

# Impedance Matching by Using a Multi-stub System

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*Abstract:* Nowadays lots of applications in telecommunications require a wide bandwidth to transmit more information and improve the quality on the transmission of this information. In order to guarantee this condition low losses systems are designed; these losses can be classified in dissipation losses and mismatch losses. There are different classic methods for impedance matching, but they reach the impedance matching in a very narrow bandwidth, whereas the bandwidth used in different applications is generally greater in accordance with the quantity of information to be transmitted. A solution proposed in this paper to reach an impedance matching into a wider bandwidth is through the connection of various stub lines in parallel all along the transmission line, so we can have a multiple variables system, with more grades of liberty. A software that automatizes the calculations to reach the impedance matching was programmed, by using the minimum squares and Nelder-Mead methods. Once the user introduces the data of Standing Wave Ratio (SWR) and bandwidth desired, the load impedance and the work frequency of the system, the program determines the minimum number of stubs necessary to match the system, and the longitudes and distances along the line where they should be located.

*Key-Words:* SWR, stub, impedance matching, optimization

## 1 Introduction

The impedance matching is a fundamental stage in the design process of any telecommunication system. The essential reasons why it is required to match the system correctly are two: To protect the generator from the effects caused by the reflected waves at the load, and to avoid the transmission losses when the load is mismatched from the line.

Generally the matching network is constructed with non dissipative elements, to avoid unnecessary power losses, and it is designed in order the network input impedance to the line to be  $Z_0$ . When this is achieved, the reflections in the transmission line disappear. This means that the reflection coefficient is null and therefore the SWR is equal to one. There are several classic techniques for impedance matching, like for example  $\lambda/4$  line and one stub matching, among others. In general, in a communication system the impedance matching is needed in a bandwidth, which depends on the application the system is designed for. The classic methods mentioned before only reach the impedance matching at one frequency.

This paper proposes to use an optimization algorithm to achieve the impedance matching into the desired frequency range, from the knowledge of the system requirements. This algorithm is based on the

connection of several stub lines in parallel all along the transmission line to reach an impedance matching into a wider bandwidth. The software automatizes the calculus to reach the impedance matching, and give to the user the minimum number of stubs necessary to match the system, and the longitudes and distances along the line where they should be located.

## 2 Impedance matching methods

There are a lot of classic methods to achieve impedance matching in a telecommunication system. Some of them are mentioned below:

- $\lambda/4$  transformer
- series-parallel matching
- parallel-series matching
- one stub matching

The problem with these methods is that they reach the impedance matching at only one frequency, instead of reaching this into the bandwidth desired by the user. However, we will explain here the one stub matching method, because from this one an other method that improves the impedance matching is originated: two stub matching, and also it is the foundation of this work.

### 2.1 One stub matching

The objective is to achieve the impedance matching for the system at one specific frequency, by connecting one stub line in parallel to the line at a distance  $d_s$  from the load, with longitude  $l_s$ , like it is shown in figure 1. In general  $d_s$  and  $l_s$  have to be calculated.

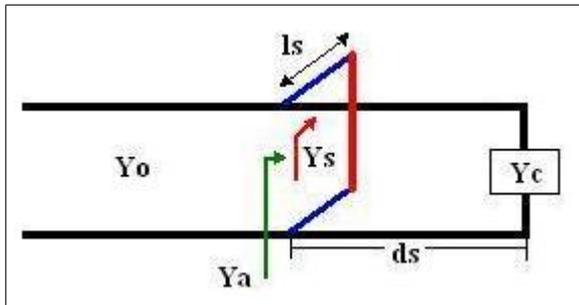


Figure 1: Connection of one stub

From figure 1 the following equations can be obtained:

$$Y_a = Y_s + Y' \tag{1}$$

where

$$Y' = Y_0 \cdot \frac{1 - \Gamma}{1 + \Gamma} \tag{2}$$

being  $\Gamma$  the reflection coefficient at a distance  $d_s$  from the load, defined by:

$$\Gamma = \Gamma_l \cdot e^{-2\gamma d_s} \tag{3}$$

$$\Gamma_l = \frac{Y_0 - Y_c}{Y_0 + Y_c} \tag{4}$$

The stub line reflection coefficient, if it is ended in a short circuit, is:

$$\Gamma_s = -e^{-2j\beta l_s} \tag{5}$$

and its input admittance is:

$$Y_s = Y_{0s} \cdot \frac{1 - \Gamma_s}{1 + \Gamma_s} \tag{6}$$

In order to reach the impedance matching  $Y_a$  must be equal to  $Y_0$ . Now,  $l_s$  and  $d_s$  values can be obtained from the previous equations. This way, the system is matched at the frequency given by:

$$\beta = \frac{2\pi}{\lambda} \tag{7}$$

where  $\lambda = v_p/f$ .

The following figure shows a typical curve of the Standing Wave Ratio obtained by matching with one stub. The impedance matching is reached only at one frequency:

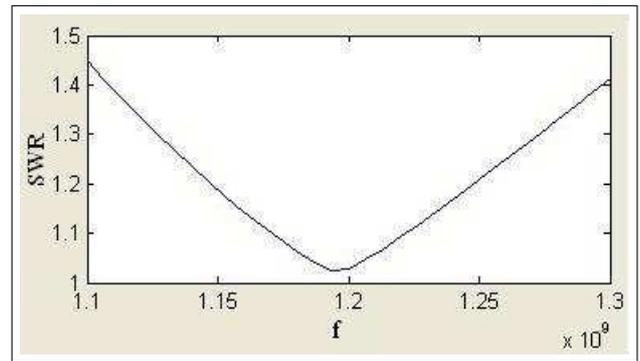


Figure 2: SWR for one stub

### 2.2 Two stub matching

This is an other method for impedance matching, where instead of connecting one stub to the line, the user adds another stub. The circuit obtained is as shown in figure 3.

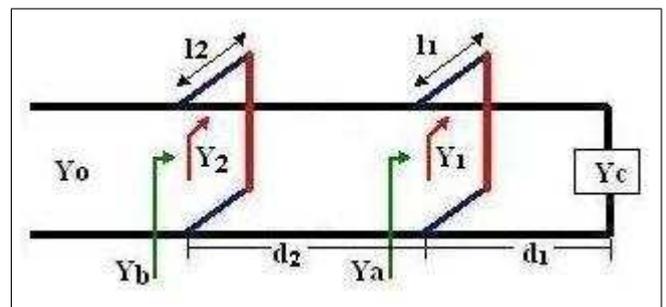


Figure 3: Circuit with two stubs

In this case, the equations to obtain the values of  $l_1, d_1, l_2$  y  $d_2$  are the followings:

$$Y_a = Y_1 + Y_1' \tag{8}$$

$$Y_1' = Y_0 \cdot \frac{1 - \Gamma_l \cdot e^{-2\gamma d_1}}{1 + \Gamma_l \cdot e^{-2\gamma d_1}} \tag{9}$$

$$\Gamma_l = \frac{Y_0 - Y_c}{Y_0 + Y_c} \tag{10}$$

$$Y_1 = Y_{0s1} \cdot \frac{1 + e^{-2j\beta l_1}}{1 - e^{-2j\beta l_1}} \tag{11}$$

$$Y_b = Y_2 + Y_2' \tag{12}$$

$$Y_2' = Y_0 \cdot \frac{1 - \Gamma_a \cdot e^{-2\gamma d_2}}{1 + \Gamma_a \cdot e^{-2\gamma d_2}} \tag{13}$$

$$\Gamma_a = \frac{Y_0 - Y_a}{Y_0 + Y_a} \tag{14}$$

$$Y_2 = Y_{0s2} \cdot \frac{1 + e^{-2j\beta l_2}}{1 - e^{-2j\beta l_2}} \tag{15}$$

The figure 4 shows a typical curve of the Standing Wave Ratio obtained by matching with two stubs. It is possible to see that the impedance matching into the

bandwidth is better than the one obtained in figure 2, by using the one stub matching method.

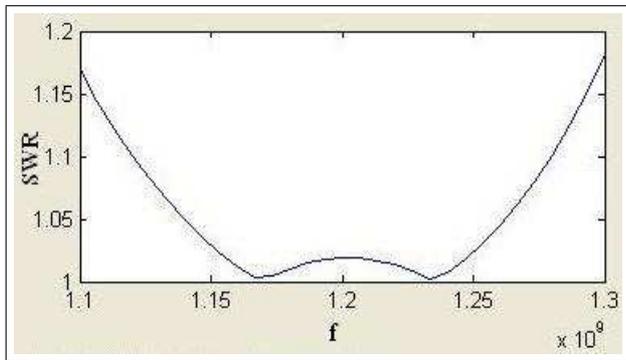


Figure 4: SWR for two stubs

### 2.3 Matching at several frequencies

Based on the previous theory, if impedance matching is wanted in a specific bandwidth, various stubs can be connected along the line. This way a SWR near to one can be obtained along all the bandwidth, as shown in figure 5. In this figure there are the system ideal and real responses, which should be into a minimum error margin. As it is very difficult to achieve a SWR equal to one (null reflection coefficient) for all the frequencies, it is necessary to impose a maximum value of it, which can not be overcome on the desired bandwidth.

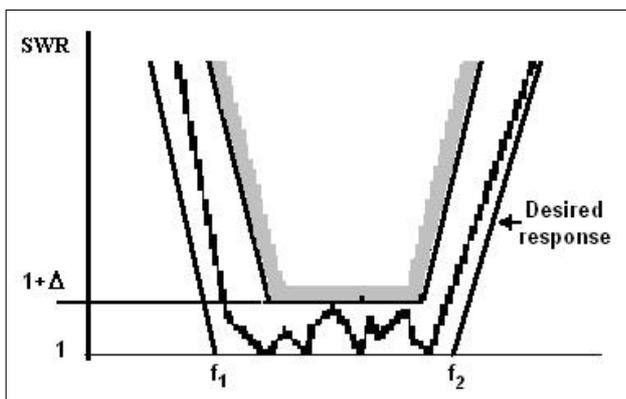


Figure 5: SWR obtained with various stubs

Due to the difficulty of the calculus for several stubs along the line, it is better to use a numerical solution than an analytical one, which is a very good reason to develop a software to solve the problem.

## 3 Numerical solution

The first thing that has to be done in an optimization problem is to define the objective function, which is

going to be maximized or minimized, depending on the case.

In this paper the function to be minimized is SWR. This relation depends on the reflection coefficient, through the following equation:

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \tag{16}$$

It is easier to work with  $|\Gamma|$ , because its range of variation is  $0 < |\Gamma| < 1$ , instead of SWR that varies between zero and infinity.

Finally, the objective function chosen was  $|\Gamma|$ , knowing that if this function is minimized, automatically SWR is minimized too.

It was decided to minimize the objective function by minimum squares, because in other works developed before the results obtained were very good [2].

### 3.1 Minimum squares

The function considered  $G(f, a_1, a_2, \dots, a_n)$  is continuous in  $f$  at the interval  $I = [f_0 - \frac{\Delta f}{2}; f_0 + \frac{\Delta f}{2}]$ , and depends on the  $N$  parameters  $a_1, a_2, \dots, a_n$ . For each  $A_i$  set there is a function  $G$  and the problem is to find the function  $G$  which better approximates into the interval  $I$  to the previous selected reference function ( $R$ ).

Having the reference function evaluated in several points into the interval  $I$ , the quadratic errors can be determinate as follows:

$$F(a_1, \dots, a_n) = \sum_{i=1}^M [G(f_i, a_1, \dots, a_n) - R(f_i)]^2 \tag{17}$$

The expression (17) is the objective function to be minimized, and that occurs when  $\nabla F = 0$ . The solution to this problem is reached when the optimum values of  $a_1, a_2, \dots, a_n$  are obtained.

### 3.2 Numerical method used

The function to be minimized is a nonlinear function, and it is also very complex. For these reasons it was important to choose an adequate optimization method, so the time used by the program to make the calculations could be minimum. In order to reach this, the Nelder-Mead method was used. It belongs to a class of methods that use comparisons between different values of the objective function and do not require the use of any derivatives, called direct search methods.

Nelder-Mead method is generally used for unconstrained optimization. In this case there are some constraints, such as the values of distances and longitudes, that have to be positive; these constraints were programmed apart.

In this method, once the function is evaluated at several points, the one that gives the worst value of the objective function is replaced for an other one, by using one of the four basic operations, shown in Figure 6. Then the function is evaluated again at this new point, if there is an improvement respect to one of the others points, then the algorithm starts again with the new worst point. If not, it is necessary to apply an other different operation that the one was applied first.

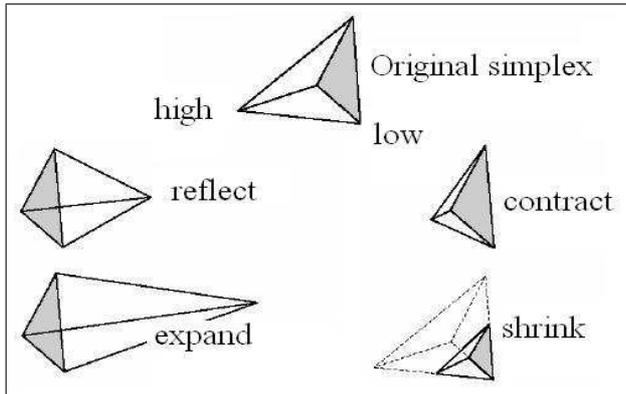


Figure 6: Basics operations in Nelder-Mead

### 4 Programming

The objective of the software is to find the number of stubs required to reach the impedance matching into the bandwidth selected by the user, and their longitudes and location along the transmission line. For this paper the stubs and transmission line have the same characteristic impedance, and they do not have attenuation losses.

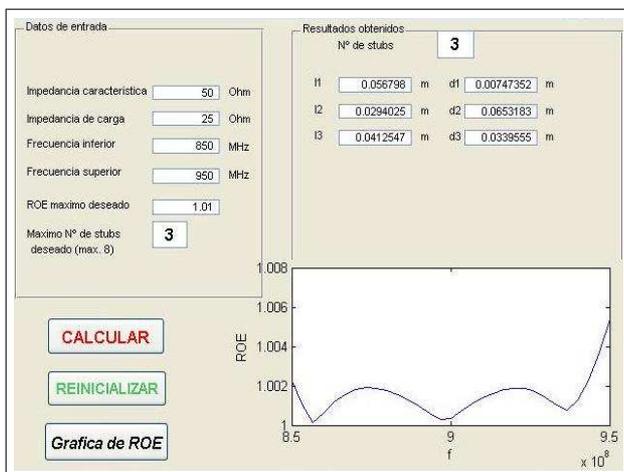


Figure 7: User Interface

The input data for the software are: load

impedance of the system, the interval of frequencies and the characteristic impedance of the system. Also the user has to put the maximum value of SWR allowed into the bandwidth as a tolerance margin, so the method convergence can be assured. It can also be chose the maximum number of stubs desired. The outputs are the longitudes and distances of the stub lines, and the quantity of stubs necessities to find the best solution. The user interface is shown in Figure 7. The function to be minimized is the quadratic error between  $\rho = |\Gamma|$  and the reference function, given in (18):

$$R = \Delta \cdot \left(\frac{2}{B}\right)^{2m} \cdot (f - f_0)^{2m} \quad (18)$$

where  $\Delta$  is adjusted in accordance with the maximum value of SWR allowed in the bandwidth [2]. The value of m adjusts the form of R to the desired form of SWR into the interval.

The equations used for  $\rho = |\Gamma|$  are the same explained in Section 2. The only difference is that the  $\Gamma_c$ , for each stub is the result of the  $\Gamma$  obtained at the connection point of the previous stub and so on.

The Nelder-Mead method can converge a local optimum, depending on the initial conditions. In order to assure the convergence to the global optimum, is very important an adequate selection of the initial conditions. For this reason a subprogram was made to calculate the best initial conditions. After that, the main program calculates the number of stubs, their longitudes and locations along the transmission line.

The program has a limit of eight possible stubs to be connected. Of course, if it is necessary this limit could be change . Once the user introduces the data, the program starts calculating the number of stubs that minimizes the quadratic error between  $\rho$  and the reference function. If the condition of maximum value of SWR introduced by the user is not satisfied, the program shows a message suggesting the change of this condition or the bandwidth.

Once the program finds the solution, the user can also see the SWR curve at the point of the last stub connection.

### 5 Results obtained

The program was validated for different examples, and the solutions obtained were always satisfactory. An example is presented here, for the following data:

- o Interval of frequencies: 1.1GHz to 1.3GHz
- o  $Z_0 = 50\Omega$
- o  $Z_c = 150 - j60\Omega$
- o maximum SWR= 1.15

stub	l (mm)	d (mm)
1	24.5371	39.9823
2	63.3895	38.8459
3	65.3817	5.83867
4	61.4128	4.07737
5	60.2661	65.0554
6	60.369	95.5695
7	64.2648	40.3593

Table 1: Results obtained with the program

The results obtained are shown in Table 1.

Each stub distance is referred to the previous stub connection point, except for the first stub, which reference point is the load, as shown in Figure 8.

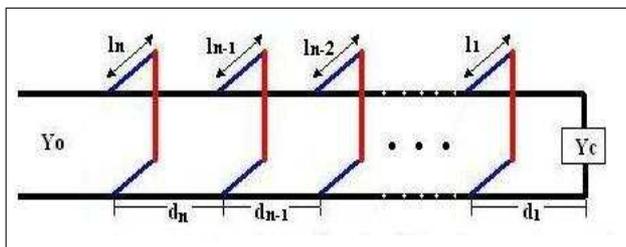


Figure 8: SWR curve for 7 stubs

Finally, the SWR obtained at the seventh stub connection point is shown in Figure 9. It is obvious that the condition of maximum SWR desired is satisfied in this case.

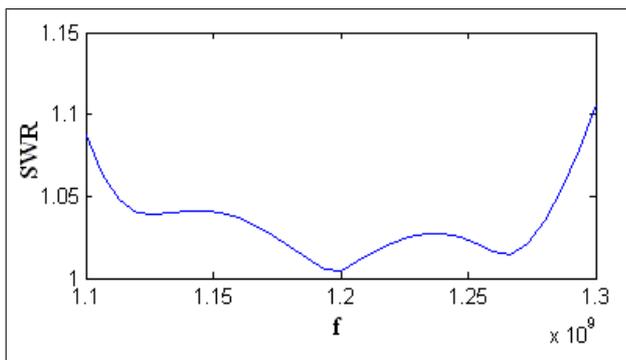


Figure 9: SWR curve for 7 stubs

## 6 Conclusion

Telecommunications systems, day by day, need a greater bandwidth, so more information can be transmitted, but it is necessary to guarantee a good quality of the system. In order to reach this, is very important

to achieve the system impedance matching. The problem with the classic impedance matching methods is that they only solve the problem at one frequency or in a very narrow band.

The software developed in this work is very useful, because it can solve the problem for an impedance matching into the bandwidth required for a specific application. This way is easier to guarantee the quality of the system, because the software finds a very good solution for a specific problem.

It is very important to emphasize that if the bandwidth is very broad, or the maximum value of SWR desired is very small, the software might not found a solution. In this case, the program shows a message and suggests the user to change one of those attributes.

Finally it would be very interesting to design and simulate a system, to check the results obtained. It could also help making a hardware to verify the results by metering the SWR.

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