Modeling of Mode-Locked Lasers

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Abstract: Growing demand of communication networks requires ultra-short pulses. Ultra-short pulses can guarantee high data transfer and power. Fiber lasers, which are actively or passively mode-locked to generate ultra-short pulses, are termed as Mode locked fiber lasers. There are a number of methods available for mode-locking of lasers. We have discussed most of them in this paper. At the end a model of actively mode-locked fiber ring laser is simulated.

Index terms: Active mode locking, Passive mode locking, Fiber lasers, Non-linear Schrodinger equation (NLSE), Saturable Absorber (SA).

1. INTRODUCTION

In many lasers the bandwidth of the laser transition is much larger than the spacing of the cavity modes. This makes it possible to make the laser oscillate at many cavity modes simultaneously. Based on the relation $\Delta v = \frac{1}{T_r}$; Δv is the cavity mode spacing and T_r the round trip time, it is clear that to produce pulses much shorter than the round trip time the laser needs to oscillate at quite a few longitudinal modes simultaneously. The gain bandwidth of typical laser dyes is several tens of nano-meters, while the cavity mode spacing is about 100 MHz. The gain bandwidth thus easily covers 10^5 cavity modes.

If the gain profile can be made sufficiently flat for the laser to oscillate simultaneously in many modes, in order to produce short pulses, the phases of the optical fields of these modes must maintain a definite relation. A way to make the laser oscillate at many modes with a definite phase relation between them is by coupling the modes together. The coupling locks the amplitudes and phases of the modes to a definite relation, therefore the technique is generally referred as mode-locking. Minimum pulse duration achievable by mode-locking can be approximated by $\frac{T_r}{n}$; where T_r is the round trip time and n is the number of longitudinal modes locked together.

Rest of the paper is organized as follows; part two describes Mode-locking techniques, part three defines Active mode locking, part four defines Passive mode locking techniques, part five defines Mode-locking master equation followed by simulation of actively mode-locked ring fiber laser in part six and conclusion in part seven.

2. MODE-LOCKING TECHNIQUES

Traditionally techniques used to mode-lock lasers fall under one of two classifications:

2.1 Active mode-locking

Intra-cavity amplitude or phase modulators are used to force the cavity modes to couple.

2.2 Passive mode-locking

The coupling is done by optical nonlinear effects in the cavity. Following Tree-diagram best explain the types of mode locking.

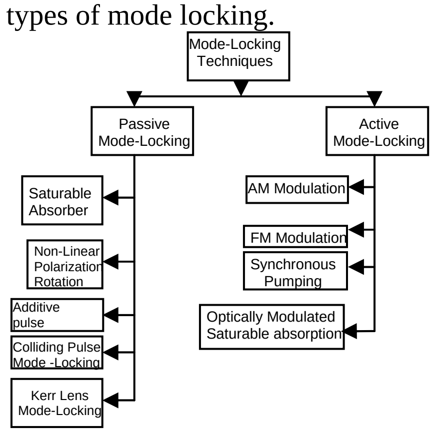

Fig.1: Types of mode locking

3. ACTIVE MODE LOCKING TECHNIQUES

The active mode locking techniques are based on the modulation of subject pulse. These techniques help in reducing the Pulse-width, in most of the cases researchers and expermenters first apply active mode-locking techniques, so as to bring the pulse width to a lower value and later on Passive Mode Locking techniques can be applied on to the pulse.

3.1 Amplitude Modulation

AM modulation, relies on either the acusto-optic or electro-optic effect in an appropriate crystal. An acusto-optic modulator is based on the fact that a suitable crystal's diffractive properties can be altered by applying a sound wave across the crystal. Acousto-optic AM mode lockers are limited to modulation frequencies less than 1 GHz making them difficult to use at high harmonic frequencies. They have stable mode-locking properties, and, because they modulate cavity loss, can be used to fix the maximum repetition frequency [2]. Electro-optic effect produces a refractive index term that varies with the applied electric field. Electro-optic devices can be modulated at rates considerably higher than their acusto-optic counterparts. As far as Mode-Locked Fiber laser design is concerned; it is preferable to focus on EO AM mode-lockers. Although there are a few different implementations for constructing an AM modulator using the electro-optic effect; I address only the fiber-based Mach-Zhender AM modulator here. A fiber-based Mach-Zhender interferometer (MZI). Such a device may be constructed by connecting two 3-dB couplers (which act as beam splitters) and ignoring one of the input/output ports of each coupler. An optical pulse passing through such a device will first be split by the input 3-dB coupler, its two halves will then travel through different sections of fiber, at which point they will recombine/interfere at the output 3-dB coupler. Wether the interference is constructive or destructive depends on the path length difference between the two arms of the device. By using an electro-optic crystal in one of the arms of this MZI, the optical path length of that arm can be changed.

3.2 Frequency Modulation Mode Lockers

It can be constructed using the same electro-optic crystals as the AM modulators described above, the difference between the two being that the resulting device does not rely on any interference. The topology for a fiber based FM modulator simply consists of placing an electro-optic crystal in series with a section of fiber. Changing the refractive index of the crystal in a time varying manner is effectively changing the cavity length. A phase modulator is usually made of a crystal with large electro-optic coefficient. In such crystals the index of refraction changes linearly with the strength of an externally applied electric field E. In such crystals the index of refraction changes linearly with the strength of an externally applied electric field E:

$$n(E) = n_o + \alpha E \text{ ---------- (1)}$$

For a sinusoidal electric field $E = E_o \cos(\omega t)$, the depth of the phase modulation is $k\alpha E_o l$, where l is the length of the modulator. Electro-optic FM mode lockers are adapted easily to microwave frequencies [4] but they do not have the stability of their AM counterparts [3]. Also, because they do not affect the gain or loss of the cavity, FM mode lockers are not well suited for preserving the repetition rate in a multiply mode-locked cavity.

The modulation depth can be calculated by analyzing the coupling between the incoming light and the deflected light through the phase grating. Because the length of the modulator is only a few centimeters, the time for the light to pass the modulator is much shorter than the period of the modulation frequency. Under this condition we only need to consider a static phase grating in the analysis. Assume the amplitudes of both the incoming light and the deflected light change slowly due to coupling as they propagate in the *z*-direction through the modulator.

$$E_i(r) = A_i(z)\exp(ik_i \cdot r) \text{ ----- (2)}$$

$$E_{d\pm}(r) = A_{d\pm}(z)\exp(ik_{d\pm} \cdot r) \text{ ---- (3)}$$

Here $k_i = k\hat{z}, k_{d\pm} = k\hat{z} \pm k_a\hat{x}, k_a \ll k$

3.3 Synchronous pumping Mode lockers

It refers to the periodic modulation of a laser's gain media at a repetition rate corresponding to a harmonic of the fundamental cavity frequency. Here mode-locking is initiated due to what we can view as a type of inverse AM modulation, where cavity's gain is modulated as opposed to AM mode-locking which corresponds to the modulation of the cavity's loss. One of the serious limitation of this approach is that; it is useful only for gain media that posses fast relaxation times.

3.4 Optically Modulated Saturable Absorption

OMSA combines aspects of both AM modulation and synchronous pumping with SBR mode-locking [1]. In this scheme a laser, which is passively mode-locked using a SBR, has its loss modulated. The difference between this loss modulation and conventional AM mode-locking is that the modulation is achieved here through the use of a time varying optical signal which pumps the lasers SBR. This time varying pumping of the lasers SBR makes its loss not only intensity dependent but now time dependent as well. This allows one to align the pulses within the cavity as well as increase the laser's repetition rate.

4. PASSIVE MODE LOCKING TECHNIQUES

Several different physical mechanisms can be used for passive mode-locking: saturable absorbtion (SA) { SA mode-locking, comes in two varieties: slow and fast.}, Nonlinear Polarization Rotation (NLPR), Additive Pulse mode-locking, Colliding Pulse mode-locking, and Kerr lens mode-locking.

4.1 Saturable Bragg Reflector

A SBR can be thought of as an intensity and wavelength-dependent mirror constructed using a semiconductor material. The master equation of passive mode-locking with a fast saturable absorber [5] is as follows:-

$$T_R \frac{\partial a}{\partial T} = (g-l)a + \left(\frac{1}{\Omega_f^2} + jD\right)\frac{\partial^2 a}{\partial t^2} + (\gamma - j\delta)|a|^2 a \quad \text{--- (4)}$$

The evolution of the pulse envelope $a(T,t)$ is written in terms of two time scales: the fast time scale t on the order of the pulse width, and the slow time scale T on the order of several roundtrip times T_R. g is the gain per pass; l is the loss per pass; $\frac{1}{\Omega_f^2}$ represents the filter strength; D is the group velocity dispersion parameter; are the coefficients of saturable absorption (Self-Amplitude Modulation) and of the self-phase modulation due to the Kerr effect (SPM).

$a(T,t)$ is used to find the position of the minimum pulse width. If the pulse stretches and compresses in one roundtrip, the nonlinear shaping action does not fit snuggly onto the pulse envelope, but appears wider in time. This changes the master equation of the stretched pulse operation [6].

$$T_R \frac{\partial a}{\partial T} = (g-l)a + \left(\frac{1}{\Omega_f^2} + jD\right)\frac{\partial^2 a}{\partial t^2} + (\gamma - j\delta)|A_o|^2\left(1-\mu\frac{t^2}{\tau^2}\right)a \quad \text{- (5)}$$

A_o = pulse amplitude; τ = pulse width;

D = Average group velocity dispersion.

Eqn#5 has a chirped Gaussian solution of the form [7]:-

$$a(T,t) = A_o \exp\left(-\frac{t^2}{\tau^2}(1+j\beta)\right)\exp - j\psi T \quad \text{----- (6)}$$

Eqn#6 explains the operation of the stretched pulse laser [8]. SBR acts as a mode-locking mechanism by making a cavity an unfavorable place for CW light to exist by breaking it into pulses. Once pulses form, the SBR will attenuate the pulse wings allowing the center of the pulse to reflect off the SBR with minimal loss. After repeated reflections, this shortens the pulse, whose final width is ultimately determined by the material properties of the SBR as well as the total cavity dispersion, nonlinearity, loss, and gain.

4.2 Slow SA mode-locking

It uses a Saturable Absorber, as just described, with the exception that it is not able to saturate on a time scale local to the pulse. Instead this slow SA saturates after some leading part of the pulse has pumped it. Since this device has a longer life time than its fast SA counterpart, it stays saturated for some finite time after the peak of the pulse has passed, thus asymmetrically shortening the pulse by only clipping off its leading edge. This effect is then combined with the saturation of the gain medium which changes the amount of available gain. If the gain medium saturation is visualized (thus reducing the available gain) after the SA saturates; it is able to clip off of the trailing edge of pulse. In reality it is the combined action of these two SA mechanisms, that forms the basis of slow SA mode-locking. As rare-earth-doped fiber-based gain media have relatively long lifetimes, the pulse widths resulting from this type of mode-locking in a fiber laser is large.

4.3 Non-Linear polarization Rotation

Intensity dependent birefringence of optical fiber results in an elliptically polarized pulse will have its ‘x’ and ‘y’ components experience different phase shifts, thus rotating the polarization ellipse. Nonlinear mechanisms of self phase modulation (SPM) and cross phase modulation (XPM) are required to be considered, whose action on a field can be approximated by:-

$$SPM \rightarrow A_x(z+\Delta z,t) = A_x(z,t)e^{i\gamma|A_x(z,t)|^2\Delta z} \quad \text{---- (7)}$$

$$XPM \rightarrow A_x(z+\Delta z,t) = A_x(z,t)e^{i\gamma\frac{2}{3}|A_y(z,t)|^2\Delta z} \quad \text{--- (8)}$$

Noting that fields propagate with an e^{ikz} dependence (where $k=\frac{2\pi n}{\lambda}$), it is realized that the SPM and XPM effects may be interpreted as introducing a polarization-dependent, intensity-dependent refractive index. Considering the superposition of two linearly polarized waves. The electric field in phasor notation can be written as:

$$E(z) \approx \begin{bmatrix} A_x(z,\tau)e^{ikz}e^{i\Delta k_x z} \\ A_y(z,\tau)e^{ikz}e^{i\Delta k_y z} \end{bmatrix} \quad \text{----- (9)}$$

Here

$$\Delta k_x = \frac{2\pi}{\lambda}\gamma\left(|A_x(z,\tau)|^2 + \frac{2}{3}|A_y(z,\tau)|^2\right) \quad \text{----- (10)}$$

and

$$\Delta k_y = \frac{2\pi}{\lambda}\gamma\left(|A_y(z,\tau)|^2 + \frac{2}{3}|A_x(z,\tau)|^2\right) \quad \text{---- (11)}$$

are due to both SPM and XPM. Total field can be written as:

$$E(z,\tau)=\begin{bmatrix}E_x(z,\tau)\\ E_y(z,\tau)\end{bmatrix}\approx \mathrm{Re}\left\{\begin{bmatrix}a_x(z,\tau)\\ a_y(z,\tau)e^{i(\Delta k_y-\Delta k_x)z}e^{i\phi}\end{bmatrix}e^{ikz}e^{i\Delta k_x z}e^{-i\omega\tau}\right\} \text{ - (12)}$$

Real fields a_x and a_y and the phase ϕ accounts for any initial phase difference between the two. Polarization of such a field is governed by the following ellipse:-

$$\left(\frac{E_x(z,\tau)}{a_x(z,\tau)}\right)^2+\left(\frac{E_y(z,\tau)}{a_y(z,\tau)}\right)^2-2\left(\frac{E_x(z,\tau)}{a_x(z,\tau)}\right)\left(\frac{E_y(z,\tau)}{a_y(z,\tau)}\right) \text{-- (13)}$$

$$\cos\left(\left[\Delta k_y-\Delta k_x\right]z+\phi\right)=\sin^2\left(\left[\Delta k_y-\Delta k_x\right]z+\phi\right)$$

Angle of the ellipse can be written as:-

$$\alpha=\frac{1}{2}\tan^{-1}\left[\left(\frac{2a_x(z,\tau)a_y(z,\tau)}{a_x^2(z,\tau)-a_y^2(z,\tau)}\right)\cos\left(\left[\Delta k_y-\Delta k_x\right]z+\phi\right)\right] \text{-(14)}$$

The polarization shown in Fig. 2a, at a fixed location L_1, as well as a polarization due to a slightly perturbed location, L_2, which is shown in Fig. 2b (assuming that $\tau_2 \approx \tau_1$) such that field amplitude's have not changed.

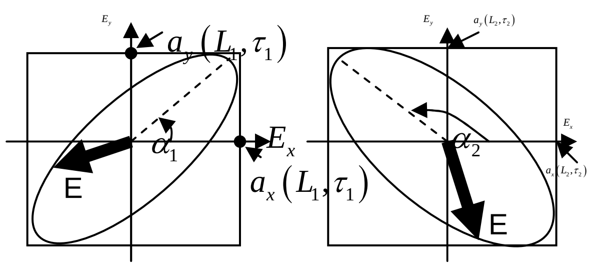

Figure 2a Figure 2b

Figure 2: Polarization diagram of a plane wave and its tilted ellipse at two different observation distances (L_2 and L_1).

By considering low-intensity light (or ignoring the nonlinearities) such that $\Delta k_x=\Delta k_y=0$ and angle, α. As far as pulse propagation in an isotropic optical fiber is concerned, thus Eqn. #14 indicates that angle is constant and hence no rotation occurs.

NLPR mechanism can be introduced by; $\Delta k_x=\Delta k_y\neq 0$ and the angle, α is dependent on 'z'. The polarization ellipse rotates with propagation in this fiber. In practice a pulse in a NLPR mode-locked laser is oriented (through some type of linear polarization controller) such that when it passes through an analyzer its wings are shed and the pulse is shortened. NLPR mode-locking is somewhat closer to saturable absorbing mechanism.

4.4 Kerr lens mode-locking

It is used most notably in Ti: Sapphire laser cavities, relies on a crystal's nonlinear refractive index to create a device that acts as an intensity-dependent lens. As a result, inserting an aperture behind a suitable Kerr material allows an experimenter to effectively filter CW light out of a laser cavity as it does not have a high enough intensity to be tightly focused. Consequently, short pulses, which possess high intensities, are focused more acutely and end up with a lower threshold than CW light.

4.5 Additive pulse mode-locking

It relies on the nonlinear process of SPM in a passive optical fiber cavity, which is coupled to the main cavity by use of a beam splitter. The pulse in the fiber cavity then interferes with the pulse circulating in the main cavity at the beam splitter. Since the SPM acts on the pulse in the fiber cavity, the pulses can be made to interfere constructively at their peaks and destructively on their wings. Hence the interference shortens the cavity pulses in a manner reminiscent to that of a SA. To put this type of mode-locking into practice, cavity lengths must be matched with inter-ferometric precision.

4.6 Colliding pulse mode-locking

It relies on pulse interference in a Saturable Absorber (SA) medium. If an SA is chosen with a saturation intensity that cannot be achieved by a single pulse in cavity, due to the available gain and loss. It can only be saturated by allowing more pulses incident on the SA at the same time. This can be made practical only in a linear or ring cavity, where counter-propagating pulses can be made to saturate a dye jet (located in the center of the cavity) when they overlap in it.

5. MODE-LOCKING MASTER EQUATION

The well-known Haus master equation [9] for describing mode-locking can be used to describe the pulse evolution in the cavity of a mode locked laser. Main points concerning the basic idea are as under:

(a). Pulse circulating in the laser cavity is described in the time domain with a complex amplitude A(t). Here "A(t)" is used for a single pulse, not a pulse train and applies for a certain position within the cavity (may be just before the output coupler). The amplitudes are normally normalized so that the squared modulus of A(t) is either the optical power or the intensity in the gain medium.

(b). Changes "$\Delta A(t)$" occurred on the pulse within a single cavity round trip due to following reasons:

(i). Laser gain (with a finite gain bandwidth).

(ii). Optical losses (time dependence introduced by an optical modulator for active mode-locking, Saturable absorber for passive mode-locking).

(iii). Dispersion & Optical non-linearities (e.g. kerr effect).

(c). Considering amplitude $A_j(t)$, where "j" indicates the number of cavity round trips. Replacing the index "j" with a second time variable $T = jT_{rt}$; (where T_{rt} is the cavity round trip time) we can get a function A(T,t).

(d). Considering "T" as a continuous variable, the evolution of which can be described with a differential equation. If the combined changes of the amplitude per round trip are ΔA, then the resulting Haus master equation will become of the form:

$$T_R \frac{\partial}{\partial T} A(T,t) = \Delta A \text{ ----- (15)}$$

$$T_R \frac{\partial}{\partial T} A(T,t) = \left(-jD\frac{\partial^2}{\partial t^2} + j\delta|A|^2 \right)A + \left(g - l + \frac{g}{\Omega_g{}^2}\frac{\partial^2}{\partial t^2} - q(T,t) \right)A \text{ - (16)}$$

$A(T,t)$ is a slowly varying electric field envelope.

T is a long-term time variable (used for slowly varying envelope approximation). t is the short-term time variable. D is the GDD. δ is the SPM coefficient. g is the small-signal electric field gain coefficient (as in *gain*=1+*g*), l is the constant small-signal cavity loss, $q(T,t)$ is the saturable loss, T_R is the cavity round trip time, Ω_g is the HWHM bandwidth of the gain medium.

The master equation can also be taken as a generalization of NLSE. The gain saturates to a constant value according to average laser power [10]:

$$g(T) = \frac{g_0}{1 + \frac{E_P(T)}{P_L T_R}} \text{ ---- (17)}$$

g_o is the unsaturated gain. P_L is the laser saturation power. E_P is the pulse energy. The absorber saturates according to the following equation:

$$\frac{dq(T,t)}{dt} = -\frac{q - q_0}{\tau_A} - q\frac{|A(T,t)|^2}{E_A} \text{ ---(18)}$$

τ_A is the absorber recovery time. E_A is the absorber saturation energy.

6. SIMULATION OF AN ACTIVELY MODELOCKED FIBER RING LASER

Higher repetition rates in fiber ring-lasers can be realized by employing active mode locking [12]. Block diagram of an actively modelocked fiber ring laser is shown in figure 3 below:

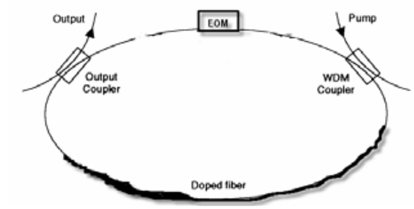

Fig.3: Block diagram of an actively modelocked fiber ring laser

The theoretical model for the erbium doped fiber ring laser is based upon the propagation of pulse through the laser cavity and is described as follows [11]:

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A \text{ --- (19)}$$

A mach zhendher interferometer with voltage application in one arm has been realized as an active modelocker. Pulse propagation through erbium doped fiber has been realized using the following complex Ginzberg Landau equation [11]:

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + igT_2^2\frac{\partial^2 A}{\partial T^2} + \frac{g}{2}A + i\gamma\left|A^2\right|A \text{ ----- (20)}$$

The input signal (chirped Gaussian pulse) and modelocked output pulse after ten round trips is shown in figure 4:

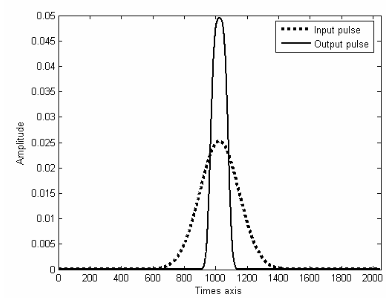

Fig. 4: seed pulse and Mode-locked output pulse

The amplitude and pulse width of output signal and seed pulse for each round trip has been monitored and are shown as follows (see Fig. 5 & 6):

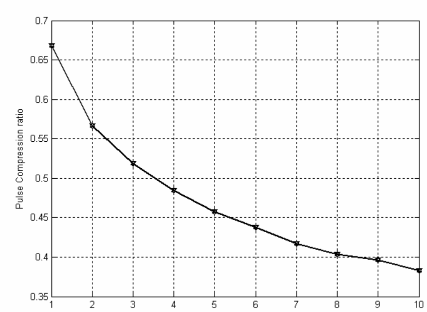

Fig. 5: Pulse compression ratio corresponding to number of round trips

It is evident from figure 5 and figure 6 that pulse amplitude is at increase along with corresponding decrease in pulsewidth. Further research in this direction is continued to get a multiwavelngth modelocked output after application of filter.

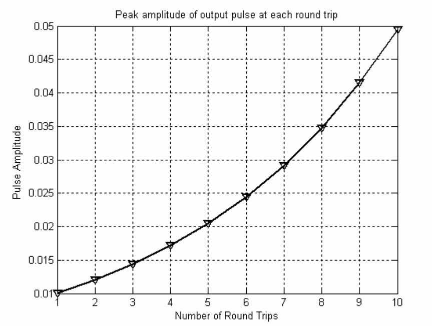

Fig.6: Pulse Amplitude corresponding to number of round trips

7. CONCLUSION

A review of mode locking methods has been presented in this paper. Synchronous pumping relies on gain media with fast relaxation times; where-as fiber based gain media have long relaxation times; so it is not relevant as far as modeling of Fiber laser is concerned. Rest of the three active mode-locking mechanisms are important. Kerr lens mode-locking is only used in bulk lasers and can not be implemented in fiber. The stringent requirement inherent to additive pulse mode-locking, that both cavities be matched with interferometric precision, introduces environmental instability into any laser using this scheme, consequently this technique will prove to be too unreliable in this case as well. Since colliding pulse mode-locking requires a dye jet; it will also not be pursued. Mode-locking using the slow SA technique cannot be developed to produce short (fs) pulses in fiber lasers owing to the long lifetimes of fiber based gain media. This leaves us with two passive mechanisms, **NLPR** and **SBR** mode-locking; which can be of direct importance in passively mode locking fiber lasers. Actively mode-locked Fiber Laser cavity equation has been designed following the rules set by H.A. Haus. Increasing bandwidth requirements of telecommunication networks, mode locked fiber lasers seems to be the future source of ultra short pulses. The literature study and simulation results are in use for generating such a source and hence verifies the importance of work carried out.

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