Creeping Flow Past an Elliptical Cylinder in the Presence of a Vortex

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Abstract: In this paper the flow generated by a vortex in the presence of an elliptical cylinder is considered. By relating the coefficients of some of the terms in the asymptotic expansion of the stream function to the force components and the torque on the combined system, together with the imposition of integral constraints, enable the boundary element method to provide a closed system of equations. Our numerical results agree with all the available analytical ones.

Key–Words: boundary element method, creeping flow, elliptical cylinder, vortex, torque.

1 Introduction

The solution of problems involving application of point forces and torques are of considerable interest in continuum mechanics. These solutions can be used in the representation of solutions of more complicated and physically realizable problems see Bedford[1] and Kim[2].

There are many known solutions of problems of the two-dimensional motion of an infinite viscous fluid disturbed by a moving solid body. Wilton[3] incorporates, in the case of the elliptic cylinder, infinite vorticity and indeterminate velocities at the ends of the axes.

Swain[4], first treats the problem as a limiting case of the motion of an ellipsoid through infinite viscous fluid. His solution fulfills all the boundary conditions, including the velocity zero at infinity. This gives, in the limiting case of the elliptic cylinder, a solution which involves the velocity being logarithmically infinite in the direction of the flow.

Subject to this condition, the solution appears to be unique. It has been obtained as a definite value for the resistance and then treated the circular cylinder as a limiting case of the elliptic cylinder.

The problem of creeping flow past elliptical cylinder in the presence of a vortex has been solved analytically in El Bashir[5], where a vortex of strength Γ is placed at a distance C from a cylinder whose boundary is $\xi = \alpha$. Here

$$C = a\cosh(\gamma), \qquad \gamma > \alpha. \tag{1}$$

The transformation from cartesian to elliptical coordinates is defined by

$$x = a\cosh(\xi)\cos(\eta), \tag{2}$$

$$y = a\sinh(\xi)\sin(\eta), \tag{3}$$

and the dimensionless variables x', y', c and β

$$x' = x/a, y' = y/a, c = C/a, \beta = \Gamma a/C, \tag{4}$$

are introduced.

For convenience the " $'$ " will be removed and the position of the vortex at $(c, 0)$ and the major and minor axes of the ellipse are fixed at $5/4$ and $3/4$, respectively (see Fig. 1).

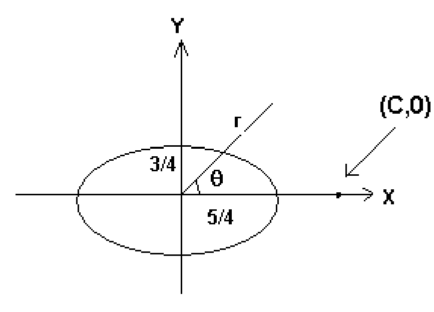

Fig. 1: The geometry of the elliptical cylinder and the position of the vortex.

Following this approach we need to solve the biharmonic equation

$$\nabla^4\psi = 0, \tag{5}$$

for the stream function $\psi(\xi,\eta)$, where $\xi \geq \alpha$ and

$$\nabla^2 = \frac{1}{h^2}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right). \tag{6}$$

Here

$$h = (\cosh^2(\xi)\sin^2(\eta) + \sinh^2(\xi)\cos^2(\eta))^{1/2}, \tag{7}$$

and (ξ,η) are elliptic coordinates.

Before embarking on a numerical solution of this problem, it is important to discuss the analytical solution obtained by El Bashir[5]. It is found that the Fourier series representation for the vortex term

$$\psi_0 = \frac{1}{2}\ln((x-c)^2 + y^2), \tag{8}$$

is given by

$$\psi_0 = -\sum_{n=1}^{\infty}\frac{e^{-n\gamma}}{n}(e^{-n\xi} + e^{n\xi})\cos(\eta) + (\xi_0 - \ln(2)) \texttt{ for } \xi < \xi_0. \tag{9}$$

Then put the complete solution as $\psi = \psi_0 + \psi_1$ where ψ_1 is the analytic function outside the ellipse. Using the general solution of $\nabla^4\psi = 0$ as $\phi_0 + (x^2+y^2)\phi_1$ where ϕ_0 and ϕ_1 are harmonic functions, we can write

$$\psi_1 = \frac{1}{2}C_0 + d_0 r^2 + \sum_{n=1}^{\infty}(C_n e^{-n\xi} + 2d_n(\sinh 2\xi + e^{-n\xi}\cos 2\eta)\cos n\eta). \tag{10}$$

The solution of $\nabla^4\psi = 0$ can be found by adding the general Fourier Series term $\cos(n\eta)$ and $\sin(n\eta)$ to $\psi_0(\xi,\eta)$. This is the solution for the vortex in the absence of the ellipse, by finding the coefficients in (10) to satisfy the no-slip conditions, and by summing the series, i.e.

$$\begin{aligned}\psi &= \frac{1}{2}\ln[(\cosh(\xi)\cos\eta - \cosh(\gamma)\cos\phi)^2 \\ &+ (\sinh(\xi)\sin\eta - \sinh(\gamma)\sin\phi)^2] \\ &- \frac{1}{2}\ln[(\cosh(\alpha)\cos\eta - \cosh(\xi+\gamma-\alpha)\cos\phi)^2 \\ &+ (\sinh(\alpha)\sin\eta - \sinh(\xi+\gamma-\alpha)\sin\phi)^2] \\ &+ (\xi-\alpha) + \sinh(\xi-\alpha)\sin(\eta) I_m[\\ &\frac{sinh(\xi+\delta-\gamma)}{sinh(\delta)(cosh(\xi+\delta-2\gamma) - cos(\eta))}] \\ &+ \sinh(\xi-\alpha)R_e[\frac{cosh(\xi+\alpha)e^{-\delta}}{sinh(\delta)cosh(2\alpha)} \\ &- \frac{cosh(\alpha) - cosh(\xi+\delta-\alpha)cos(\eta)}{sinh(\delta)(cosh(\xi+\delta-2\gamma) - cos(\eta))}],\end{aligned} \tag{11}$$

where

$\delta = \alpha - i\phi,\ i = \sqrt{-1},\ I_m$ and R_e are the imaginary and real parts in (11), respectively. At large distances from the ellipse (11) can be written in the form

$$\psi \simeq Ar^2, \tag{12}$$

where

$$A = \frac{cos(2\phi) - e^{-2\gamma}}{cosh(2\alpha)[cosh(2\gamma) - cos(2\phi)]}. \tag{13}$$

Hence a solid body rotation is induced at large distances from the ellipse except at $\phi = \cos^{-1}(e^{-2\gamma})/2$ which gives a uniform flow when a vortex at this point is placed see Fig. 2. In fact, it is possible to obtain uniform flow at large distances if the vortex is placed at any one of four points due to the symmetry of the problem.

To simplify the analytical solution in order to compare it with the numerical solution we put it in the form

$$\Psi = \psi_B + \psi^*, \tag{14}$$

where

$$\begin{aligned}\psi_B &= b_1 r\cos(\vartheta) + b_2 r\sin(\vartheta) + b_3 + b_4\sin(2\vartheta) \\ &+ b_5\cos(2\vartheta) + b_6\ln(r) + b_7 r^2 + b_8 r\ln(r)\cos(\vartheta) \\ &+ b_9 r\ln(r)\sin(\vartheta) + \ln(r^2 + c^2 - 2rc\cos(\vartheta)) \\ &+ b_{10}\cos(\vartheta)/r + b_{11}\sin(\vartheta)/r + b_{12}\cos(3\vartheta)/r \\ &+ b_{13}\sin(3\vartheta)/r,\end{aligned} \tag{15}$$

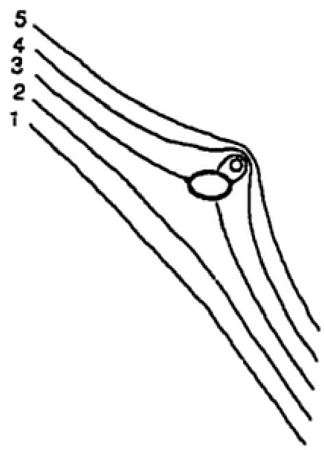

Fig. 2: The uniform flow obtained when a vortex at the position (2.236,0.758). The streamlines labeled $1,\ 2,\ 3,\ 4$ and 5, correspond to $\Psi = -1.0,\ -0.5,\ 0.0,\ 0.5$ and 1.0, respectively.

and ψ_B is the asymptotic expansion of ψ and ψ^* is the perturbation value about this expansion that tends to zero as $r \to \infty$. Then, if we use sufficiently large r, say, $r \geq 30$ and we use (11), we find the value of ψ. By repeating this 13 times we can get 13 equations with 13 unknowns $b_j,\ j = 1, 2, \ldots, 13$. By using the required Gauss elimination method we find the values of b_j as

$$\begin{aligned} &b_1 = 0.500,\ b_2 = 0.000,\ b_3 = -1.092,\ b_4 = 0.000,\\ &b_5 = 0.208,\ b_6 = 0.000,\ b_7 = 0.056,\ b_8 = 0.000,\\ &b_9 = 0.000,\ b_{10} = -0.015,\ b_{11} = -2.585,\\ &b_{12} = 0.105,\ b_{13} = -0.002. \end{aligned} \tag{16}$$

Equation (11) can be transformed to the solution obtained by Dorrepaal[6] by replacing the variables $\xi+\alpha$ by ξ. The ellipse itself is given by $\xi = 0$ and by making $\alpha \to \infty,\ a \to 0$, so that acosh$\alpha \to A$, asinh$\alpha \to A$, where A is the radius of the cylinder see Swain[4].

2 Forces and Moment on the Ellipse

The components of the force (F_x, F_y) and the moment M acting on a volume V of a fluid which is enclosed within the surface S can be expressed as

$$F_x = \int_S (\sigma_{\xi\xi}\partial x/\partial\xi - \sigma_{\xi\eta}\partial y/\partial\xi) dS/h, \tag{17}$$

$$F_y = \int_S (\sigma_{\xi\xi}\partial y/\partial\xi + \sigma_{\xi\eta}\partial x/\partial\xi) dS/h, \tag{18}$$

$$\begin{aligned} M = \int_S [(&\sigma_{\xi\xi}\partial x/\partial\xi - \sigma_{\xi\eta}\partial y/\partial\xi) y \\ &- (\sigma_{\xi\xi}\partial y/\partial\xi + \sigma_{\xi\eta}\partial x/\partial\xi) x]\, dS/h, \end{aligned} \tag{19}$$

where x and y are the elliptic co-ordinates defined in (2) and (3), respectively, and

$$\sigma_{\xi\xi} = -p + 2\mu(\frac{\partial V_\xi}{\partial\xi}/h - V_\eta\frac{\partial(1/h)}{\partial\eta}), \tag{20}$$

$$\sigma_{\xi\eta} = \sigma_{\eta\xi} = \mu(\frac{\partial}{\partial\xi}\left[\frac{V_\eta}{h}\right] + \frac{\partial}{\partial\eta}\left[\frac{V_\xi}{h}\right]), \tag{21}$$

$$h = \left(\cosh^2(\xi)\sin^2(\eta) + \sinh^2(\xi)\cos^2(\eta)\right)^{1/2}, \tag{22}$$

Inserting these values of $\sigma_{\xi\xi},\ \sigma_{\xi\eta}$ into expressions (17), (18) and (19) and using the Stokes equations of motion, we obtain, after simplification,

$$F_x = \int_S \left[\frac{\partial\omega}{\partial n}\sinh\alpha\sin\eta - \omega\cosh\alpha\sin\eta/h\right] ds, \tag{23}$$

$$F_y = -\int_S \left[\frac{\partial\omega}{\partial n}\cosh\alpha\cos\eta - \omega\sinh\alpha\cos\eta/h\right] ds, \tag{24}$$

$$\begin{aligned} M = \int_S &\left[\frac{\partial\omega}{\partial n}\left\{\cosh^2\alpha\cos^2\eta + \sinh^2\alpha\sin^2\eta\right\}/2\right. \\ &+\omega\sinh\alpha\cosh\alpha/h]\, ds, \end{aligned} \tag{25}$$

where the vorticity $\omega = \nabla^2\psi$. The integrals, which occur in equations (23), (24) and (25) are next evaluated numerically using Simpson$'s$ rule.

3 The Governing Equations

For slow steady, two-dimensional flow of an incompressible Newtonian fluid the Navier-Stokes and continuity equations reduce to

$$\nabla p = \mu\nabla^2\mathbf{u}, \tag{26}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{27}$$

where the Reynolds number, $\rho(\Gamma/C)a/\mu$, is assumed to be very small. On introducing the stream function, ψ say, such that $\partial\psi/\partial x = -v_y$ and $\partial\psi/\partial y = v_x$, then ψ satisfies the biharmonic equation, see Batchelor[7],

$$\nabla^4\psi = 0. \tag{28}$$

Upon introducing the vorticity, ω, equation (28) may be written in the following form

$$\nabla^2\psi = \omega, \tag{29}$$

$$\nabla^2\omega = 0. \tag{30}$$

In order to solve (29) and (30) in the domain Ω we use the Boundary Element Method (BEM). For any $p = (x, y) \in \Omega \cup \partial\Omega$ and $q = (x_0, y_0) \in \partial\Omega$, let

$$f(p, q) = \ln \mid p - q \mid, \tag{31}$$

$$g(p, q) = \mid p - q \mid^2 (\ln \mid p - q \mid -1), \tag{32}$$

where $\mid p-q \mid = \{(x-x_0)^2+(y-y_0)^2\}^{1/2}$. Applying Green's second identity we obtain

$$\begin{aligned} &\eta(p)\psi(p) = \\ &\int_{\partial\Omega} \psi(q)f'(p, q)dq - \int_{\partial\Omega} \psi'(q)f(p, q)dq \\ &+ \frac{1}{4}\int_{\partial\Omega} \omega(q)g'(p, q)dq - \frac{1}{4}\int_{\partial\Omega} \omega'(q)g(p, q)dq, \end{aligned} \tag{33}$$

$$\begin{aligned} &\eta(p)\omega(p) = \\ &\int_{\partial\Omega} \omega(q)f'(p, q)dq - \int_{\partial\Omega} \omega'(q)f(p, q)dq. \end{aligned} \tag{34}$$

The boundary is first calculated by using Runge Kutta method and is found to be 6.386 which agrees with Weast[8], then it is subdivided into N segments, $\partial\Omega_j$, by solving the differential equation

$$d\eta/ds = 1/h. \tag{35}$$

The stream function, ψ, its derivative, ψ', the vorticity, ω, and its derivative, ω', are approximated by piecewise constant functions. This results in the following system of algebraic equations

$$\left.\begin{aligned} \sum_{j=1}^{N} E_{ij}\psi_j - G_{ij}\psi'_j + L_{ij}\omega_j - M_{ij}\omega'_j &= 0 \\ \sum_{j=1}^{N} E_{ij}\omega_j - G_{ij}\omega'_j &= 0 \\ i = 1, \ldots, \quad n, & \end{aligned}\right\} \tag{36}$$

where $G_{ij},\ E_{ij},\ L_{ij}$ and M_{ij} are given by

$$\begin{aligned} G_{ij} &= \int_{\partial\Omega_j} f(p_i, q)dq, \\ E_{ij} &= \int_{\partial\Omega_j} f'(p_i, q)dq - \eta(p_i)\delta_{ij}, \\ L_{ij} &= \frac{1}{4}\int_{\partial\Omega_j} g'(p_i, q)dq, \\ M_{ij} &= \frac{1}{4}\int_{\partial\Omega_j} g(p_i, q)dq, \end{aligned} \tag{37}$$

Equations (36) represents $2N$ equations in $4N$ unknowns. With the application of the appropriate conditions the system of equations (36) can be solved and then equations (33) and (34) are used to find the value of the stream function, ψ, at any point within the solution domain, Ω.

4 Numerical Solution

In the present work the fluid flow passing through an elliptical cylinder in the presence of a vortex is investigated. In the mathematical model the fluid flow is assumed to be two-dimensional, the ellipse has major and minor axes of lengths a and b, respectively, and the vortex is at a distance c from the center of the ellipse as shown in Fig. (1).

In order to solve numerically the Navier-Stokes equations in an exterior region, it is very advantageous to use the BEM because there is a simple fundamental solution, which enables one to convert the equations into integral equations. This involves only boundary integrals. Further, these integral equations are appropriate when dealing with the infinite boundary condition, see for example Jaswon[9] and Brebbia[10].

Here we use the BEM with constant elements. It is convenient to separate the stream function and the vorticity into two parts, namely,

$$\psi = \psi_A + \psi^*, \tag{38}$$

$$\omega = \omega_A + \omega^*, \tag{39}$$

where

$$\begin{aligned} \psi_A &= \lambda_1 r\cos(\vartheta) + \lambda_2 r\sin(\vartheta) + \lambda_3 + \lambda_4\sin(2\vartheta) \\ &+ \lambda_5\cos(2\vartheta) + \lambda_6\ln(r) + \lambda_7 r^2 + \lambda_8 r\ln(r)\cos(\vartheta) \\ &+ \lambda_9 r\ln(r)\sin(\vartheta) + \frac{\beta}{2}\ln(r^2 + c^2 - 2rc\cos(\vartheta)), \end{aligned} \tag{40}$$

$$\begin{aligned} \omega_A &= -4\lambda_4\sin(2\vartheta)/r^2 - 4\lambda_5\cos(2\vartheta)/r^2 - 4_7\lambda \\ &- 2\lambda_8\cos(\vartheta)/r - 2\lambda_9\sin(\vartheta)/r. \end{aligned} \tag{41}$$

ψ_A and ω_A are the asymptotic expansions of ψ and ω as $r \to \infty$, and ψ^* and ω^* are the perturbations values about these expansions that tend to zero as $r \to \infty$,

Considering the surface of the ellipse as the streamline $\psi = 0$, then ψ^* and ω^* satisfies

$$\nabla^2\psi^* = \omega^*, \tag{42}$$

$$\nabla^2\omega^* = 0, \tag{43}$$

with

$$\psi^* = -\psi_A \text{ and } \psi^{*\prime} = -\psi_A' \text{ on the boundary}. \quad (44)$$

Equations (42) and (43) result in $2N$ equations, and the number of unknowns is $2N+9$ which means that we need another 9 conditions to close the system. The extra unknowns λ_j, where $j = 1, \ldots, 9$, will require extra conditions:

Case (a)

Let us assume for the moment that the values of the drag, lift and moment are all zeros. Using the expressions (23), (24) and (25) where we have replaced ω by $\omega_A + \omega^*$, the parts of the integrals involving ω_A were evaluated analytically producing linear expressions in the unknowns λ_j. The other parts involving ω^* are expressed after numerical integrations using Simpson 's method as linear expressions in ω_j^* and $\omega_j^{*\prime}$. The resulting expressions for the drag, lift and moment can be written as

$$\sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) \sinh(\alpha)\sin(\eta_j) - (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \cosh(\alpha)\sin(\eta_j)/h_j](2\pi/3N) = 0, \quad (45)$$

$$\sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) \cosh(\alpha)\cos(\eta_j) - (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \sinh(\alpha)\cos(\eta_j)/h_j](2\pi/3N) = 0, \quad (46)$$

$$\sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) h_j^2/2 + (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \sinh(\alpha) coh(\alpha)/h_j](2\pi/3N) = 0, \quad (47)$$

where

$$h_j = (\cosh^2(\alpha)\sin^2(\eta_j) + \sinh^2(\alpha)\cos^2(\eta_j))^{1/2}, \quad (48)$$

$$\omega_1^* = \omega_N^*, \quad (49)$$

$$\omega_1^{*\prime} = \omega_N^{*\prime}, \quad (50)$$

Equations (45), (46) and (47) gives three conditions plus one extra condition arising from the fact that the pressure distribution is single valued (51).

$$-\sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1})](2\pi/3N) = 0. \quad (51)$$

Hence, equations (42) and (43), together with (45), (46), (47) and (51) result in $(2N+4)$ equations in terms of the $(2N+9)$ unknowns. Therefore we need five extra conditions to complete the system of equations.

In order to obtain the other equations, we make use of the fact that $\eta(p)$ vanishes outside the domain, $i.e$ when p is inside the elliptical cylinder, namely $p(\xi_{Ij}, \eta_j)$, where $\xi_{Ij} < \alpha$ for $j = 1, 2, 3, 4, 5$.

Case (b)

In this situation we use the asymptotic expansion of ψ given in equation (40). Hence, equations (45), (46) and (47) have to be replaced with

$$-4\pi\lambda_9 + \sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) \sinh(\alpha)\sin(\eta_j) - (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \cosh(\alpha)\sin(\eta_j)/h_j](2\pi/3N) = 0, \quad (52)$$

$$4\pi\lambda_8 - \sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) \cosh(\alpha)\cos(\eta_j) - (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \sinh(\alpha)\cos(\eta_j)/h_j](2\pi/3N) = 0, \quad (53)$$

$$4\pi\lambda_6 - \sum_{j=1}^{(N-1)/2} [(\omega^{*\prime}_{2j-1} + 4\omega^{*\prime}_{2j} + \omega^{*\prime}_{2j+1}) h_j^2/2 + (\omega^*_{2j-1} + 4\omega^*_{2j} + \omega^*_{2j+1}) \sinh(\alpha) coh(\alpha)/h_j](2\pi/3N) = 0. \quad (54)$$

We proceed in the same way as described in case (a). However in this situation we do not assume that the forces and the moment are zero. This however gives rise to certain computational difficulties, which we will discuss in some detail in the results section. To overcome these difficulties we developed case (c):

Case (c)

Here we used the same methodology adapted in case (b) except that we have replaced the lift condition (53) by one point inside the ellipse. This gave us six points inside the ellipse, one integral condition (51) together with the expressions for the drag (52) and for the moment (54).

5 Numerical Results

Numerical details for situations when the position of the vortex at (2.236,0) is presented for three cases

Initially in case (a) the drag, the lift and the moment are all enforced to be zero. The values of $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7,\ \lambda_8$ and λ_9 which, analytically are 0.500, 0.000, -1.092, 0.000, 0.208, 0.000, 0.056, 0.000 and 0.000 see El Bashir[5] numerically are 0.477, 0.000, -1.091, -0.024, 0.204, 0.000, 0.056, 0.032 and 0.006, respectively. The above inaccuracies arising in the coefficients are overcome by using case (c).

In case (b) the asymptotic expansion for the drag, the lift and the moment at infinity are included. The values of $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7,\ \lambda_8$ and λ_9 are 0.477, 0.000, -1.091, -0.024, 0.204, 0.000, 0.056, 0.032 and 0.006, respectively which are also inaccurate as in case (a).

In case (c) six points inside the elliptical cylinder were used beside the asymptotic expansion at infinity of the drag and the moment and the integral condition. This case was chosen because the lift appears to be inaccurate in case (b) and is very sensitive to the discretization. The values of $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ and λ_9 for $N = 600$ are 0.501, 0.000, -1.091, 0.001, 0.207, 0.000, 0.056, 0.000 and 0.000, respectively

Fig. 2 represents the non-dimensional streamline pattern. The streamline shows a uniform flow at large distances from the elliptical cylinder when a vortex is at $(2.236, 0.758)$. The streamlines for $-1.0,\ -0.5,\ 0.0,\ 0.5$ and 1.0 are presented. In fact, it is possible to obtain uniform flow at large distances if the vortex is placed at any one of four points $(2.236, 0.758)$, $(-2.236, 0.758)$, $(2.236, -0.758)$ and $(-2.236, -0.758)$ due to the symmetry of the problem.

Fig. 3 represents the non-dimensional streamline pattern and the vorticity pattern. The streamlines are symmetric about the x-axis and show a rotational flow about the elliptical cylinder and form closed contours around the cylinder and the vortex, together with a stagnation point on the x-axis opposite to the position of the vortex. However, due to the different values of c the position of the stagnation points differ. The streamlines for $-0.65,\ 0.00$ and 0.19 and the non-dimensional vorticity for $-0.07,\ 0.0,\ 0.22,\ 0.3$ and 0.4 are presented in Fig. 3(b).

Fig. 4 represents the non-dimensional streamline pattern and the vorticity pattern. The streamlines are symmetric about the y-axis and show a rotational flow about the elliptical cylinder and also form closed contours around the cylinder and the vortex, together with a stagnation point on the y-axis. However, due to the different values of c the positions of the stagnation points differ. The streamlines for $-1.5,\ -0.75,\ 0.0,\ 0.75$ and 1.5 and the non-dimensional vorticity for $-0.5,\ -0.4,\ 0.0$ and 0.2 are presented in Fig. 4(b).

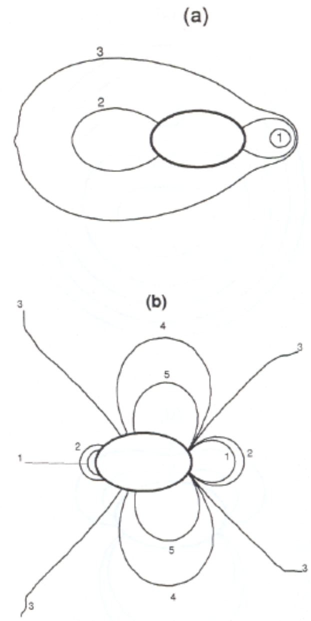

Fig. 3: The numerically obtained streamlines and vorticity pattern for $c = 2.236$ with $N = 800$. (a) the streamlines labeled $1,\ 2$ and 3 correspond to $\psi = -0.65,\ 0.00$ and 0.19, respectively, (b) the vorticity lines labeled $1,\ 2,\ 3,\ 4$ and 5 correspond to $\omega = -0.07,\ 0.0,\ 0.22,\ 0.3$ and 0.4, respectively.

6 Conclusion

The relation of the coefficients of some of the terms in the asymptotic expansion of the stream function to the force components and the torque on a body, together with the imposition of an integral constraint, enables the BEM to provide a closed system of equations for the flow generated by a vortex in the presence of an elliptical cylinder. It is found that the numerical results are in reasonable agreement with those obtained analytically.

It is also found that the determination of the lift is very sensitive to the form of the discretization. In order to overcome this problem we found it necessary to used six points inside the elliptical cylinder, instead of the expected five points, with the constraint on the lift being omitted.

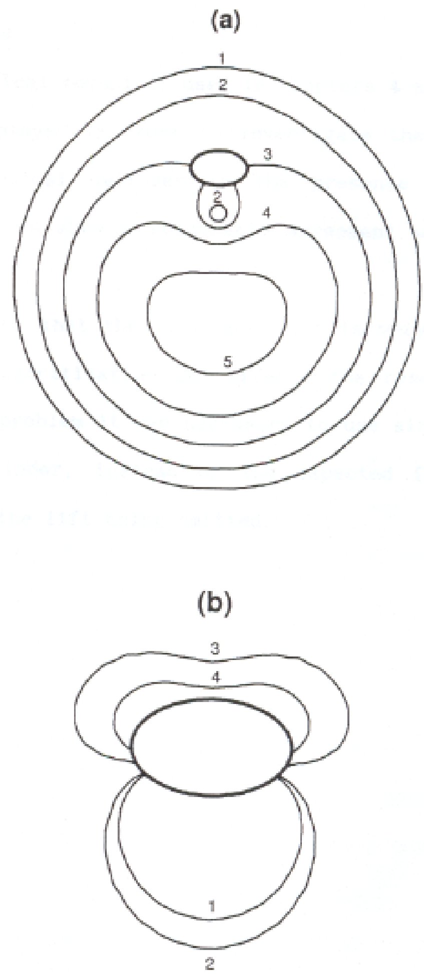

Fig. 4: The numerically obtained streamlines and vorticity pattern for $c = 2.236$ with $N = 800$. (a) the streamlines labeled $1,\ 2,\ 3,\ 4$ and 5 correspond to $\psi = -1.5,\ -0.75,\ 0.0,\ 0.75$ and 0.5, respectively, (b) the vorticity lines labeled $1,\ 2,\ 3$ and 4 correspond to $\omega = -0.5,\ -0.4,\ 0.0$ and 0.2, respectively.

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