A new distance relay based on a weighted recursive least-square algorithm

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Abstract: - A new algorithm is presented for distance protection based on the weighted recursive least-square method. This approach operates directly on voltage and current samples, without employing, as it usually happens, the phasor approach applied in the complex space. In comparison with other solutions, the algorithm is very robust and allows high speed in the fault localization on both HV and MV systems. Capabilities and performances of the algorithm are properly analysed and discussed in the paper.

Key-Words: Power system protection; Recursive least square method, Distance relaying.

1 Introduction

A distance protection is based on the estimate of the line direct impedance ($\overline{Z}_d$) between the relay and fault [1], [2], [3], [4], [5]. Since symmetrical electrical power lines exhibit constant kilometric impedance, the fault distance can be promptly evaluated once R_d and L_d line fault parameters are known.

The most frequently adopted protection procedure is usually derived from the symmetrical component theory [1], [3], [5], which allows processing voltage and current signals in order to determine the fault distance. These procedures, for instance that based on the Phase-Modified Fourier Transform (PMFT), [4], employ the simplified line equivalent circuit of Fig. 1.

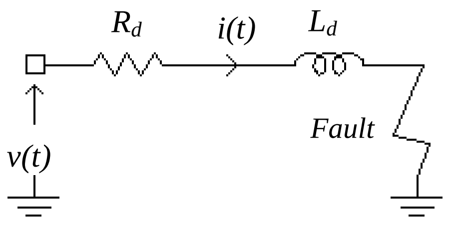

Fig. 1. Simplified faulted-line equivalent circuit.

In this case, the following differential equation can be written:

$$R_d \cdot i(t) + L_d \cdot \frac{di(t)}{dt} = v(t) \qquad (1)$$

where the unknowns can be expressed as $\vartheta = [R_d \quad L_d]$.

With reference to the PMFT, the problem is solved by using a variable observation window (T_W) which moves during the sampled signal processing. Once the voltage and current signals are acquired, the PFMT can be applied as follows [4]:

$$\begin{cases} i(t,\tau) = \int_{-\infty}^{+\infty} I(t,\omega_e)\cdot e^{j\omega_e\tau} d\omega \\ i'(t,\tau) = \int_{-\infty}^{+\infty} I'(t,\omega_e)\cdot e^{j\omega_e\tau} d\omega \\ e(t,\tau) = \int_{-\infty}^{+\infty} V(t,\omega_e)\cdot e^{j\omega_e\tau} d\omega \end{cases} \Leftrightarrow \begin{cases} I(t,\omega_e) = e^{j\omega_e(t-\frac{T_W}{2})}\cdot\frac{1}{2\pi}\int_{t-T_W}^{t} i(\tau)\cdot e^{-j\omega_e\cdot\tau} d\tau \\ I'(t,\omega_e) = e^{j\omega_e(t-\frac{T_W}{2})}\cdot\frac{1}{2\pi}\int_{t-T_W}^{t} \frac{di(\tau)}{d\tau}\cdot e^{-j\omega_e\cdot\tau} d\tau \\ E(t,\omega_e) = e^{j\omega_e(t-\frac{T_W}{2})}\cdot\frac{1}{2\pi}\int_{t-T_W}^{t} e(\tau)\cdot e^{-j\omega_e\cdot\tau} d\tau \end{cases}$$

By separating the real and imaginary parts, the following two integral equations with the two $\vartheta = [R_d \quad L_d]$ unknowns can be written:

$$\begin{cases} R_d \cdot K \int_{t-T_W}^{t} i(\tau)Cos(\omega_e(t-\tau))d\tau + L_d \cdot K \int_{t-T_W}^{t} \frac{di(\tau)}{d\tau} Cos(\omega_e(t-\tau))d\tau = K \int_{t-T_W}^{t} e(\tau)Cos(\omega_e(t-\tau))d\tau \\ R_d \cdot K \int_{t-T_W}^{t} i(\tau)Sin(\omega_e(t-\tau))d\tau + L_d \cdot K \int_{t-T_w}^{t} \frac{di(\tau)}{d\tau} Sin(\omega_e(t-\tau))d\tau = K \int_{t-T_W}^{t} e(\tau)Sin(\omega_e(t-\tau))d\tau \end{cases}.$$

This system can be expressed also in a matrix form as follows:

$$\begin{bmatrix} I_f(t,\omega_e) & I'_f(t,\omega_e) \\ I_q(t,\omega_e) & I'_q(t,\omega_e) \end{bmatrix} \begin{bmatrix} R_d \\ L_d \end{bmatrix} = \begin{bmatrix} E_f(t,\omega_e) \\ E_q(t,\omega_e) \end{bmatrix}$$

The corresponding system determinant is:

$$\Delta(t,\omega_e) = I_f(t,\omega_e) \cdot I'_q(t,\omega_e) - I'_f(t,\omega_e) \cdot I_q(t,\omega_e)$$

When this determinant is different from zero, the system gives the following solution:

$$\begin{bmatrix} R_d(t,\omega_e) \\ L_d(t,\omega_e) \end{bmatrix} = \frac{1}{\Delta(t,\omega_e)} \begin{bmatrix} I'_q(t,\omega_e) & -I'_f(t,\omega_e) \\ -I_q(t,\omega_e) & I_f(t,\omega_e) \end{bmatrix} \begin{bmatrix} V_f(t,\omega_e) \\ V_q(t,\omega_e) \end{bmatrix}$$

Since the kilometric resistance and inductance of the line are known, the fault distance can be quickly estimated.
The symmetrical component approach operates by extracting medium physical effects through signal filtering; in this case the use of a simple $R_d L_d$ circuit and an analysis in the complex space allow to achieve the fault parameter estimation.
The most recent digital procedures have greatly improved this approach allowing better precision and reliability in the estimated results [4], [6], but the need for a phasor representation has always prevented some of the capabilities offered by digital technology, for instance the possibility to operate directly on the samples of the acquired quantities instead of on medium-filtered, pre-processed signals. In order to exploit the latter possibility, in the following the use of a Weighted Recursive Least-Square (WRLS) approach is proposed to estimate fault line parameters.

2 The WRLS approach for line parameter estimation

Once the type of a short circuit is identified, each relay must estimate the distance between its own position and the fault in order to achieve the required selectivity for a correct line trip either in the instantaneous operation or in reserve (second or third step).

The problem is introduced by assuming the validity of the (1) differential equation and the availability from A/D conversion devices of sampled and digitalized signals.
In addition, the following assumptions are established:

- $T_s = \frac{1}{f_s}$: sampling step (f_s : sampling frequency).
- $t = k \cdot T_s$: the discretized real time, where $k \in N^+$. For simplicity reasons in the following this time will be indicated as: $t = k$.
- The derivative operation is approximated using the centered Euler method.

According to the above assumption, the following relation can be written:

$$\frac{di(t)}{dt} \approx \frac{i(k+1) - i(k-1)}{2T_S} = D\, i(k).$$

As a consequence, the (1) differential equation can be rewritten as:

$$\tilde{R}_d \cdot i(k) + \tilde{L}_d \cdot \frac{i(k+1) - i(k-1)}{2T_s} = v(k).$$

Finally, by adopting the matrix notation, the same relation takes the following form:

$$\begin{bmatrix} i(k) & D\, i(k) \end{bmatrix} \begin{bmatrix} \tilde{R}_d(k) \\ \tilde{L}_d(k) \end{bmatrix} = \begin{bmatrix} v(k) \end{bmatrix} \qquad (2).$$

A heuristic solution can be obtained by writing the (2) relation for two subsequent instants and solving a system with two equations and two unknowns $\tilde{\vartheta} = [\tilde{R}_d \ \ \tilde{L}_d]$. Unfortunately, this method supplies solutions oscillating around the right value. For this reason, by assuming that the samples of voltages and current must always satisfy the discrete (2) relation, the same equation can be written m times so as to obtain the following equation system:

$$\begin{bmatrix} i(k-m+1) & D\,i(k-m+1) \\ \dots & \dots \\ i(k-h) & D\,i(k-h) \\ \dots & \dots \\ i(k) & D\,i(k) \end{bmatrix} \cdot \begin{bmatrix} \tilde{R}_d(k) \\ \tilde{L}_d(k) \end{bmatrix} = \begin{bmatrix} v(k-m+1) \\ \dots \\ v(k-h) \\ \dots \\ v(k) \end{bmatrix} \quad (3)$$

In a compact form, the same system can be written as:

$A(k) \cdot \tilde{\vartheta}(k) = Y(k)$, where $A(k) \in R^{(m\times 2)}$, $\vartheta(k) \in R^2$ and $Y(k) \in R^m$.

The problem is here to solve the (3) overdimensioned system defined inside the *k* time interval. In order to obtain an optimal solution, objective criteria must be established for evaluating the reliability of the estimate. As a matter of fact, this means to establish an objective, either cost, weight or merit function. It is evident that for each objective criterion established, a different optimal solution will be obtained. Since a continuous evaluation of the goodness of the estimate can be obtained from the computation error, assuming the error as $\varepsilon = Y - A\tilde{\vartheta}$, the most frequently adopted criterion refers to the norm of the ε error. In this case, the objective function refers to the minimum value of the norm of ε, which is defined as:

$$\|\varepsilon(k)\| \overset{\Delta}{=} \sqrt{\varepsilon(k)^T W(k) \varepsilon(k)}$$

where $W(k)$ is named the weight matrix, that is defined as symmetrical and positive. The associated optimal $\hat{\vartheta}(k)$ estimate is obtained by solving the following optimization problem:

$$\min_{\hat{\vartheta}\in R^2}[(Y(k) - A(k)\tilde{\vartheta}(k))^T W(k)(Y(k) - A(k)\tilde{\vartheta}(k))]$$

The procedure exhibits a single solution only when the rank of the $A(k)$ matrix has the same dimension as the unknown vector. This solution is obtained by setting the gradient of the previous relation as equal to zero:

$$-2A(k)^T W(k)(Y(k) - A(k)\tilde{\vartheta}(k))\Big|_{\hat{\vartheta}} = 0$$

The Weighted Least-Square estimate can be written as:

$$\hat{\vartheta}(k) = A_W^{\dagger}(k) \cdot Y(k) \quad (4)$$

where

$A_W^{\dagger}(k) = (A(k)^T W(k) A(k))^{-1} A(k)^T W(k))$;

$A_W^{\dagger}(k)$ is a pseudo-matrix.

This formulation involves a static problem since the $A(k)$ matrix is built until the *m* dimension is reached; once the estimate is computed, if a new estimate is required further samples must be acquired and more equations must be added to the $A(k)$ matrix.

The dynamic or recursive formulation is a technique that uses a reduced numerical complexity and minor data storage capabilities, allowing to calculate an optimal estimate from the knowledge of only both the estimate computed in the previous instant and the equation in the current time instant. To this purpose, it is useful to define the following matrix:

$$S(k) \overset{\Delta}{=} (A(k)^T W(k) A(k))^{-1} \quad (5)$$

In this case, taking into account the (4) equation, the optimal estimate in the instant *k* is:

$$\hat{\vartheta}(k) = S(k) A(k)^T W(k) Y(k).$$

The problem can be solved by establishing the following positions:

$$A(k+1) = \begin{bmatrix} A(k) \\ a(k+1) \end{bmatrix},$$

$$W(k+1) = \begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix} \quad (6)$$

and $Y(k+1) = \begin{bmatrix} Y(k) \\ y(k+1) \end{bmatrix}$

At this point, specific relations must be searched for the computation of the following two quantities:

- $S(k+1) = (A(k+1)^T \cdot W(k+1) \cdot A(k+1))^{-1}$ (when $S(k)$ is known).
- $\hat{\vartheta}(k+1) = S(k+1) \cdot A(k+1)^T \cdot W(k+1) \cdot Y(k+1)$ (when $\hat{\vartheta}(k)$ is known).

The recursive relation between $S(k+1)$ and $S(k)$ can be found by developing the following linked block matrices:

$$S(k+1) = (A(k+1)^T \cdot W(k+1) \cdot A(k+1))^{-1} =$$
$$= \left(\begin{bmatrix} A(k)^T & a(k+1)^T \end{bmatrix} \begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix} \begin{bmatrix} A(k) \\ a(k+1) \end{bmatrix} \right)^{-1} =$$
$$= \left(\left[A(k)^T \cdot W(k) \cdot A(k) + a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right] \right)^{-1} =$$
$$= \left(S(k)^{-1} + a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right)^{-1} =$$
$$= \left(I + S(k) \cdot a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right)^{-1} S(k)$$

$$S(k+1) = \left(I + S(k) \cdot a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right)^{-1} S(k) \quad (7)$$

The computation of the second quantity requires a derivative operation of the recursive relation between $\hat{x}(k+1)$ and $\hat{x}(k)$:

$$\hat{\vartheta}(k+1) = S(k+1) \begin{bmatrix} A(k)^T & a(k+1)^T \end{bmatrix} \begin{bmatrix} W(k) & 0 \\ 0 & w(k+1) \end{bmatrix} \begin{bmatrix} Y(k) \\ y(k+1) \end{bmatrix} =$$
$$= \left(I + S(k) \cdot a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right)^{-1} \cdot \hat{\vartheta}(k) + \quad (8)$$
$$+ \left(I + S(k) \cdot a(k+1)^T \cdot w(k+1) \cdot a(k+1) \right)^{-1} \cdot S(k) \cdot a(k+1)^T w(k+1) \cdot y(k+1)$$

Finally, the fault distance can be accurately estimated by means of relations (3), (4), (6), (7) and (8).

3 The proposed estimate algorithm

Based on the above demonstrations, the following computation algorithm can be implemented:

0. Initial conditions:
   - ► Sampling time of the signals observed by relay: Ts= 0.4 ms.
   - ► Dimension of the initial observation matrix: m= 10.
   - ► Dimension of the post-fault observation matrix: mG= 2.
1. Initialization process (first estimate by means of the Minimum Square method):
   - 1.1 Identification of the matrices: $A(k) \in R^{(m \times 2)}$, $Y(k) \in R^m$ and $I(k) = W(k) \in R^{(m \times m)}$ with the first *m* samples (before the fault occurrence).
   - 1.2 Computation of the matrix: $S(k) = (5)$.
   - 1.3 First estimate computation of the unknown parameters: $\hat{\vartheta}(0) = (8)$.
2. Pre-fault (Recursive-Least-Square method):
   - 2.1 Identification of the new equations: $a(k+1)$, $w(k+1) = 1$, $y(k+1)$.
   - 2.2 Computation of the unknown parameter estimates: $\hat{\vartheta}(k+1) =$ (8).
   - 2.3 Computation of the matrix: $S(k+1) =$ (7).
   - 2.4 GOTO 2.1 in case there is no fault, otherwise continue.
3. Post-fault (Least-Square method):
   - 3.1 Identification of the matrices $A_{(1x2)}$, $Y_{(1x1)}$, $I_{(1x1)} = W_{(1x1)}$ at the sampling time immediately after the fault detection.
   - 3.2 FOR $j = 2$: mG.
     - ► $A_{(jx2)}$, $Y_{(jx1)}$, $I_{(jxj)} = W_{(jxj)}$
     - ► Computation of the matrix: $S_{(2xj)} =$ (5).
     - ► Computation of unknown parameter estimates: $$\hat{\vartheta}_g(k) = (4) = S(k)A(k)^T W(k)Y(k)$$
4. Post-fault (Weighted Recursive Least-Square method):
   - 4.1 $a(k+1)$, $w(k+1) = 1000$, $y(k+1)$
   - 4.2 $\hat{\vartheta}_g(k+1) =$ (8)
   - 4.3 $S(k+1) =$ (7)
   - 4.4 GOTO 4.1 until the simulation is over.

4 Conclusions

The above described Weighted Recursive Least-Square method has proved a valid alternative to other techniques currently adopted in the parameter estimation for distance protection. The method operates directly on voltage and current samples acquired by relays, allowing a fast, precise estimate of fault distances. The proposed algorithm can be directly implemented in the presently used apparatuses for distance protection, since no changes are required in the hardware of the adopted relays.

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