# Numerical Analysis of Low-order Modes in Thermally Diffused Expanded Core (TEC) Fibers

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*Abstract:* - Optical intensity distributions, cut off frequencies and propagation constants for the low-order modes of Thermally-Diffused Expanded Core (TEC) Fibers are demonstrated by numerical analysis based on Galerkin's method. A set of orthogonal Laguerre-Gauss functions is used to calculate the spectral dependence of effective indices and mode fields of LP-modes. Results are compared with and shown to be accurately approximated by those obtained by an one-parameter variational method.

*keywords*:-propagation characteristics, thermally diffused expanded core fibers, Garlekin's method, variational methods.

## 1 Introduction

Transformation of modal field diameters (MFD) in single-mode fibers (SMF) is an important technology [1] in the area of optical communications and optical sensing. For example, the matching of field diameters between dispersion-shifted fibers and erbium doped optical fibers in amplifiers, is highly desirable to reduce splicing losses. Local expansion of field diameters is essential for the integration of fiber in various optical devices such as isolators, filters and optical switches [2,3]. The technology is also useful for efficient coupling between SMF and laser diodes [4]. Thermally-diffused expanded core (TEC) optical fibers have been widely used for such applications [5].

A TEC fiber has an enlarged mode field diameter (MFD) obtained by heating a single-mode fiber (SMF) locally at a high temperature ($\sim 1300 - 1650\,^oC$) and diffusing the germanium dopant into the core [6]. The core expansion rate depends on the heating temperature, the heating time and the dopant intensity in the fiber core. The TEC fiber has the feature that although thermal diffusion changes the refractive-index profile, the normalized frequency does not change and hence the single mode condition is maintained through the process. The maximum MFD ever reported is 40 $\mu m$, at the wavelength $\lambda = 1.55\ \mu m$, without reduction in the cladding diameter of the fiber [7]. Modal field profiles, diffraction losses in a single-mode TEC fiber as well as splice losses between two TEC fibers have been experimentally obtained [8] and theoretically analyzed[9,10]. There is a little information, however, concerning the propagation characteristics of low-order modes in a TEC fiber, if they are excited.

In this paper, we analyze the modal properties of low-order modes in TEC fibers using Galerkin's numerical method as well as a variational method. The paper is organized as follows: In Section 2 we briefly present the refractive index profile in a TEC fiber under isotropic thermal diffusion. In Sections 3 and 4, the Galerkin's method and the variational analysis are described, respectively, in order to calculate the propagation constants and modal fields of low-order modes in a TEC fiber. Results of the above calculations are presented in Section 5 using normalized parameters that can be used in designing TEC-fiber integrated optical devices under the condition of excitation of low-order modes. We conclude with a summarizing Section 6.

## 2 Refractive Index Profile of a TEC Fiber

A schematic illustration of a thermally expanded core fiber is given in Fig. 1. This shows that a TEC fiber has core diameter 2A and is fabricated from an initial step index fiber with core diameter $2\alpha$. The heating region is considered very long and the expanded core region uniform in the longitudinal direction. We assume that the refractive-index profile in the expanded core region is proportional to the dopant profile and can be expressed as:

$$n^2(r) = n_{cl}^2 + (n_{co}^2 - n_{cl}^2)u(r,t) \quad (1)$$

where $u(r,t)$ is the Germanium dopant concentration as a function of the radial distance $r$ and heating time $t$. Also, $n_{co}$ and $n_{cl}$ are the refractive indices of the core and the cladding respectively.

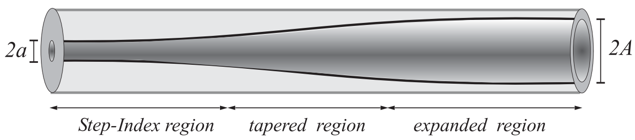


Fig. 1: Schematic illustration of a thermally expanded core fiber.

A Gaussian distribution function of the dopant $u(r,t)$, after the heat treatment, can be obtained as an exact solution of the diffusion equation in cylindrical coordinates, assuming an infinite heating region with a line source and as initial dopant concentration a delta function [6]. Also the mass conservation of the diffusing atoms is taking into account. Finally, following the formalism used in the study of graded index fibers, the refractive index profile of the TEC fiber can be written as [10]:

$$n^2(r) = n_{co}^2[1 - 2\Delta f(r)] \quad (2)$$

where $\Delta = (n_{co}^2 - n_{cl}^2)/2n_{co}^2$ and

$$f(r) = 1 - (\alpha^2/A^2)e^{-(r^2/A^2)} \quad (3)$$

where $A = \sqrt{Dt}$ is defined as the core radius of the fiber after the heat treatment and $D$ is the diffusion coefficient of Ge dopant. Therefore, the variation of refractive index profile under conditions of isotropic thermal diffusion has a Gaussian radial dependence.

## 3 Galerkin's method

In the weak-guidance approximation, when the variation in refractive index is small, the fields and propagation constants are determined by the scalar wave equation, rather than the full set of Maxwell's equations. The scalar wave equation of a circularly symmetric optical fiber for a given azimuthal mode number $m$ can be written in polar coordinates as [11]

$$\frac{d^2E(r)}{dr^2} + \frac{1}{r}\frac{dE(r)}{dr} + \Big(k_0^2n^2 - \beta^2 - \frac{m^2}{r^2}\Big)E(r) = 0 \quad (4)$$

where $E(r)$ is the amplitude of the scalar field, $k_0 = 2\pi/\lambda$ is the free space wave number, $n(r)$ is the refractive index profile of the given fiber and $\beta$ is the propagation constant. Since there is no analytic solution of Equation (4) for an arbitrary refractive index profile $n(r)$, a numerical Galerkin's method is used to find approximate solutions.

In a numerical Galerkin's method [12], the solution of a differential equation is expanded with a linear combination of analytically differentiable orthogonal basis functions inside finite boundary. In the present paper we use as basis functions the well known associated Laguerre-Gauss functions $\phi_i^m(x)$

$$\phi_i^m(x) = \sqrt{\frac{i!}{(i+m)!}}x^{m/2}\exp^{-x/2}L_i^m(x) \quad (5)$$

where $L_i^m(x), i = 0, 1, 2, \cdots N-1$ are the associated Laguerre polynomials and N is the basis number.

For the solution of Eq.(4) we define a normalized dimensionless parameter

$$x = \sigma r^2/\alpha^2 \quad (6)$$

where $\alpha$ is the core radius and $\sigma$ is an arbitrary positive number that affects the convergence, accuracy and computational time. In our computation, $\sigma$ is chosen to be 1/8. We also define a profile function $h(r)$, a normalized frequency $V$ and a normalized propagation constant $b$ as:

$$\begin{aligned} h(r) &= \frac{n^2(r) - n_{cl}^2}{n_{co}^2 - n_{cl}^2}, \quad V^2 = k_0^2\alpha^2(n_{co}^2 - n_{cl}^2) \\ b &= \frac{(\beta/k_0)^2 - n_{cl}^2}{n_{co}^2 - n_{cl}^2} \end{aligned} \quad (7)$$

Inserting Eqs.(6) and (7) into Eq.(4) yields

$$x\frac{d^2E}{dx^2} + \frac{dE}{dx} + \frac{1}{4}\Big(\frac{V^2h - V^2b}{\sigma} - \frac{m^2}{x}\Big)E = 0 \quad (8)$$

By expanding the mode field $E$ in terms of the orthogonal Laguerre-Gauss functions $\phi_i^m(x)$

$$E(x) = \sum_{i=0}^{N-1} \alpha_i \phi_i^m(x) \tag{9}$$

we obtain

$$\begin{aligned} &\frac{\sigma}{V^2} \sum_{i=0}^{N-1} \alpha_i [x - 2(1 + 2i + m)] \phi_i^m(x) \\ &+ \sum_{i=0}^{N-1} \alpha_i h\Big(\sqrt{x/\sigma}\Big) \phi_i^m(x) = b \sum_{i=0}^{N-1} \alpha_i \phi_i^m(x) \end{aligned} \tag{10}$$

If we multiply both sides of Eq.(10) by $\phi_j^m(x)$ and integrate in the whole space from 0 to $\infty$ then Eq. (10) transforms to a system of eigenvalue equations

$$[M][Y] = b[Y] \tag{11}$$

where $[Y]$ is the coefficient eigenvector and $[M]$ is a square matrix with dimension of $N \times N$.

For a given wavelength, the propagation constants of all allowed modes are calculated. The accuracy and validity of our algorithm is verified by comparing our numerical results for step-index fibers with the available analytical results [11]. The Galerkin's method is generally stable and gives accurate results when the mode is away from the cut off region. The main advantage of the method is its versatility in treating circular fibers with arbitrary index profiles without significant modifications.

# 4 Variational method

In this section we describe a variational method in order to obtain analytic approximations for the modal fields and propagation constants of the low-order modes of TEC fibers. Usually the variational process is applied with respect to the propagation constant $\beta$. Instead of $\beta$ we consider here the dimensionless modal parameter $U$ defined in step index fiber theory as

$$U = \alpha(k_0^2 n_{co}^2 - \beta^2)^{1/2} \tag{12}$$

Then our variational expression for $U^2$ yields[11]

$$U^2 = \frac{\alpha^2 \int_0^\infty \left( \Big(\frac{m^2}{r^2} + \frac{V^2}{\alpha^2} f\Big) E - \frac{d^2E}{dr^2} - \frac{1}{r}\frac{dE}{dr} \right) E r dr}{\int_0^\infty E^2 r dr}$$

Table 1

The values of the coefficients $A_\rho^{l-1}$

| | | | |
|---|---|---|---|
| $A_0^0$ | $1$ | $A_0^2$ | $\frac{1}{4}(m+1)^2(m+2)^2$ |
| $A_0^1$ | $(m+1)^2$ | $A_1^2$ | $-(m+1)(m+2)^2$ |
| $A_1^1$ | $-2(m+1)$ | $A_2^2$ | $\frac{1}{2}(5+3m)(m+2)$ |
| $A_2^1$ | $1$ | $A_3^2$ | $-(m+2)$ |
| | | $A_4^2$ | $1/4$ |

We choose as trial functions the Laguerre-Gauss functions [13]

$$E(r) = C_{lm} (r/r_0)^m \exp^{-r^2/2r_0^2} L_{l-1}^m (r^2/r_0^2) \tag{13}$$

where $C_{lm}$ is the normalization constant given by

$$C_{lm} = \sqrt{\frac{2}{r_0^2} \frac{(l-1)!}{(l-1+m)!}}, \quad l = 1, 2, 3 \cdots$$

and $r_0$ is the variational parameter. The value of $r_0$ is found by solving the equation for the extremum value of $U^2$

$$\frac{\partial U^2}{\partial r_0} = 0$$

which finally leads to the relation

$$2l + m - 1 = \frac{r_0^2 V^2}{2\alpha^2} \int \frac{df}{dr} [E(r)]^2 r^2 dr \tag{14}$$

Eq.(14) can be easily written in closed form for the first three modes $l = 1, 2, 3$ that is:

$$\begin{aligned} 2l + m - 1 &= V^2 C_{lm}^2 (cr_0)^4 \frac{r_0^2}{2(1 + c^2 r_0^2)^{m+2}} \\ &\times \sum_{\rho=0}^{2l-2} A_\rho^{l-1} \frac{(m + \rho + 1)!}{(1 + c^2 r_0^2)^\rho} \end{aligned} \tag{15}$$

where $c = 1/A$. The coefficients $A_\rho^{l-1}$ are given in Table 1. Given the solution of Eq. (15) for $r_0$, the corresponding value of U and hence the propagation constant $\beta$, can be found.

# 5 Results and Discussion

The TEC fiber under consideration has a thermally expanded core with radius $A = 8\ \mu m$. The parameters of the standard SI-SMF before heat treatment are: relative refractive index difference $\Delta = 0.3\%$, refractive index of the cladding $n_{cl} = 1.46$ and core radius $\alpha = 4\ \mu m$. The number of guided modes depends on the value of the normalized frequency V which remains constant during the heat treatment.

We applied Galerkin's method to determine the propagation constants, effective indices and modal fields of low-order linearly-polarized modes $LP_{ml}$. The number of basis functions (along the radial direction) used in our calculations was $N = 100$ for each azimuthal number $m$ in order to reach convergence. We define the cut off normalized frequency $V_c$ to be the smallest value of the modal parameter V for which a particular mode can propagate. At this cut off value, $U = V = V_c$. There is no cut off for the fundamental mode $LP_{01}$. The values of $V_c$ for the next few excited modes are given in Table 2.

Table 3 gives the calculated values of the modal parameter U for some low-order modes based on both Garlekin's and variational methods and the relative errors are determined with respect to the results by Galerkin's method. Our Variational results are generally larger than the numerical ones obtain by Galerkin's method and the relative error for each mode decreases as the normalized frequency increases.

Table 2
Cutoff values of the normalized frequency $V_c$ calculated by Galerkin's method.

| Mode | $V_c$ | Mode | $V_c$ | Mode | $V_c$ |
|---|---|---|---|---|---|
| $LP_{02}$ | 3.328 | $LP_{03}$ | 5.923 | $LP_{11}$ | 2.594 |
| $LP_{12}$ | 5.081 | $LP_{13}$ | 7.577 | $LP_{21}$ | 4.339 |
| $LP_{22}$ | 6.761 | $LP_{23}$ | 9.216 | $LP_{31}$ | 6.027 |
| $LP_{32}$ | 8.425 | $LP_{33}$ | 10.856 | | |

In Figures 2 and 3, we plot the modal parameter U as a function of the normalized frequency V for the first three $LP_{1l}-$modes and the first three $LP_{2l}-$ modes, respectively. The effective indices as a function of wavelength for a few low-order modes are shown in Fig. 4. When the modes are close to the cut off, which is shown in the figure as the region of lowest refractive index, the modes are close to the radiation modes and discrepancies between the two methods appear.

The mode intensity distribution of the mode fields for the fundamental mode $LP_{01}$ (Fig. 5) as well as for the low-order modes $LP_{11}$,$LP_{21}$ (Fig. 6 and Fig. 7, respectively) are also calculated by the two methods and compared. As it is seen from Fig.5, as the wavelength increases towards the cut off value, the variational approach gives less accurate modal fields.

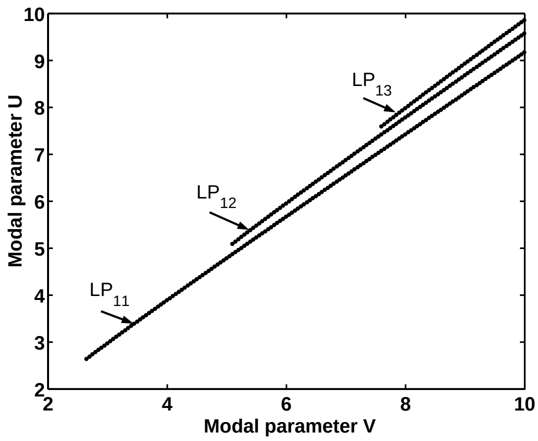


Fig. 2: Plots of the modal parameter U as a function of the normalized frequency V for the first three $LP_{1l}-$modes.

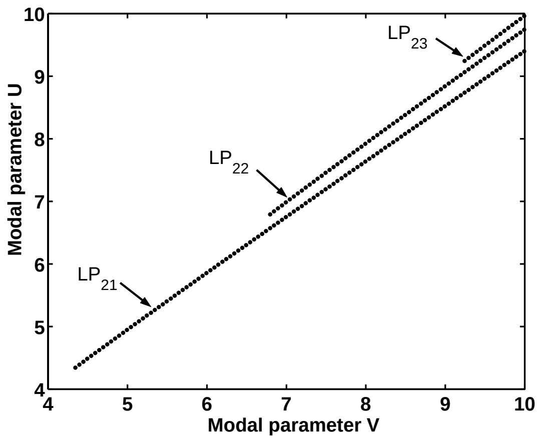


Fig. 3: Plots of the modal parameter U as a function of the normalized frequency V for the first three $LP_{2l}-$modes.

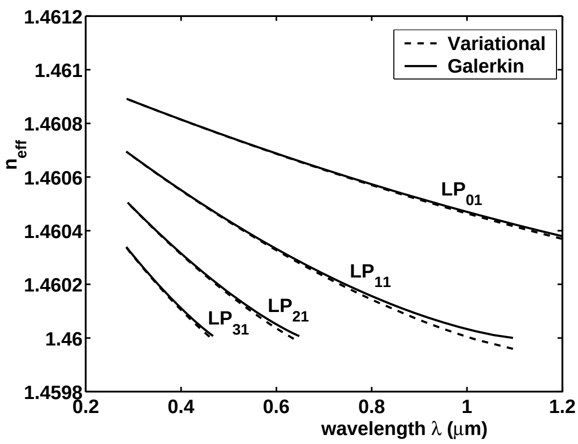


Fig. 4: Effective indices $n_{eff}$ of the first three $LP_{m1}-$modes as a function of the wavelength.

Table 3
Comparison of the values of $U$ obtained using Galerkin's and Variational methods for TEC fibers

| Mode | $V$ | $U$(Galerkin) | $U$(Variational) | Error% |
|---|---|---|---|---|
| | 1.5 | 1.4739 | 1.4790 | 0.347 |
| | 2.0 | 1.9328 | 1.9365 | 0.190 |
| $LP_{01}$ | 2.5 | 2.3823 | 2.3848 | 0.108 |
| | 3.0 | 2.8266 | 2.8284 | 0.066 |
| | 3.5 | 3.2678 | 3.2692 | 0.043 |
| | 3.0 | 2.9802 | 2.9884 | 0.274 |
| | 3.5 | 3.4418 | 3.4474 | 0.162 |
| $LP_{11}$ | 4.0 | 3.8955 | 3.8996 | 0.105 |
| | 4.5 | 4.3444 | 4.3475 | 0.072 |
| | 5.0 | 4.7900 | 4.7925 | 0.052 |
| | 4.5 | 4.4903 | 4.5000 | 0.215 |
| | 5.5 | 5.4083 | 5.4134 | 0.094 |
| $LP_{21}$ | 6.5 | 6.3081 | 6.3114 | 0.052 |
| | 7.5 | 7.1985 | 7.2008 | 0.033 |
| | 8.5 | 8.0831 | 8.0849 | 0.022 |

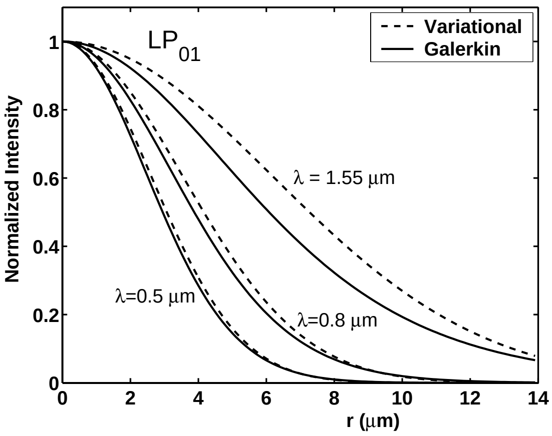


Fig. 5: Normalized intensity distribution of the $LP_{01}$ mode of the TEC-fiber for different wavelengths. The radius of the expanded core of the fiber is $A = 8\mu m$.

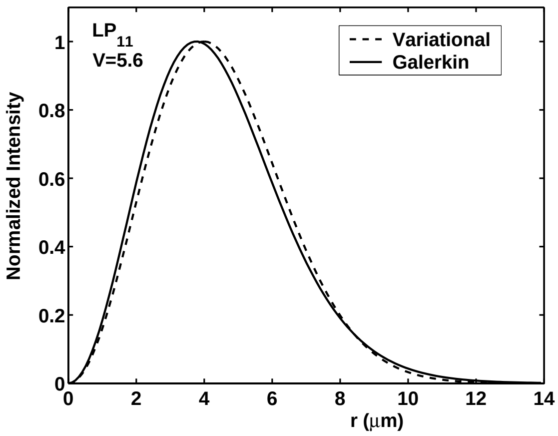


Fig. 6: Normalized intensity distribution of the $LP_{11}$ mode of the TEC-fiber for $V = 5.6$. The radius of the expanded core of the fiber is $A = 8\mu m$.

## 6 Concluding Remarks

We have presented an efficient numerical analysis of propagation constants, dispersion relations and mode fields for the low-order modes of Thermally-Diffused Expanded Core (TEC) Fiber. Our analysis is based on Galerkin's method in cylindrical coordinates. We expanded the field of a guided mode with orthogonal Laguerre-Gauss functions along the radial direction.

We have also developed a variational approach in which the mode fields are well approximated by simple analytic functions depending on a variational parameter. Our comparative study for low-order modes, shows that the variational results agree well with the exact numerical results obtained by Galerkin's method. On the other hand, for the variational analysis of higher order modes, a series expansion of Lagueere-Gauss functions as trial function is needed [13]. Our numerical analysis can be used in calculations of coupling co-

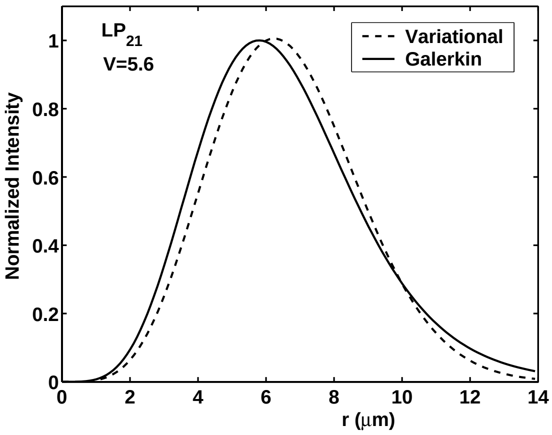


Fig. 7: Normalized intensity distribution of the $LP_{21}$ mode of the TEC-fiber for $V = 5.6$. The radius of the expanded core of the fiber is $A = 8\mu m$.

efficients, misalignment losses and source to fiber coupling when low-order modes in TEC-fibers are excited. Such calculations are applicable to the designing of various fiber-integrated type optical devices and micro-optical switching systems.

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