of boundary conditions) within the given maximum iteration, the corresponding chromosomes are called feasible chromosomes (B.C flag = 1) and their fitness is given as

$$fitness = COPR - \textit{magnitude of acoustic power loss and energy at the end of the resonator,}$$

otherwise (B.C. flag = 0)

$$fitness = 0.$$

Note that only genes ω and U_1 are altered while in the shooting loop. Moreover, since we use the real chromosomes in this paper, this makes the fusion of the GA and the shooting method even more natural. There is no need of a binary-real conversion step.

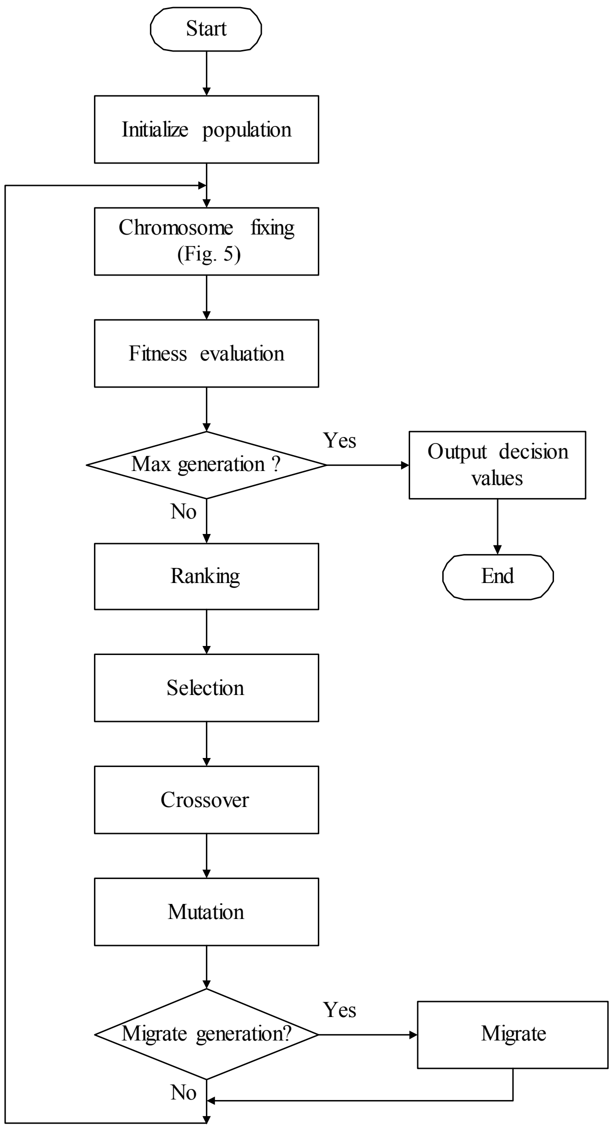


Fig. 4 Modified multi-population GAs

Note that only genes ω and U_1 are altered while in the shooting loop. Moreover, since we use the real chromosomes in this paper, this makes the fusion of the GA and the shooting method even more natural. There is no need of a binary-real conversion step.

6 Development of the modified GA

6.1 Decision Variables and Cost Function

Referring to section 4, the decision variables consist of the half-plate spacing of the stack (h) and the stack length (Δx). The search range for h is limited to 0.01 ≤ h ≤ 10 mm, search range for Δx is limited to 0 ≤ Δx ≤ 90 mm. The cost function for maximizing is

$$J = COPR = \frac{COP}{COP_C}$$

6.2 Chromosome

We use real chromosome in this paper. Referring to section 5, the chromosome of each individual contains both the decision variables and the system's unknown parameters (i.e. $chromosome = [\,h, \Delta x, \omega, U_1\,]$). The search range for ω is limited to 200π ≤ ω ≤ 1400π Hz, and the search range for U_1 is limited to $0 \le U_1 \le 0.01$ m^3/sec.

6.3 Fitness Assignment

As mention earlier, the design objective is to maximize the relative coefficient of performance (COPR). Since we are dealing with a boundary-value problem, the fitness is set such that the violation of the boundary conditions will penalty the fitness. In addition, if the violation is too high (i.e., the shooting does not converge to the pre-specified neighborhood of the boundary conditions), the fitness will set to zero. The detail is described in section 5.

6.4 Recombination (Crossover) and Mutation Methods

A discrete recombination is used. The recombination performs an exchange of genes between the individuals. In addition, the mutation is done by randomly created value to add to the genes with a low probability.

6.5 Communication Method

The communication between subpopulations is based on unrestricted migration topology.

The parameters setting for the modified GA is summarized in Table 1.

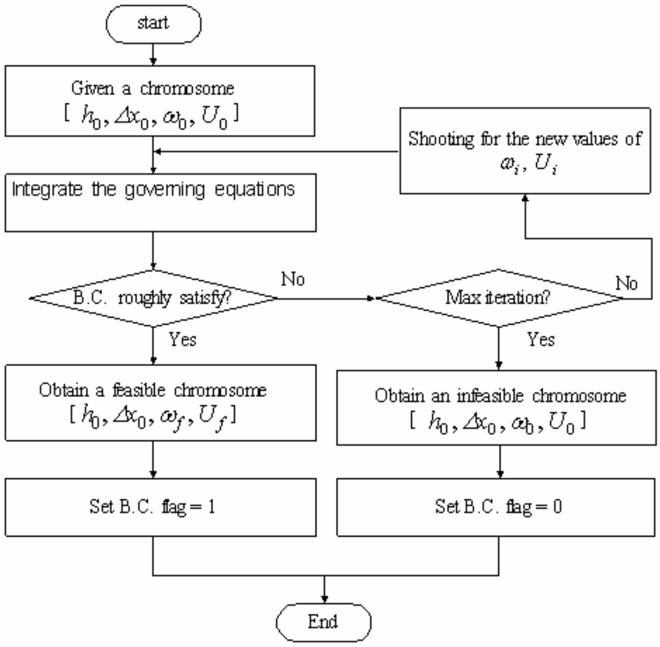


Fig. 5 The proposed chromosome fixing

Table 1. Parameter setting

Parameter	*Value*
Population size	200
Subpopulation size	40
Real chromosome length	3
Crossover probability	1
Mutation probability	0.14
Migration probability	0.2
Number of generations between migration	20
Number of generations	40

7. Results

The maximum COPR that is the result of the modified GA search is 0.17530224, which agrees with the results of Hofler. The history of COPR in each generation is shown in Fig. 6. Note that the convergence occurs after the 27th generation.

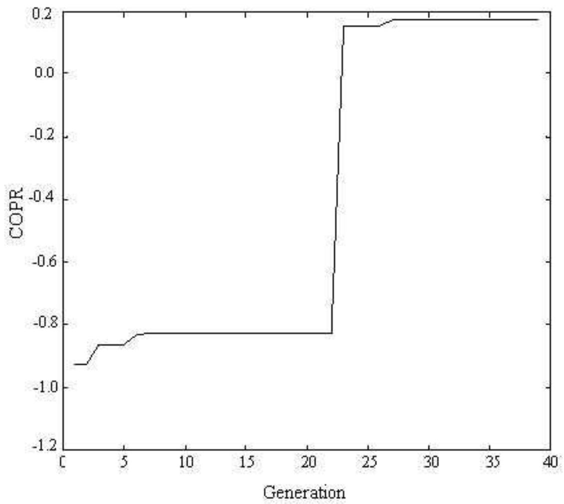


Fig. 6 Convergence rate

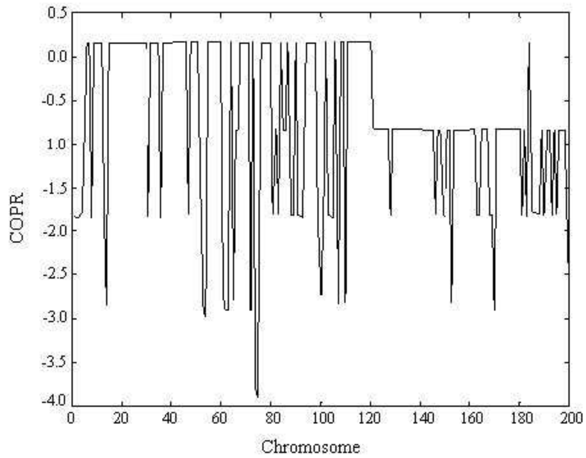


Fig. 7 Final distribution of COPR

Fig. 7 shows the distribution of COPR of the final generation. It shows that there are many chromosomes yielded the maximum COPR. This implies that we can reduce the computational time by reduce the number of population.

7 Conclusion and Future Work

A modified genetic algorithm (GA) that allows us to solve a boundary-value problem and do the optimization at the same time was presented in this paper. The algorithm reduces the computational time by relaxing the convergence of the shooting method. The good result was obtained.

Future work will attempt to apply this algorithm to more complicated problems and investigate faster and better algorithms.

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