Identification of dynamic non-linear models of aircrafts with big incidence angles

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Abstract- Starting from a non-linear description of the longitudinal move of aircrafts with big incidence angles, in this paper one makes an identification structure for the model's unknown parameters using the second method Liapunov algorithm. One also presents the simulation program and a characteristics family obtained with this program. These express state variables' variation in time as response of the system to longitudinal command of the aircraft and identification errors variation. The simulation result differ from the ones obtained by others authors in papers.

Key-Words: identification, longitudinal move, non-linear model, incidence angle

1 Introduction

The subject of the study is identification of aircrafts with big incidence angles (very maneuverable aircrafts and agile rackets), described by non-linear equations. The linear state vector is replaced by one formed by non-linear functions of state variables. The model matrix, obtained by identification, tends to the system's matrix.

An algorithm based on second Liapunov method is used for non-linear model's identification. The system input and model input is elevator deflection. State variables are: flying velocity, incidence angle, pitch angular velocity, lift coefficient and pitch aerodynamic coefficient.

Simulation program is made in Matlab medium. Simulations results are state variables' variations of the aircraft and model and time variations of state variables of the model towards the ones of aircraft.

The system structure and obtained results differ from the ones presented in references [4].

2 Model's identification of the aircraft longitudinal move

An algorithm for unknown parameters' identification of the non-linear dynamic multivariable systems [1] may be used with good results to non-linear longitudinal dynamic's identification for an aircraft with big incidence angle. The algorithm is based on second Liapunov method. An algorithm based on this method is presented in [2].

Non-linear dynamic of the aircraft is described by state equation [3]:

$$\dot{x} = Af(x,u), x(t_0) = x_0 , \qquad (1)$$

where x is the state vector $(n \times 1)$, u - the command vector $(m \times 1)$, f - non-linear known vectorial function and A - unknown matrix; the system's parameters may be obtain experimentally. The non-linear model of the aircraft's move is described by the state equation:

$$\dot{x}_M = A_M f(x_M, u), x_M(t_0) = x_{M_0} , \qquad (2)$$

the coefficients of matrix A_M will be determinated.

The matrix A_M may be calculated from the convergence condition:

$$\lim_{t\to\infty} \|\Delta A(t)\| = \lim_{t\to\infty} \|A(t) - A_M(t)\| = 0. \qquad (3)$$

Noting with $\Delta x = x - x_M$ and with:

$$\Delta f(x, x_M, u) = f(x,u) - f(x_M, u) \qquad (4)$$

and decaying equation (2) from equation (4) one results:

$$e = \Delta\dot{x} - A_M \Delta f(x, x_M, u) = \Delta A f(x,u), \qquad (5)$$

where ΔA is the solution of equation [4]:

$$\Delta\dot{A} = -\sum_{i=0}^{\eta} e^{\frac{2i+1}{2j+1}} f^T(x,u) M, \Delta A(t_0) = \Delta A_0 . \qquad (6)$$

Using the second Liapunov method, in [4] one demonstrates that A_M verifies equation

$$\dot{A}_M = \dot{A} + \sum_{i=0}^{\eta} e^{\frac{2i+1}{2j+1}} \cdot f^T(x,u) M, A_M(t_0) = A_{M_0}; \qquad (7)$$

the initial values of A_{M_0}'s components may be chosen zero; matrix M is defined as follows:

$$M = \mathrm{diag}[m_k], m_k(t) = 0.05 + \exp(-10t). \qquad (8)$$

Longitudinal move's dynamic of an aircraft with big incidence angle is described by state equation:

$$\dot{x} = Af(x) + Bu, \qquad (9)$$

where:

$$f^T(x) = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_2x_5 \ x_1^{1/3} \ x_1^3 \ x_2^{1/3} \ x_2^2 \ x_2^3 \ x_3^{1/3} \ x_3^3 \ x_4^3 \ x_5^3], \qquad (10)$$

$x_1 = \Delta V, x_2 = \Delta\alpha, x_3 = \Delta\omega_y, x_4 = \Delta c_p$ (lift coefficient's variation), $x_5 = \Delta C_m$ (aerodynamic moment coefficient's variation), $u = \delta_p$ (the elevator deflection) and:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & a_{35} & a_{36} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & a_{47} & a_{48} & a_{49} & a_{410} & a_{411} & a_{412} & a_{413} & a_{414} & 0 \\ a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 & a_{57} & a_{58} & a_{59} & a_{510} & a_{511} & a_{512} & a_{513} & 0 & a_{515} \end{bmatrix}$$

$$B^T = [b_{11} \ b_{12} \ b_{13} \ b_{14} \ b_{15}]. \qquad (11)$$

Function $f(x_M)$ has the form:

$$f^T(x_M) = [x_{1M} \ x_{2M} \ x_{3M} \ x_{4M} \ x_{5M} \ x_{2M}x_{5M} \ x_{1M}^{1/3} \ x_{1M}^3 \ x_{2M}^{1/3} \ x_{2M}^2 \ x_{2M}^3 \ x_{3M}^{1/3} \ x_{3M}^3 \ x_{4M}^3 \ x_{5M}^3]. \qquad (12)$$

Using $(1) \div (11)$ one obtains the block scheme (Fig. 1).

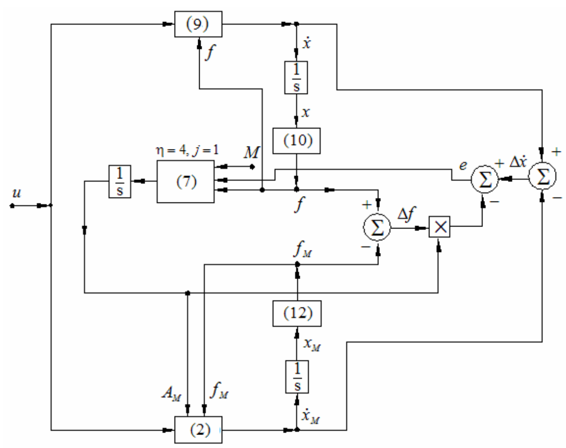

Fig.1.

3 Numerical simulation results

For example, the elements of matrix A are chosen as values closed the ones from [5], [6]:

$$\begin{aligned} & a_{1,1} = -1.85; a_{1,2} = -2.9; a_{1,3} = -0.85; a_{2,1} = -5; \\ & a_{2,2} = -3; a_{2,3} = 18.1; a_{2,4} = 30; a_{3,1} = -0.25; \\ & a_{3,2} = 8; a_{3,3} = -0.1; a_{3,5} = -14; a_{3,6} = -350; \\ & a_{4,1} = -0.15; a_{4,2} = 0.08; a_{4,3} = -0.2; a_{4,4} = -3.9; \\ & a_{4,7} = -0.1; a_{4,8} = 0.19; a_{4,9} = 0.34; a_{4,10} = 0.25; \\ & a_{4,11} = 0.15; a_{4,12} = -0.12; a_{4,13} = 0.2; a_{4,14} = -0.12; \\ & a_{5,1} = 0.1; a_{5,2} = -0.04; a_{5,3} = -0.1; a_{5,5} = -4.3; \\ & a_{5,7} = 0.058; a_{5,8} = 0.16; a_{5,9} = 0.29; a_{5,10} = 0.2; \\ & a_{5,11} = -0.05; a_{5,12} = 0.12; a_{5,13} = -0.05; a_{5,15} = -0.09 \end{aligned} \qquad (13)$$

and:

$$B^T = [3.1 \quad 2.9 \quad 1 \quad 1 \quad 1], \delta_p = 1\,\mathrm{grd}. \qquad (14)$$

The algorithm for longitudinal move's identification is:

- matrices A , B and elevator deflection (δ_p) presentation;

- matrix A_M initialization (for example $A_M(t_0)=0$);
-vectors f and f_M initialization (for example $f=f_M=0$);

for k=1:step_number,
begin

- $\dot{x}$ calculus using equation (9);
- x determination through $\dot{x}$ integration;
- f calculus using (10);
- $\dot{x}_M$ calculus using an equation of type (9) with $A=A_M$ and $f=f_M$;
- x_M determination through $\dot{x}_M$ integration;
- f_M calculus using an equation of type (11);
- Δf calculus using (4);
- error e's calculus using (5);
- matrix M calculus using (8);
- $\dot{A}_M$ calculus using (7);
- A_M determination through matrix $\dot{A}_M$'s integration;

end

The Matlab program based on this algorithm is:

```
close all; clear all;
a11=-1.85;a12=-2.9;a13=-0.85;a21=-5;a22=-3;
a23=18.1;a24=30;a31=-0.25;a32=8;a33=-0.1;
a35=-14;a36=-350;a41=-0.15;a42=0.08;a43=-0.2;
a44=-3.9;a47=-0.1;a48=0.19;a49=0.34;a410=0.25;
a411=0.15;a412=-0.12;a413=0.2;a414=-0.12;a51=0.1;
a52=-0.04;a53=-0.1;a55=-4.3;a57=0.058;a58=0.16;
a59=0.29;a510=0.2;a511=-0.05;a512=0.12;a513=-
0.05;
a515=-0.09;t=0;u=1;B=[3.1;2.9;1;1;1];p=0.01;
A=[a11 a12 a13 0 0 0 0 0 0 0 0 0 0 0 0;
 a21 a22 a23 a24 0 0 0 0 0 0 0 0 0 0 0;
 a31 a32 a33 0 a35 a36 0 0 0 0 0 0 0 0 0;
 a41 a42 a43 a44 0 0 a47 a48 a49 a410 a411
 a412 a413 a414 0;  a51 a52 a53 0 a55 0 a57
 a58 a59 a510 a511 a512 a513 0 a515];
for i=1:5,
   for j=1:15,
      AM(i,j)=0;
   end
end
for i=1:15,
    fM(i,1)=0;f(i,1)=0;
end
alfa(1)=f(2);V(1)=f(1);omegay(1)=f(3);cp(1)=f(4);
cm(1)=f(5); alfaM(1)=fM(1);VM(1)=fM(2);
omegayM(1)=fM(3);cpM(1)=fM(4);cmM(1)=fM(5);
for k=1:20,
xp=A*f+B*u; x=xp*p;
x1=x(1);x2=x(2);x3=x(3);x4=x(4);x5=x(5);x6=x2*x5;
x8=x1^3;x10=x2^2;x11=x2^3;x13=x3^3;
x14=x4^3;x15=x5^3;
     if x1<0
       x7=-((abs(x1))^(1/3));
     else
       x7=x1^(1/3);
     end  %if x1
     if x2<0
       x9=-((abs(x2))^(1/3));
     else
       x9=x2^(1/3);
     end  %if x2
     if x3<0
       x12=-((abs(x3))^(1/3));
     else
        x12=x3^(1/3);
     end  %if x3
f=[x1;x2;x3;x4;x5;x6;x7;x8;x9;x10;x11;x12;x13;x14;
x15]; xMp=AM*fM+B*u;  xM=xMp*p;
xM1=xM(1);xM2=xM(2);xM3=xM(3);xM4=xM(4);x
M5=xM(5);xM6=xM2*xM5;
xM8=xM1^3;xM10=xM2^2;xM11=xM2^3;xM13=x
M3^3;xM14=xM4^3;xM15=xM5^3;
     if xM1<0
       xM7=-((abs(xM1))^(1/3));
    else
       xM7=xM1^(1/3);
    end  %if xM1
     if xM2<0
       xM9=-((abs(xM2))^(1/3));
    else
       xM9=xM2^(1/3);
    end  %if xM2
     if xM3<0
       xM12=-((abs(xM3))^(1/3));
     else
       xM12=xM3^(1/3);
     end  %if xM3
fM=[xM1;xM2;xM3;xM4;xM5;xM6;xM7;xM8;xM9;
xM10;xM11;xM12;xM13;xM14;xM15];
alfa(k+1)=f(2);V(k+1)=f(1);omegay(k+1)=f(3);cp(k+1
)=f(4);cm(k+1)=f(5);
lfaM(k+1)=fM(1);VM(k+1)=fM(2);
omegayM(k+1)=fM(3);cpM(k+1)=fM(4);cmM(k+1)=f
M(5);  dxp=xp-xMp; df=f-fM; e=dxp-AM*df;
m=0.05+exp(-10*t); t=t+p;
for i=1:15,
    for j=1:15,
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        if i==j
          M(i,j)=m;
        else
          M(i,j)=0;
       end  %if
     end  %for j
   end  %for i
 S=zeros(5,1);
   for b=1:5,
      if e(1)<0
        ee1=-((abs(e(1)))^((2*b-1)/3));
      else
        ee1=e(1)^((2*b-1)/3);
      end     % if ee1
      if e(2)<0
        ee2=-((abs(e(2)))^((2*b-1)/3));
      else
        ee2=e(2)^((2*b-1)/3);
      end     % if ee2
      if e(3)<0
        ee3=-((abs(e(3)))^((2*b-1)/3));
      else
        ee3=e(1)^((2*b-1)/3);
      end     % if ee3
      if e(4)<0
        ee4=-((abs(e(4)))^((2*b-1)/3));
      else
         ee4=e(4)^((2*b-1)/3);
      end     % if ee4
      if e(5)<0
        ee5=-((abs(e(5)))^((2*b-1)/3));
      else
        ee5=e(5)^((2*b-1)/3);
      end     % if ee5
   ee=[ee1;ee2;ee3;ee4;ee5]; S=S+ee;
    end  %for b
AMp=S*transpose(f)*M; AM=AMp*p;subplot(231);
plot(alfa);hold on;
plot(alfaM,'r');grid;xlabel('Timp[s]');
subplot(232);plot(V);hold on;plot(VM,'r');grid;
xlabel('Timp[s]');subplot(233);plot(omegay);hold
on;plot(omegayM,'r');grid;xlabel('Timp[s]');
subplot(234);plot(cp);hold on;plot(cpM,'r');grid;
xlabel('Timp[s]');subplot(235);plot(cm);hold on;
plot(cmM,'r');grid;xlabel('Timp[s]'); h=figure;
a1=alfa-alfaM;a2=V-VM;a3=omegay-omegayM;
a4=cp-cpM;a5=cm-cmM;subplot(231);plot(a1);
grid;xlabel('Timp[s]');subplot(232);plot(a2);
grid; xlabel('Timp[s]');subplot(233);plot(a3);grid;
xlabel('Timp[s]');subplot(234);plot(a4);grid;
xlabel('Timp[s]'); subplot(235);plot(a5);grid;
xlabel('Timp[s]');
```

The program identifies longitudinal move of an aircraft and represents the curves that express the variables $x_i(t)$ and $x_{M_1}(t)\left(i=\overline{1,5}\right)$ in fig.1 and the errors Δx_i $\left(i=\overline{1,5}\right)$ in fig.2. For fig.1 ΔV (fig.1.a), $\Delta\alpha$ (fig.1.b), $\Delta\omega_y$ (fig.1.c), Δc_p (fig.1.d), ΔC_m (fig.1.e) are represented with continuous blue line, while the variables ΔV_M , $\Delta\alpha_M$, , Δc_{p_M} and ΔC_m are represented with dash dot red line. Comparing these characteristics with the ones from [1], obtained for $u = 0$ which express the instability of the longitudinal move of an aircraft with big incidence angle, one observes that the ones from fig.1 and fig.2 for $u=\delta_p \neq 0$ express the stability of the longitudinal move.

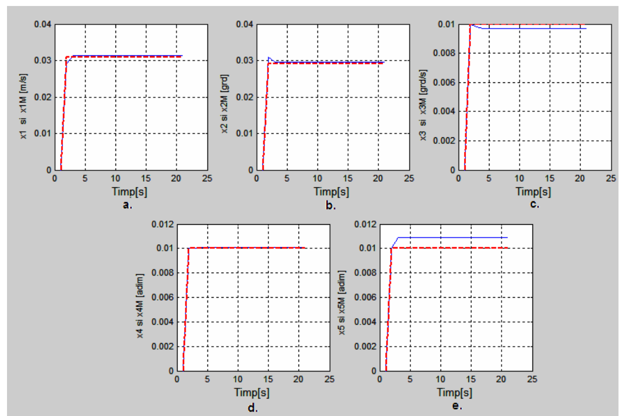

Fig.2.

In fig.2 the five components of the error vector $\Delta x = x - x_M$ has been represented (Δx_1 (fig.2.a), Δx_2 (fig.2.b), Δx_3 (fig.2.c), Δx_4 (fig.2.d), Δx_5 and (fig.2.e)).

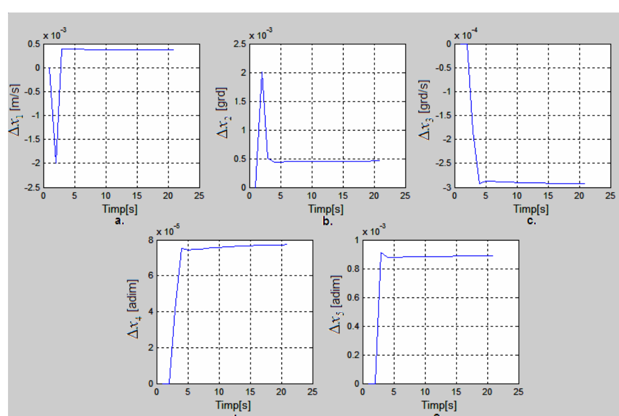

Fig.3.

4 Conclusion

This work presets the structure of a system for non-linear model identification for the longitudinal move of the aircrafts with big incidence angles, instable in generally because of the aerodynamic turbulences. The simulation program and the results of it are also presented (the time variations of state variables of the model as response of the system to longitudinal command of the aircraft and the functions that express identification errors). The obtained result differs from the ones obtained by others authors.

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