Interaction of External Disturbances with Boundary Layer

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Abstract. Longitudinal structures generated by external vortical and thermal waves in subsonic and supersonic boundary layers are studied in the paper. Particular attention is paid to the boundary conditions at the boundary layer outer edge. It was established that longitudinal velocity and mass flow disturbances inside the boundary layer can exceed the amplitude of external vortical wave in several times. Excitation efficiency decreases with increasing Mach number. Influence of thermal external waves on the flow structure in the boundary layer is much weaker.

Key words: Mach number, turbulence, supersonic boundary layers, disturbances, waves.

1. Introduction

Now at research of a problem of originating of turbulence in a boundary layer the special place is allocate to a problem about originating disturbances in a boundary layer by external waves. Morkovin was the first who has paid attention to this phenomenon, which one become to call as a problem of a receptivity of a boundary layer [1]. There is a lot of experimental and theoretical activities on analysis of a subsonic boundary layer. The detail review of these activities can be found in [2,3]. Much less papers are dedicated to a case of a supersonic boundary layer. Mainly, the interaction of external acoustic waves with a supersonic boundary layer was studied [4-6]. At the same time, at supersonic flow together with acoustic (or vortex-free) waves there are vortical and thermal ones. In [7] the interaction of hydrodynamic waves with a supersonic boundary layer was investigated. In it the vortex-free external disturbances, damping in streamwise direction are considered, basically. Even in case of subsonic speeds there are few papers, in which the interaction of vortical disturbances with a boundary layer was studied theoretically. This paper is dedicated to research to the excitation of disturbances in subsonic and supersonic boundary layers by external vortical and thermal waves.

2. Formulation and basic equations.

The linear statement is considered. The flow of a compressible gas in a boundary layer on a flat plate is taken as an initial undisturbed flow. Disturbances in a boundary layer we shall consider in orthogonal coordinate system (ξ,ψ,z) [9,10] connected with stream-surfaces of basic flow and look like $\tilde{a}(\xi,\psi)\exp(i\alpha\xi+i\beta z-i\omega t)$. Here ψ - flow function; for a plate $\xi=x+O(\mathrm{Re}^{-2})$; $\mathrm{Re}=\sqrt{u_\infty x/\nu_\infty}$; u_∞,ν_∞ - speed and kinematical viscosity of a ram airflow; x,y,z - longitudinal, normal to a wall and transversal co-ordinates of the Cartesian system with the beginning on an edge of a plate. Gas is perfect with a constant Prandtl number Pr. Resulting a set of Navier-Stokes equations to a linear view, using estimations on the whole degrees of a Reynolds number Re, rejecting the members order Re^{-2} respect to the main ones, the properties of a critical layer [9,10]

$$\partial_2\tilde{v}=-(\partial_2\ln\rho)\tilde{v}-\left[i\alpha-(\partial_1\ln u)+\partial_1\right]\tilde{u}-i\beta\tilde{w}-u_c\tilde{\rho}/\rho-g_m u\partial_1\tilde{p}+u\partial_1(\tilde{T}/T),$$

$$\partial_2\left[\tilde{p}+2\mu\left(i\alpha\tilde{u}+i\beta\tilde{w}-2\tilde{e}_0/3\right)\right]=-\rho(h_1u+d_t)\tilde{v}+i\alpha\tilde{\tau}_{12}+i\beta\tilde{\tau}_{23},$$

$$\partial_2\tilde{\tau}_{12}=(i\alpha+\partial_1)\tilde{p}+\rho(\partial_2u)\tilde{v}+\rho(\partial_1u+d_t)\tilde{u}+u(\partial_1u)\tilde{\rho}-i\alpha\tilde{\tau}_{11}-i\beta\tilde{\tau}_{13},$$

$$\partial_2\tilde{u}=-(i\alpha+\partial_1)\tilde{v}-(\partial_2u)\tilde{\mu}/\mu+\tilde{\tau}_{12}/\mu,$$

$$\partial_2\tilde{\tau}_{23}=i\beta\tilde{p}+\rho d_t\tilde{w}-i\alpha\tilde{\tau}_{13}-i\beta\tilde{\tau}_{33}, \quad (1)$$

$$\partial_2\tilde{w}=-i\beta\tilde{v}+\tilde{\tau}_{23}/\mu,$$

$$\partial_2\tilde{q} = i\omega\tilde{p} + \left[\rho\left(\partial_2 H\right) - i\alpha\mu\left(\partial_2 u\right)\right]\tilde{v} + \left(\partial_1 H\right)\left(\rho\tilde{u} + u\tilde{\rho}\right) + \left(\alpha^2 + \beta^2\right)\mu\tilde{h}/\mathrm{Pr} + \rho d_t\tilde{H} - u\left(i\alpha\tilde{\tau}_{11} + i\beta\tilde{\tau}_{13}\right),$$

$$\partial_2\tilde{h} = -\mathrm{Pr}\left(\partial_2 u\right)\tilde{u} - \left(\partial_2 h\right)\tilde{\mu}/\mu + \mathrm{Pr}\left(\tilde{q} - u\tilde{\tau}_{12}\right)/\mu,$$

where: $\partial_1 = \partial/\partial\xi$; $\partial_2 = \rho u\,\partial/\partial\psi$; $d_t = u_c + u\partial_1$; $u_c = i\alpha u - i\omega$; $h_1 = -\partial_1\ln(\rho u)$; $\tilde{\tau}_{11} = 2\mu(i\alpha\tilde{u} - \tilde{e}_0/3)$; $\tilde{\tau}_{13} = \mu(i\alpha\tilde{w} + i\beta\tilde{u})$; $\tilde{\tau}_{33} = 2\mu(i\beta\tilde{w} - \tilde{e}_0/3)$; $\tilde{e}_0 = -(\partial_2\ln\rho)\tilde{v} - u_c\tilde{\rho}/\rho$; u –velocity; T – temperature; ρ - density; p – pressure; $H = h + u^2/2$ - full enthalpy; μ - viscosity; $\tilde{v}$, $\tilde{w}$ - complex amplitudes normal to a surface and transversal components of velocity disturbances; $\tilde{\rho}/\rho = g_m\tilde{p} - \tilde{T}/T$; $\tilde{T} = g_{m1}\tilde{h}$; $\tilde{H} = \tilde{h} + u\tilde{u}$; $g_m = 1/p$; $g_{m1} = 1/c_p$; c_p - specific heat of gas at constant pressure. The view of equations will not change after normalizing with the help of following scales: ν_∞/u_∞ - length, ν_∞/u_∞^2 - time , μ_∞ - viscosity and flow function, u_∞ - velocity and its disturbances, T_∞ - temperature, ρ_∞ - density , u_∞^2 - enthalpy, $\rho_\infty u_\infty^2$ - pressure and disturbances of viscous stresses, $\rho_\infty u_\infty^3$ - value $\tilde{q}$, u_∞^2/T_∞ - specific heat (the index ∞ corresponds to values in the ram airflow). In this case: $g_m = \gamma\mathrm{M}^2$, $g_{m1} = (\gamma - 1)\mathrm{M}^2$, where $\gamma = c_p/c_V$ - relation of heat capacities; M – Mach number.

Entering independent variables $\mathrm{Re} = \sqrt{\xi}$, $d\eta = d\psi/u\,\mathrm{Re}$ and using notations: $\partial_1\tilde{a} = (1/\mathrm{Re})(\partial\tilde{a} + f_1\tilde{a}')$, $\partial_2\tilde{a} = \rho\tilde{a}'/\mathrm{Re}$ (where $\partial = 0{,}5\,\partial/\partial\mathrm{Re}$; the stroke means a derivative on η , $f_1 = -\psi/\left(2\,\mathrm{Re}^2 u\right)$) equations, founded on estimations of a critical layer, are adduced to a view:

$$\tilde{v}' = -g_m uT\partial\tilde{p} + \rho T'\tilde{v} - T\left(f_0 u' + \partial\right)\tilde{u} - \tilde{u}_w - i_c T\tilde{r} - \left(f_2\rho T' - u\partial\right)\tilde{T} - f_1 T\tilde{u}' + f_2\tilde{T}',$$

$$\tilde{p}' = -\left(i_c + r_h u\right)\tilde{v} + i_x\tilde{\tau}_{12} + i_z\tilde{\tau}_{23} - 2\mu_r\tilde{u}_w',$$

$$\tilde{\tau}_{12}' = \left(i_x + T\partial\right)\tilde{p} + \left(i_c + f_1 u' + u\partial\right)\tilde{u} + f_2 u'\tilde{r} - \tilde{i}_T + f_2\tilde{u}' + \rho u'\tilde{v}$$

$$\tilde{u}' = -i_x\tilde{v} - u'\mu_t\tilde{T} + \tilde{\tau}_{12}/\mu_r,$$

$$\tilde{\tau}_{23}' = i_z\tilde{p} + \left(i_c - \mu_a + u\partial\right)\tilde{w} - i_z\mu_r\tilde{u}_w + f_2\tilde{w}',$$

$$\tilde{w}' = -i_z\tilde{v} + \tilde{\tau}_{23}/\mu_r, \qquad (2)$$

$$\tilde{q}' = i\omega RT\,\tilde{p} + \rho H'\tilde{v} + f_2 H'\tilde{r} - u\tilde{i}_t + \left(i_c u + f_1 H' + f_2 u' + u^2\partial\right)\tilde{u} + f_2\tilde{h}' + f_2 u\tilde{u}' + \left(i_c - \mu_a/\mathrm{Pr} + u\partial\right)\tilde{h},$$

$$\tilde{h}' = -\mathrm{Pr}\,u'\tilde{u} - h'\mu_t\tilde{T} + \mathrm{Pr}\left(\tilde{q} - u\tilde{\tau}_{12}\right)/\mu_R,$$

where $\tilde{u}_w = i_x\tilde{u} + i_z\tilde{w}$;

$$\tilde{i}_t = i_x\mu_r\tilde{u}_w + \mu_a\tilde{u};\ \mu_a = \left(i_x^2 + i_z^2\right)\mu_r;$$

$$\tilde{p} = \tilde{\pi} - 2\mu\left(i\alpha\tilde{u} + i\beta\tilde{w} - 2\tilde{e}_0/3\right);$$

$$\tilde{r} = \tilde{\rho}/\rho = g_m\tilde{p} - \rho\tilde{T};$$

$$i_c = \mathrm{Re}\,u_c = i\,\mathrm{Re}(u\alpha - \omega);\ i_x = i\alpha\,\mathrm{Re}\,T;$$

$$i_z = i\beta\,\mathrm{Re}\,T;\ r_h = \mathrm{Re}\,h_1 = f_0 u' + f_1\rho T';$$

$$f_0 = -f_1/u;\ f_2 = f_1 u;\ \mu_r = \mu\rho/\mathrm{Re};$$

$$\mu_T = d\ln\mu/dT.$$

Equations (2) have the following structure:

$$\mathbf{Z}' = \left(A + D\partial\right)\mathbf{Z}.$$

Here prime stands for derivative with respect to η, $d\eta = (d\psi/u)/\mathrm{Re}$, $\mathrm{Re} = \sqrt{\xi}$, $\partial = 0{,}5\,\partial/\partial\mathrm{Re}$, $\mathbf{Z} = \left(\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}\right)$, A, D—quadratic matrixes of given functions of Re, η. $\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}$ are amplitudes of pressure; normal to a surface of a plate, longitudinal and transversal speeds; enthalpy disturbances. The expressions for $\tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}$ can be found in the indicated papers [7,9,10]. Reduced above a parabolized set of equations is solved at for the flowing boundary conditions. The disturbances of speeds and temperature on a surface are equals to zero, $\tilde{v}(0) = \tilde{u}(0 = \tilde{w}(0) = \tilde{T}(0) = 0$. Outside of a boundary layer the disturbances are determined by the correspondent values in free (in a model absence) flow.

3. Numerical scheme and boundary conditions.

Using approximating $\partial\tilde{a}/\partial R \approx (\tilde{a} - \tilde{a}_0)/\Delta R$ ($\Delta R = R - R_0$ - step of the marching scheme, the index 0 here and below correspond to the previous step) a parabolized set of equations will be converted to a system of ordinary differential equations: $Z' = AZ + B(Z - Z_0)$. The common solution of a system is constructed as follows. At the boundary layer edge four solutions ensure damping of disturbances outside of a boundary layer are selected. Inside of a boundary layer they are satisfied to a system of homogeneous equations. The fifth solution is agreed with the external wave, and inside a boundary layer it is satisfied to an inhomogeneous set of equations. The common solution is constructed as superposition, $Z = \sum_{m=1}^{4} C_m(x) Z_m + Z_5$, which factors are determined on the base of the boundary conditions on the plate.

If to consider propagated downstream disturbances on homogeneous mean flow behind a grid, naturally to introduce a wave in view $\exp[iky + i\beta z + i\alpha x - i\omega t]$, where k, β, ω - real. As it is established in [10], for vortical and thermal waves the values α can be determined from a ratio $i(\omega - \alpha) = \alpha^2 + \beta^2 + k^2$, $i(\omega - \alpha) = \alpha^2 + \beta^2 + k^2$ accordingly. Numerous experiments and the analytical investigations at subsonic speeds demonstrate, that under the influence of external turbulence in boundary layer the longitudinal structures develop. It means, what the greatest concern is introduced by low frequency disturbances with a longitudinal vorticity ($\tilde{u} = 0$) provided that $|\alpha/\beta| << 1$. Using [7] and mentioned above, conforming partial solution in a homogeneous flow: $Z_5^2 = (0, -i\beta, 0, ik, 0, 0, -k^2 + \beta^2, 0)$. Vector, basically, thermal disturbances: $Z_5^3 = (0, ikB_1, 0, i\beta B_1, \mathrm{Pr}, 0, -2\beta k B_1, ik)$. The vector of vortex-free disturbances looks like: $Z_5^4 = (0, \lambda_4 A_1, 0, i\beta A_1, 0, 0, -2i\beta\lambda_4 A_1, 0)$ (increasing at $\lambda_4=\beta$, fading at $\lambda_4=-\beta$). Respect to [7] $B_1 = (\gamma-1)M^2$, $A_1 \approx M^2$. A case vortex-free disturbance is studied in [7] explicitly. Therefore here external vortical and thermal disturbances will be reviewed.

The indispensable solutions of a homogeneous set of equations on the edge of a boundary layer we shall receive from an asymptotics of locally - parallel approaching at coordinate $\eta \to \infty$.

In a homogeneous flow $u' = T' = 0$, $u = T = 1$. Therefore at $\eta = \delta$ there are four fading basic solutions at $\eta = \delta$:

$$Z_1 = (0, 0, -ik, 0, 0, -k^2, 0, -k^2).$$

$$Z_2 = (0, -i\beta, 0, -ik, 0, 0, -k^2 + \beta^2, 0)$$

$$Z_3 = (0, -ikB_1, 0, i\beta B_1, \mathrm{Pr}, 0, -2\beta k B_1, -ik)$$

$$Z_4 = (0, 1, 0, -1, 0, 2i\beta, 0, 1)$$

4. Results.

The calculations were conducted for a boundary layer on a flat plate for Mach numbers M=0 and 2.0, frequency $\omega= 10^{-6}$. The adopted frequency rate satisfies to steady conditions. Viscosity-temperature relation, adopted in calculations, was determined by the Sutherland; formula, Prandtl number Pr=0.72.

The results obtained for external vortical disturbances, were set norms on value of amplitude of speed u= $(\tilde{v}^2 + \tilde{w}^2)^{1/2}$ nearly by to position of a grid. Parameters of a problem were α_i , Re, x_0, where α_i –damping intensity of external disturbances along longitudinal coordinate x_0 - dimensionless spacing interval from a grid up to a leading edge of a plate, Re = $(x)^{1/2}$, and x- dimensionless spacing interval from a leading edge of a plate. The values x_0 is oriented on activities [11.12], in which the grid was placed apart 1.6 m/sec and 1 m/sec. The maximum approach stream velocity was equal to 12 m/sec. and minimum – 2 m/sec. Thus, $0.80\cdot10^5 \le x_0 \le 1.28\cdot10^6$. At a set value α_i wave numbers β and k in z и y-directions were taken real, satisfied to $\alpha_i = \beta^2 + k^2$ for vortical disturbances and $\alpha_i = (\beta^2 + k^2)/\mathrm{Pr}$ for thermal waves. All calculations are conducted for a boundary layer thickness $\delta= \eta_1=6$.

Let's discuss, first of all, results obtained for a Mach number M=0.

In a Fig.1 the distributions of the pressure (Ap), speeds (Av, Au, Aw) and enthalpy (Ah) (proportional disturbance of temperature) disturbances amplitudes are shown at Re = 760, α_i

$=10^{-8}$, $\beta= 3\cdot10^{-4}$. Let's remark, that the view of distribution of a longitudinal speed disturbance is conservative to change of problem parameters, after normalization on maximum rating it is resulted practically to the view, which is coincided with obtained dependences in other papers, including [8].

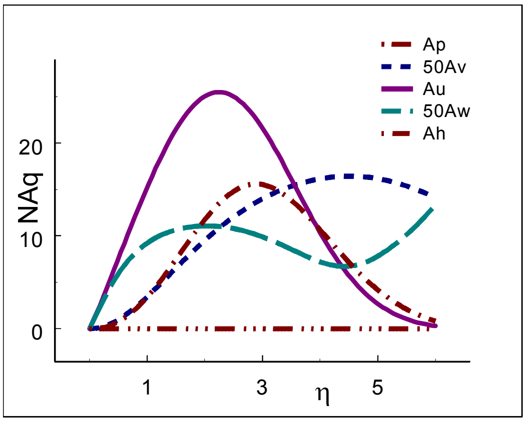

Fig.1.

In Fig. 2 the comparison our results with data of[8,13] is shown. ($U_{max}= |\tilde{u}|_{max}$). The main results were obtained for $k=\beta/3$: The line 1 - data [8], lines 2,3 - our results (obtained in a locally-parallel approaching and on the basis of parabolized equations accordingly), the line 4 is obtained on the basis of analytical expression of the paper[13]. The line 5 - data of the given paper at $k=\beta$, conforming to maximum values U_{max} at changing β. The checkmark ♥ corresponds to experimental value of the paper [14], the checkmark ◆ to [12]. Normalization in [8] differs from ours on value $\sqrt{2}A_{v,}$ where A_v corresponds under the order of values to amplitude

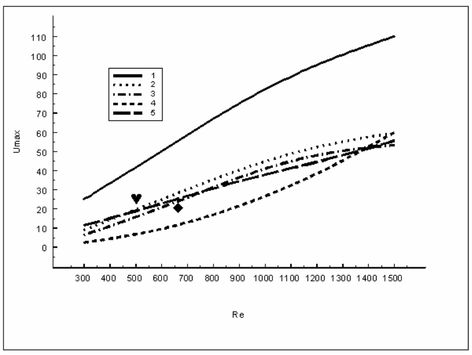

Fig.2

of disturbances of an external flow. So, in order to result in conformity data of [8] to ours they were divided by $\sqrt{2}A_v$.

The comparison of our data with results [8] for two values of a Reynold's number (Re=500: lines 1,2; Re=1000: lines 3,4) is given in Fig.3. Lines 1,3 — data [8], line 2,4 — results of the present paper, the experimental value β of the papers[12,14] is marked by the checkmark ♥.

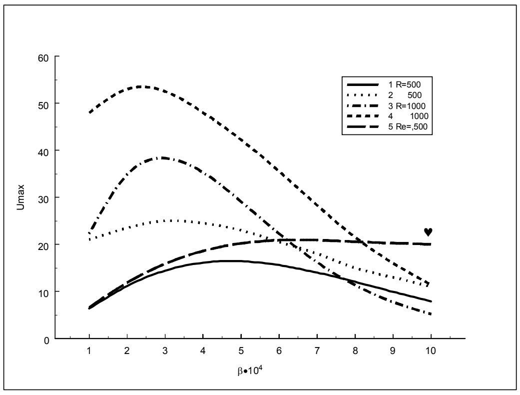

Fig.3

In Fig.4 the values $U_{max}= |\tilde{u}|_{max}$ in depending on Reynolds number at $\alpha_i=10^{-6}$, $b=\beta\cdot10^4 =2.0$; 4.0; 6.0; 8.0; 10 for $x_0= 6.4\cdot10^5$ are shown. In Fig.5 the dependences U_{max} on the wave vector β are adduced for Re=600, $-\alpha_i=10^{-6}$ and different values x_0. It is visible, what with reduction x_0 value $|\tilde{u}|_{max}$ increases. It is augmented because of low damping of external disturbances on more short spacing intervals x_0. Incidentally, notably some displacement of a maximum of relation in the party of large values β. However this displacement not strong, and β_{max},

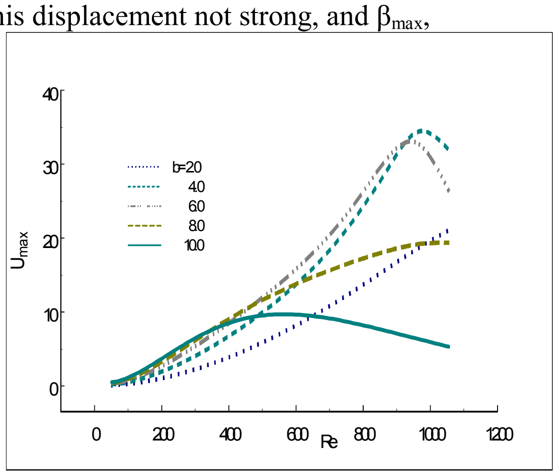

Fig.4

conforming to maximum ratings $|\tilde{u}|_{max}$, is equal approximately to $0.7\cdot10^{-3}$ and it is agreed with he data obtained in [12].

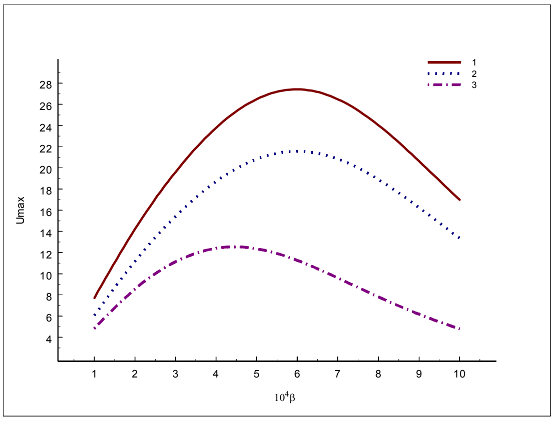

Fig.5

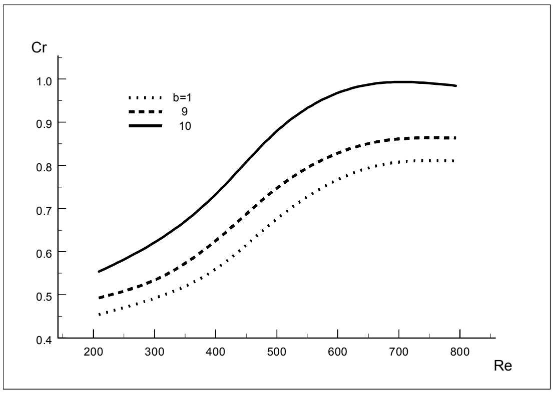

Fig.6

The dependence of a phase velocity Cr on Reynolds number is adduced in Fig. 6. The data are obtained at M=0.0, $-\alpha_i=10^{-6}$, $\omega=10^{-5}$, $x_0=.64\cdot10^6$ and three values β. The small change of Cr from a wave number β is visible. On the other hand, Cr essentially depends on a Reynolds number and varies within the limits 0.5<Cr < 1.0 at change Re from 250 up to 700. It is interesting to address to experiments (see [3]) on the turbulent spots originating. It was established there, that the leading front of a spot located in the field of large numbers Re, it propagated with speed 0.9, while back one— with speed 0.5. These results are in good, at least, qualitative conformity with our data.

Let's proceed to the data for a case of supersonic speeds. Our results demonstrate, that relation U_{max} from a Reynolds number at M = 2.0 (for vortical external disturbances) is similar to the case of Mach number M=0. The low efficiency of the excitation of the longitudinal speed

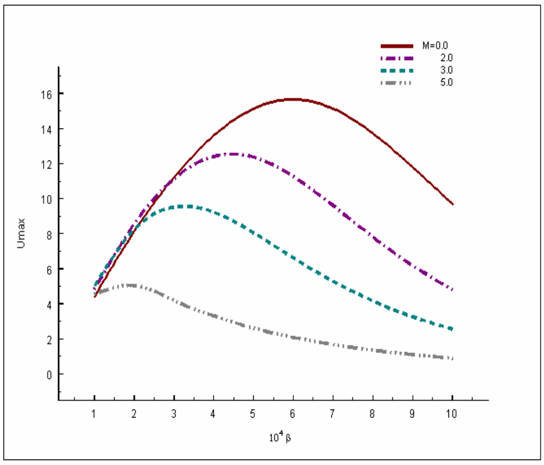

Fig.7

disturbances inside a boundary layer in comparison with a case of subsonic speeds is watched, that is agreed with the conclusion of paper [9]. By calculations it is established, that the relation U_{max} from a Mach number is monotonically decreasing. This concluding is demonstrated in Fig.7 for Re=600, $-\alpha_i=10^{-6}$, $x_0= 6.4\cdot10^5$.

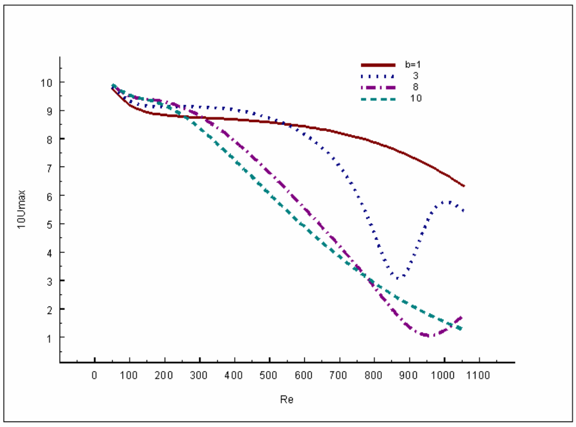

Fig.8

At last, in Fig.8 the dependence U_{max} (for a case of thermal external disturbances) on Reynold's number is resulted at $-\alpha_i=10^{-6}$ and different values of a wave number β for $x_0= 6.4\cdot10^5$. Idiosyncrasy of these relations is the strong decreasing of longitudinally speeds disturbances amplitude, at least, in area Re < 800. At large values of a Reynolds number the increase of disturbances inside a boundary layer can be seen.

Conclusions

Thus, the conducted researchers shown, that the external vortical wave can excite longitudinal disturbances inside a boundary layer. Their intensity depends, basically, on position of a grid, spectrum of disturbances on wave numbers, Mach number. At these given parameters there are reference values of wave numbers $\beta = \beta^*$, at which one the amplitude of a longitudinal speed disturbance inside a boundary layer is maximum. It explains the appearance of longitudinal structions, observed in experiments, with reference periodicity in a lateral direction. The phase velocity of disturbances at value of normal coordinate conforming to a maximum of a longitudinal speed amplitude varies within the limits from 0.5 at Re=250 up to 0.95 at Re=700. With increase of a Mach number the intensity of longitudinal speed disturbance inside a boundary layer exited by external vortical wave decreases. Apparently, the efficiency of flow deformation inside a boundary layer by external thermal waves is lower in comparison with a case of vortical ones. The results of the present paper agreed satisfactorily with the experiments and qualitatively with theoretical paper [13,15], but they differ quantitatively from data of Bertolotti, [8]. Reasons of this difference remain are unknown.

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