A New Method for Induction Motor Stability Analysis When Supplying at Variable Frequency

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Abstract: - In this paper a new method for the induction motors stability analysis when supplying at variable frequency is presented in details. It is insisted on the mathematical model, on the quantitative results and on the graphics obtained with the help of a special achieved Matlab program. Finally the conclusions resulting from the study are presented.

Key-Words: - stability, induction motor, variable frequency, parameters, simulation, Matlab

1 Introduction

The problem of the induction motors stability analysis when they operate at variable frequency is a present problem [2], [4], [6], etc. The quantitative conclusions presented in outstanding papers from this field, aiming to the induction motors parameters influences on the stability, are generally known in this case. Unfortunately, the methods used for the analyses, beside the fact that it is very difficult to implement them numerically, they also have the drawback that they do not allow to study the inertia moment influence on the stability, a very important thing especially in the case of the low power machines.

In order to eliminate these drawbacks a new method for the stability study has been conceived, with the help of the equations with representative phasors written in relative values.

2 Establishment of the used mathematical model

The equations system that is used have the following form [1]:

$$\omega_s^* = s_{ks}(\underline{\Psi}_s^* - k\underline{\Psi}_r^{/*}) + \frac{d\underline{\Psi}_s^*}{dt^*} + j\omega_s^*\underline{\Psi}_s^*$$

$$0 = s_{kr}(\underline{\Psi}_r^{/*} - k\underline{\Psi}_s^*) + \frac{d\underline{\Psi}_r^{/*}}{dt^*} + j(\omega_s^* - \omega^*)\underline{\Psi}_r^{/*} \quad (1)$$

$$h\cdot\frac{d\omega^*}{dt^*} = -\frac{k}{x_{rt}^{/*}}\operatorname{Im}\left[(\underline{\Psi}_s^*)^*\underline{\Psi}_r^{/*}\right] - m_r^*$$

These equations are linearized further on.

In order to do this thing it is considered that the pulsation modifies in saltus with a very low value. This variation will lead implicitly to a voltage modification, in saltus too, with the same value, so that the two quantities ratio to remain constant.

In this hypothesis the system (1) will modify as follows.

$$\omega_s^* + \Delta\omega_s^* = s_{ks}\left[\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^* - k(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right] + \frac{d(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)}{dt^*} + j(\omega_s^* + \Delta\omega_s^*)(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)$$

$$0 = s_{kr}\left[\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} - k(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)\right] + \frac{d(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})}{dt^*} + j(\omega_s^* + \Delta\omega_s^* - \omega^* - \Delta\omega^*)(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*}) \quad (2)$$

$$h\cdot\frac{d(\omega^* + \Delta\omega^*)}{dt^*} = -\frac{k}{x_{rt}^{/*}}\cdot \operatorname{Im}\left\{\left[(\underline{\Psi}_s^*)^* + \Delta(\underline{\Psi}_s^*)\right]\cdot(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right\} - m_r^*$$

By applying Laplace transformation to the first two equations of the systems (1) and (2), by subtracting member by member and by neglecting the products of the form $\Delta\cdot\Delta$, one obtains:

$$\Delta\omega_s^* = (s_{ks} + j\omega_s^* + s)\cdot\Delta\underline{\Psi}_s^* - s_{ks}\cdot k\cdot\Delta\underline{\Psi}_r^{/*} + j\cdot\underline{\Psi}_s^*\cdot\Delta\omega_s^* \quad (3)$$

$$0 = -s_{kr}\cdot k\cdot\Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} + j(\Delta\omega_s^* - \Delta\omega)\underline{\Psi}_r^{/*}$$

$$h\frac{d(\Delta\omega^*)}{dt} = -\frac{k}{x_{st}^*}\operatorname{Im}\left[\left(\underline{\Psi}_s^*\right)\cdot\Delta\underline{\Psi}_r^{/*} + \underline{\Psi}_r^{/*}\cdot\Delta\left(\underline{\Psi}_s^*\right)^*\right]$$

where the operational variable is noted with s.
It must also be noticed that for simplifying the writing and for not producing confusions, both in the previous relation and in the following ones, it has been given up both to indicate the quantities depending on s ($\Delta\omega_s^*(s)$, $\Delta\omega^*(s)$ etc.) and to note them with capitals.
If it is considered that $\Delta\omega^*$ is not less than 0,1 in the previous relations the following approximations may be made:

$$j\underline{\Psi}_s^* = 1 \quad \text{and} \quad j\underline{\Psi}_r^{/*} = k . \tag{4}$$

This way, the first two relations from (3) become:

$$\begin{aligned} 0 &= (s_{ks} + j\omega_s^* + s)\Delta\underline{\Psi}_s^* - s_{ks} \cdot k \cdot \Delta\underline{\Psi}_r^{/*} \\ k\left(\Delta\omega^* - \Delta\omega_s^*\right) &= -s_{kr} \cdot k \cdot \Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} \end{aligned} \tag{5}$$

The analysis of these relations can be simplified if it is considered $R_s \cong 0$. But this simplifying hypothesis leads to satisfactory results only in the field $\omega_s^* \in (0{,}5 \div 1)$.
So it is imposed to analyze the situation when $R_s \neq 0$, but considering that the studied phenomenon is linearized.
For this, it is considered that the motor operated no-load before modifying the frequency. In this situation, owing to the low frequency of the rotor current, its active component may be neglected.
Thus, one can write:

$$\Delta\underline{i}_r^{/*} = \Delta i_{dr}^{/*} + j\Delta i_{qr}^{/*} \cong \Delta i_{dr}^{/*} = \frac{\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*}{dx_s^*} . \tag{6}$$

By solving the system (5) relatively to $\Delta\Psi_s^*$ and $\Delta\Psi_r^{/*}$ by replacing these relations in (6), after computations, it is obtained:

$$\Delta i_{dr}^{/*} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k(\Delta\omega^* - \Delta\omega_s^*) \tag{7}$$

where the following notation has been used:

$$\varepsilon = (1 - k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{/*}} . \tag{8}$$

When $\omega_s^* \geq 0{,}1$ it results that it can be considered (with approximation):

$$\left(\underline{\Psi}_s^*\right)^* = 1 \quad \text{and} \quad \underline{\Psi}_r^{/*} = -jk \tag{9}$$

In these conditions, by applying Laplace transformation to the relation (9), it is obtained:

$$hs \cdot \Delta\omega^* = -\frac{k}{x_{st}^*}\mathrm{Re}(\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*) , \tag{10}$$

or, equivalently

$$hs \cdot \Delta\omega^* = -\frac{k}{x_{st}^*}\mathrm{Re}(\Delta\underline{\Psi}_{dr}^{/*} - k\Delta\underline{\Psi}_{ds}^*) , \tag{11}$$

respectively

$$hs \cdot \Delta\omega^* = -k\Delta i_{dr}^{/*} . \tag{12}$$

3 Quantitative Results

Further on, for the study of the induction motor stability, the relations (7) and (12) established before are used. The first relation can be written in the form:

$$\Delta\omega^* = -\frac{k}{hs} \cdot \Delta i_{dr}^{/*} \Leftrightarrow \Delta\omega^* = G_1(s) \cdot \Delta i_{dr}^{/*} , \tag{13}$$

with

$$G_1(s) = -\frac{k}{hs} \tag{14}$$

The second relation is processed analogously:

$$\Delta i_{dr}^{/*} = G_2(s) \cdot (\Delta\omega_s^* - \Delta\omega^*) , \tag{15}$$

where

$$G_2(s) = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k \tag{16}$$

By using (13) and (15) the following configuration can be drawn.

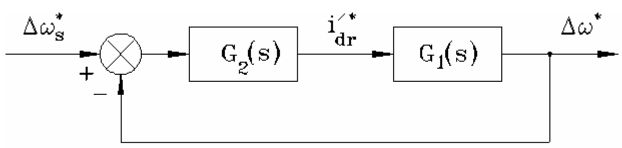
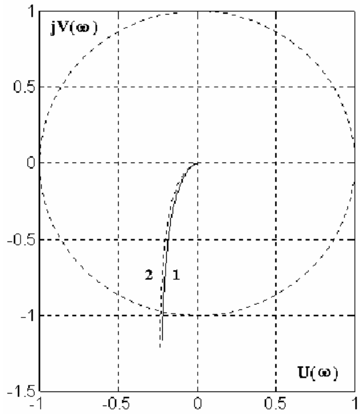
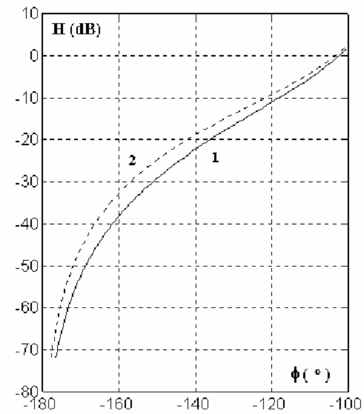


Fig.1. Block scheme of the machine in the mentioned situation.

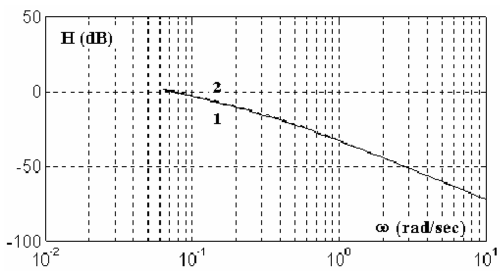
Further on it is possible to pass to the stability study in our concrete case by using all these introductive notions. This analysis will be made with the help of a Matlab program achieved on the basis of the scheme depicted in the figure 1 and of the relations (13), (14), (15) and (16).
By running this program the following graphics have been obtained.



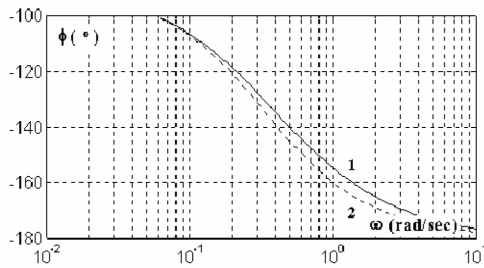
a) Transfer locus.



b) Amplitude – phase characteristics.

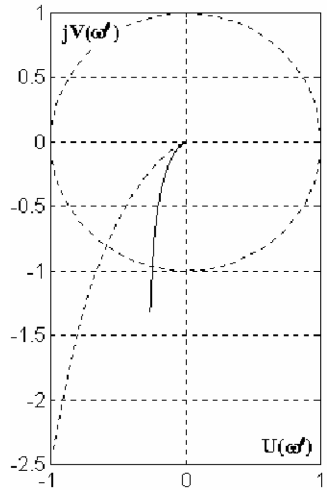


c) Pulsation-amplitude characteristics.

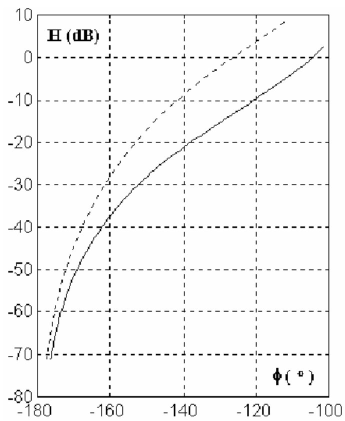


d) Phase-pulsation characteristics.

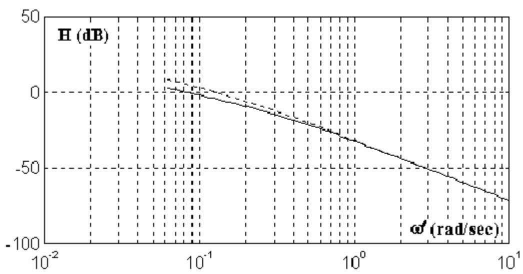
Fig.2. Graphic dependences corresponding to the cases R_S=7,5 Ω (continuous line) and R_S=2,5 Ω (dot line).



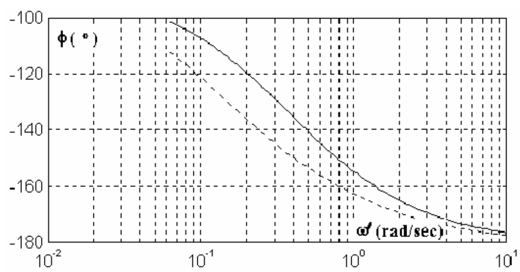
a) Transfer locus.



b) Amplitude – phase characteristics.



c) Pulsation-amplitude characteristics.



d) Phase-pulsation characteristics.

Fig.3. Graphics dependences corresponding to the cases R'_r=5,5 Ω (continuous line) and R'_r=4,5 Ω (dot line).

Observation 1
In order to determine the characteristics depicted in the previous figures it has been considered that the induction motor has the following parameters:
R_s=7,5 Ω; $R_r^{/}$=5,5 Ω; Ls=0,529 H; $L_r^{/}$=0,528 H; L_{sh}=0,498 H; J=0,004 kgm^2,
that leads to the following per unit values:
r_s^*=0,0989; $r_r^{/*}$=0,0725; x_s^*=2,1907; $x_r^{/*}$=2,1865; x_{1m}^*=2,0623; x_{st}^*=0,2456; x_{rt}^*=0,2451; s_{ks}=0,4026; s_{kr}=0,2958; k=0,9414; h=32,4; ε=0,0458.

Observation 2
With the help of a special achieved Matlab program and of the characteristics corresponding to the cases when a parameter from the ones depicted in the second column of the table 1 is successively modified (over the initial case), the margins of phase depicted in the third column of the same table are obtained.

Table 1

Par.	Abs. value [Ω], [H], [kgm^2]	Per unit par.	Per unit value	Phase margin [degree]
R_s	7,5	r_s^*	0,0988	75,54
	2,5		0,0330	74,20
$R_r^{/}$	5,5	$r_r^{/*}$	0,0725	75,54
	4,5		0,0593	53,71
L_s	0,529	x_s^*	2,1907	75,54
	0,549		2,2735	69,13
$L_r^{/}$	0,528	$x_r^{/*}$	2,1865	75,54
	0,548		2,2694	67,31
L_{sh}	0,498	x_{1m}^*	2,0623	75,54
	0,438		1,8138	75,76
J	0,004	h	32,4	75,54
	0,003		24,3	47,65

4 Conclusion

The following conclusions can be emphasized, by analyzing the previous results:
- the decrease of the stator winding resistance leads to the stability decrease;
- the rotor resistance decrease has also as an effect, the decrease of the machine stability and conversely;
- the increase of the stator winding inductivity leads to the stability decrease;
- at the same time with the rotor inductivity increase the system stability decreases;
- the main inductivity increase has a non-stabilizing effect;
- the inertia moment increase contributes to the stability increase.

In order to catch quantitatively these interdependences, the following table can be filled.

Table 2

Parameter	Per cent variation of the parameter	Per cent variation of the phase margin
R_s	66,6	2,04
$R_r^{/}$	18,2	28,89
L_s	3,64	8,48
$L_r^{/}$	3,93	10,89
L_{sh}	12,04	0,29
J	25	36,92

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