Upon Parametric Sensitivity Used in Damping Active Control for Human Body Protection Against Vibrations

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Abstract: - The topic of human body protection against vibrations is of high interest in the scientific world, due to the occupational diseases vibration induces. This paper presents the work performed in this field at Transilvania University of Brasov, having as final objective to develop advanced models of the vibration phenomenon and its influence on the human body, in order to further propose active control systems for protection against vibrations. Different vibration isolation techniques are discussed and the use of parametric sensitivity in damping active control is proposed.

Key-Words: - Human body vibration, Active vibration isolation, Sensitivity analysis, Mathematical model, Prediction model.

1 Introduction

Understanding and analyzing the dynamic behavior of structures and structural components represent an issue of utmost importance in the design process. Taking into account the present market features and the strong concurrence, the products economic and ecologic assessment has advanced to such an extent that over-dimensioned or pour performance projects are not tolerated anymore.

One sector that should give priority to enhancing the vibration isolation is the one related to human body vibrations. The research in the field revealed three frequency intervals with negative influence on the human body: range 0-2 Hz, with the following exposed occupations: aeronautic, maritime transport staff; range 2-20 Hz, with the following exposed occupations: drivers of trucks, vehicles (for intra-factory transport), tractors (agricultural, forest), escavators, bulldozers, concrete platforms, workers around fixed machines that transmit vibrations through the floor; range 20-200 Hz, with the following exposed occupations: all occupations that use vibratory machines and tools, acting on the hand-arm system: miners (pneumatic hammers for rock boring, hydro-electric power plants, railways, etc., workers form machine building (rivetting, casted parts cleaning, etc.), forestry workers (using mechanical sews), roads construction workers, etc.

The occupational diseases due to long term vibration exposure divide this problem in two distinct categories [1]:

- ***W**hole **B**ody **V**ibration (known by the acronym WHB)* – with effects on the entire body;
- ***H**and **A**rm **V**ibrations (known by the acronym HAV)* – transmit significant accelerations and displacements only to the hand-arm system.

A joint research team from Transilvania University of Brasov, Romania, consisting of engineers, physicians and computer scientists is preoccupied to develop advanced models of the vibration phenomenon and its influence on the human body, in order to further propose active control systems for protection against vibrations.

2 The Research Approach

The research approach, presented in Fig. 1, aims to develop systems for reducing vibrations induced uncomforting. The new design strategies, by seat and cabin dampers changes, together with a thorough understanding of the operation condition for the aimed vehicles or machine tools, may lead to diminishing vibration effects on human operators.

The research strategy illustrated uses four approaches to acquire information. The arrows connecting the approaches depict the information flows. The black thin arrows correspond to validations of the prediction model of discomfort and the thick arrows correspond to the modification procedure for vehicle designs when using the prediction model of discomfort [2].

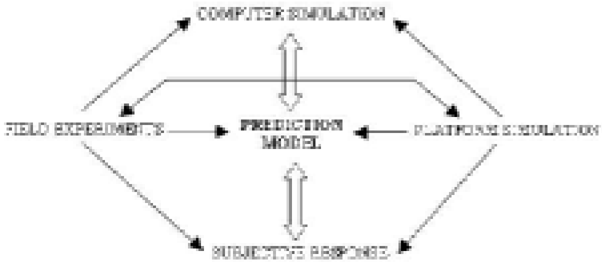


Fig. 1 The research approach for analyzing and diminishing the discomfort induced by human body vibration

3 Active Vibration Isolation

Starting from the lumped parameter model proposed by Rosen and Arcan [3], a study regarding active vibration isolation has been performed, in order to develop active control systems for protection against vibrations.

There are a number of different damping isolation techniques to use such as passive and active damping. Active damping isolators are usually expensive and have high energy consumption [2]. Therefore they are least favored by most vehicle designers, for examples. Passive damping systems are often categorized as linear and non-linear e.g. progressive damping isolators. Speaking again about vehicles vibrations, in the literature a linear damping isolator with a low damping ratio was found to offer the best vibration protection on smooth surfaces and higher damping ratio was found to preferable on a rough track. The progressive damping systems are preferable on rough roads and forklifts are most often driven on smooth surfaces, except for occasional transients. In general, a damping ratio between 0.2 and 1.0 is used (e.g. [2])

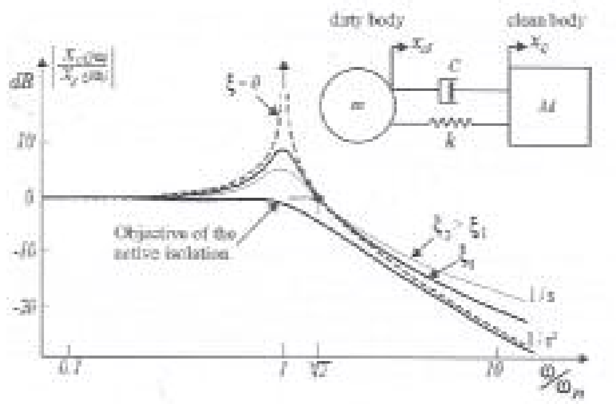
3.1 Passive Isolation

Considering the model of 'dirty body/clean body' isolation problem [4] presented in Fig. 2, x_d denotes the dirty body motion that represents the disturbances and x_c is the clean body displacement and represents the system output. The passive isolation system involves a spring and a damper, the system transmissibility being defined as:

$$\frac{X_c(s)}{X_d(s)} = \frac{1+2\xi_s/\omega_n}{1+2\xi_S/\omega_n + s^2/\omega_n^2} \qquad (1)$$

As it is presented in Fig. 2, the amplitude diagram yields to the following observations:

- for $\omega < \sqrt{2}\omega_n$, all the curves are larger than 1 and become smaller than 1 for $\omega > \sqrt{2}\omega_n$; the critical frequency $\sqrt{2}\omega_n$ delimits the isolator attenuation and amplification domains;
- for $\xi = 0$, the high frequency decay rate is $1/s^2$, while very large amplitudes occur near ω_n (the natural frequency of the spring-mass system);
- the damping reduces the amplitude at resonance and tends to reduce the effectiveness at high frequency; the high frequency decay rate becomes



1/s

Fig. 2 FRF of passive isolator transmissibility for different values of the damping [4]

The above observations lead to the conclusion that the design of a passive isolator involves a 'trade-off' between the resonance amplification and the high frequency attenuation. The ideal isolator should have a frequency dependent damping, which is high for values below critical frequency $\sqrt{2}\omega_n$ - to reduce the amplification peak, and low for values above $\sqrt{2}\omega_n$ - to improve the decay rate.

As a consequence, the objective of the active isolation system can be formulated: to achieve no amplification below ω_n and an appropriate decay rate at high frequency (Fig. 2).

3.2 The 'sky-hook' damper

Considering the single axis isolator presented in Fig. 3a, it consists of a spring k acting in parallel with the force actuator F_a (with no damping in the isolator). The absolute acceleration of the clean body $\ddot{x}_c$ is measured with an accelerometer and an integral controller is used so that:

$$F_a = -g \cdot s \cdot X_c \qquad (2)$$

It yields the resulting force control is proportional to the clean body absolute velocity – this is the reason why this type of control is called 'sky-hook damper' (Fig. 3b).

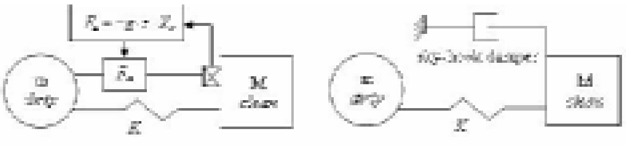


Fig. 3 a) Isolator with acceleration feed-back; b) equivalent 'sky-hook' damper [4]

The closed-loop transmissibility is given by the following relation:

$$\frac{X_c(s)}{X_d(s)} = \left[\frac{M}{K}s^2 + \frac{g}{K}s + 1\right]^{-1}. \qquad (3)$$

From this relation it may be observed that the corner frequency is at $\omega_n = \sqrt{K/M}$, the high frequency decay rate is $1/s^2$ and the control gain g can be chosen so that the isolator is critically damped ($\xi = 1$): $g = 2\sqrt{KM}$. This transmissibility fulfils the objective of the active isolation (Fig. 2).

3.3 Force Feedback

If the clean body is rigid, the acceleration is proportional to the total force transmitted by the interface: $F=F_a+F_k$. Therefore, the sky-hook damper can be obtained from the control configuration presented in Fig. 4.

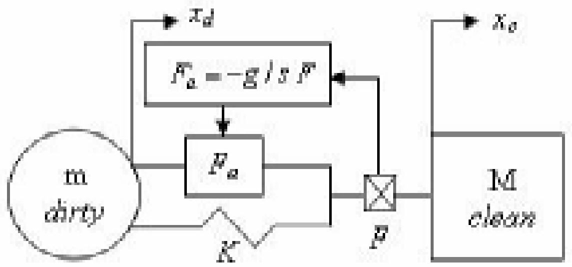


Fig. 4 Force feedback isolator [4]

In this case, the acceleration sensor from Fig. 3 has been substituted with the force sensor. The control methods based on acceleration feedback and on force feedback seems to be equivalent for the isolation of rigid bodies. The literature emphasizes two advantages of force feedback [4]:

- **Sensitivity**: force sensors with a sensitivity of 10^{-3}N are common and commercially available; accelerometers with sensitivity needed for our research purposes are more difficult to find.
- **Stability** when the clean body is flexible: the sky-hook damper seems to be stable only for small gain (conditionally stable) when the clean body becomes flexible, so that the corner frequency of the isolator overlaps with the natural frequency of the clean body.

The total force transmitted by the isolator is the sum of the control force F_a and the spring force $K \cdot \Delta x$, where Δx is the relative displacement of the two structures along the isolator axis:

$$F = K \cdot \Delta x - F_a, \qquad (4)$$

or

$$\frac{F(s)}{F_a(s)} = K\frac{\Delta X(s)}{F_a(s)} - 1 \qquad (5)$$

From relation (5) yields the open-loop transfer function F/F_a is the sum of $K\Delta x / F_a$ and a negative unit feedthrough. The transfer function given by the ration between the output and the input, $\Delta x / F_a$, has all its residues positive and its poles and zeros alternate along the imaginary axis. The FRF $F(\omega)/F_a(\omega)$ (obtained for $s = j\omega$, which is purely real when the system is undamped) yields from the FRF, by moving it along the ordinate axis by the amount of feedthrough; this changes the location of zeros, without changing the interlacing property.

4 Sensitivity Analysis

Sensitivity analysis represents a relatively new concept which has been introduced in structural dynamics; it consists of determination of modal parameters change as a result of the change of the system mass, stiffness or damping [5]. The mathematical basis are in Taylor's series development, so the method is an approximate one: just one or two serial terms are used for estimation of parameters change.

This approach is proposed to be used in the damping isolators design process, for enhancing the human body protection against vibrations.

4.1 The Mathematical Model

Let us consider a multidimensional linear viscous damping system [6], with n degrees of freedom, described by equation (6):

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}. \qquad (6)$$

By applying the Laplace transform for null initial conditions, the equation (6) becomes:

$$\left(p^2[M] + p[C] + [K]\right)\{X(p)\} = \{F(p)\} \qquad (7)$$

This last equation can be transformed in a general eigenvalue problem [6]:

$$\left(p[A]+[B]\right)\{Y\}=\{F'\} \quad (8)$$

where $[A]=\begin{bmatrix}[0] & [M]\\ [M] & [C]\end{bmatrix}$, $[B]=\begin{bmatrix}-[M] & [0]\\ [0] & [K]\end{bmatrix}$, $\{Y\}=\begin{Bmatrix}p\{X\}\\ \{X\}\end{Bmatrix}$, $\{F\}=\begin{Bmatrix}\{0\}\\ \{F\}\end{Bmatrix}$.

The eigenvalue equation corresponding to equation (8), respectively relation (7):

$$|\lambda\cdot[A]+[B]|=0 \quad , \quad (9)$$

results in N eigenvalues (the system poles) $\lambda_i=\sigma_i+j\omega_i$ and their complex conjugates, assembled in the diagonal matrix $\lfloor{}^\setminus\Lambda_\setminus\rfloor$, together with the corresponding eigenvectors $\{\Phi\}_i=\begin{Bmatrix}\lambda_i\{\Psi\}_i\\ \{\Psi\}_i\end{Bmatrix}$, assembled in the modal vectors matrix $[\Phi]$.

Considering the eigenvalue problem for a certain eigenvalue i:

$$\left(\lambda_i\cdot[A]+[B]\right)\cdot\{\Phi\}_i=\{0\} \quad (10)$$

in order to asses the influence of one modal parameter u on the system dynamic behavior, it is necessary to determine the eigenvalues and corresponding eigenvectors partial derivatives with respect to that parameter u: $\frac{\partial\lambda_i}{\partial u}$ si $\frac{\partial\{\Phi\}_i}{\partial u}$.

By multiplying equation (10) at left with the transpose of the eigenvector corresponding to eigenvalue i, it yields:

$$\{\Phi\}_i^t\cdot\left(\lambda_i\cdot[A]+[B]\right)\cdot\{\Phi\}_i=0 \quad (11)$$

The derivative of this equation with respect to modal parameter u is the following:

$$\frac{\partial\{\Phi\}_i^t}{\partial u}\cdot\left(\lambda_i\cdot[A]+[B]\right)\cdot\{\Phi\}_i+\{\Phi\}_i^t\cdot\frac{\partial\left(\lambda_i\cdot[A]+[B]\right)}{\partial u}\cdot\{\Phi\}_i+\{\Phi\}_i^t\cdot\left(\lambda_i\cdot[A]+[B]\right)\cdot\frac{\partial\{\Phi\}_i}{\partial u}=0 \quad (12)$$

Taking into account relation (10), equation (12) becomes:

$$\{\Phi\}_i^t\cdot\frac{\partial\left(\lambda_i\cdot[A]+[B]\right)}{\partial u}\cdot\{\Phi\}_i=0\,, \quad (13)$$

or:

$$\{\Phi\}_i^t\cdot\left(\frac{\partial\lambda_i}{\partial u}\cdot[A]+\lambda_i\cdot\frac{\partial[A]}{\partial u}+\frac{\partial[B]}{\partial u}\right)\cdot\{\Phi\}_i=0\,. \quad (14)$$

If the orthogonality conditions are considered [6], the sensitivity of pole i with respect to the modal parameter u change is given by the following relation:

$$\frac{\partial\lambda_i}{\partial u}=-\frac{1}{a_i}\cdot\{\Phi\}_i^t\cdot\left(\lambda i\cdot\frac{\partial[A]}{\partial u}+\frac{\partial[B]}{\partial u}\right)\cdot\{\Phi\}_i\,. \quad (15)$$

Since the eigenvectors set-up a base for the $2N\times 2N$ vector space, the partial derivative of an eigenvector i with respect to the parameter u is a linear combination of these eigenvectors:

$$\frac{\partial\{\Phi\}_i}{\partial u}=\sum_{r=1}^{2N}g_{ir}\{\Phi\}_r\,, \quad (16)$$

where: g_{ir} are coefficients and the last N eigenvectors represent the complex conjugates of the first N eigenvectors.

The identification of the g_{ir} coefficients allows us to further calculate the modal shapes sensitivity. By calculating the derivative of relation (10) and combining it with relation (16), it yields:

$$\left(\lambda_i\cdot[A]+[B]\right)\cdot\left(\sum_{r=1}^{2N}g_{ir}\cdot\{\Phi\}_r\right)+\frac{\partial\left(\lambda_i\cdot[A]+[B]\right)}{\partial u}\cdot\{\Phi\}_i=0 \quad (17)$$

By multiplying at left equation (17) and taking into account the orthogonality conditions, it result the expression of g_{im} coefficients, where $i\neq m$:

$$g_{im}\cdot\{\Phi\}_m^t\cdot\left(\lambda_i\cdot[A]+[B]\right)\cdot\{\Phi\}_m=-\{\Phi\}_m^t\cdot\frac{\partial\left(\lambda_i\cdot[A]+[B]\right)}{\partial u}\cdot\{\Phi\}_i\,, \quad (18)$$

$$g_{im}\cdot a_m\cdot\left(\lambda_i-\lambda_m\right)=-\{\Phi\}_m^t\cdot\left(\frac{\partial\lambda_i}{\partial u}\cdot[A]+\lambda_i\cdot\frac{\partial[A]}{\partial u}+\frac{\partial[B]}{\partial u}\right)\cdot\{\Phi\}_i\,, \quad (19)$$

$$g_{im}=\frac{1}{\left(\lambda_m-\lambda_i\right)}\cdot\frac{1}{a_m}\cdot\{\Phi\}_m^t\cdot\left(\lambda_i\cdot\frac{\partial[A]}{\partial u}+\frac{\partial[B]}{\partial u}\right)\cdot\{\Phi\}_i \quad (20)$$

By derivation of the orthogonality condition based on matrix [A] [6], the g_{ii} are obtained:

$$\frac{\partial\{\Phi\}_i^t}{\partial u}\cdot[A]\cdot\{\Phi\}_i+\{\Phi\}_i^t\cdot\frac{\partial[A]}{\partial u}\cdot\{\Phi\}_i+\{\Phi\}_i^t\cdot[A]\cdot\frac{\partial\{\Phi\}_i}{\partial u}=0 \quad (21)$$

By introducing relation (16) in the above equation it yields:

$$+2\cdot\{\Phi\}_i^t\cdot[A]\cdot\sum_{r=1}^{2N}g_{ir}\cdot\{\Phi\}_r=-\{\Phi\}_i^t\cdot\frac{\partial[A]}{\partial u}\cdot\{\Phi\}_i \quad (22)$$

By applying the orthogonality conditions, relation (22) becomes:

$$g_{ii} = -\frac{1}{2a_i} \cdot \{\Phi\}_i^t \cdot \frac{\partial[A]}{\partial u} \cdot \{\Phi\}_i \,. \qquad (23)$$

By combining equations (16), (20) and (23), it finally results the modal shapes sensitivity, given by the relation:

$$\frac{\partial\{\Phi\}_i}{\partial u} = -\frac{1}{2a_i} \cdot \{\Phi\}_i^t \cdot \frac{\partial[A]}{\partial u} \cdot \{\Phi\}_i \{\Phi\}_i + \sum_{r=1, r\neq i}^{2N} \frac{1}{\lambda_r - \lambda_i} \cdot \cdot \frac{1}{a_r} \cdot \{\Phi\}_r^t \cdot \left(\lambda_i \cdot \frac{\partial[A]}{\partial u} + \frac{\partial[B]}{\partial u}\right) \cdot \{\Phi\}_i \{\Phi\}_r \qquad (24)$$

By expressing relations (15) and (24) in terms of initial mode shapes vectors (unexpended – see relation 21), of mass, stiffness and damping matrices, these equations become:

$$\frac{\partial\lambda_i}{\partial u} = -\frac{1}{a_i} \cdot \{\Psi\}_i^t \cdot \left(\lambda_i^2 \frac{\partial[M]}{\partial u} + \lambda_i \frac{\partial[C]}{\partial u} + \frac{\partial[K]}{\partial u}\right) \cdot \{\Psi\}_i \qquad (25)$$

and, respectively,

$$\frac{\partial\{\Psi\}_i}{\partial u} = -\frac{1}{2a_i} \cdot \{\Psi\}_i^t \cdot \left(2 \cdot \lambda_i \cdot \frac{\partial[M]}{\partial u} + \frac{\partial[C]}{\partial u}\right) \cdot \{\Psi\}_i \cdot \cdot \{\Psi\}_i + \sum_{r=1, r\neq i}^{2N} \frac{1}{\lambda_r - \lambda_i} \cdot \frac{1}{a_r} \cdot \{\Psi\}_r^t \cdot \cdot \left(\lambda_i^2 \cdot \frac{\partial[M]}{\partial u} + \lambda_i \cdot \frac{\partial[C]}{\partial u} + \frac{\partial[K]}{\partial u}\right) \cdot \{\Psi\}_i \{\Psi\}_r \qquad (26)$$

If structural changes are taking place in the system, such as local change of mass, stiffness switch between two degrees of freedom of certain linear elastic elements, or linear viscous damping switch between two degrees of freedom, it is important to know the mass, stiffness or damping matrices, respectively.

– if u parameter is a local mass with respect to k d.o.f.:

$$\frac{\partial\lambda_i}{\partial m_k} = -\lambda_i^2 \cdot \frac{\Psi_{ki}^2}{a_i}$$

$$\frac{\partial\Psi_{ji}}{\partial m_k} = -\lambda_i \cdot \frac{\Psi_{ki}^2}{a_i} \cdot \Psi_{ji} + \Psi_{ki} \cdot \sum_{r=1, r\neq i}^{2N} \frac{\lambda_i^2}{\lambda_r - \lambda_i} \cdot \frac{\Psi_{kr} \cdot \Psi_{jr}}{a_r} \qquad (27)$$

– if u is a linear viscous damping between two d.o.f.s de k and l:

$$\frac{\partial\lambda_i}{\partial c_{kl}} = -\lambda_i \cdot \frac{(\Psi_{ki} - \Psi_{li})^2}{a_i}$$

$$\frac{\partial\Psi_{ji}}{\partial m_k} = -\frac{1}{2} \cdot \frac{(\Psi_{ki} - \Psi_{li})^2}{a_i} \cdot \Psi_{ji} + (\Psi_{ki} - \Psi_{li}) \cdot \sum_{r=1, r\neq i}^{2N} \frac{\lambda_i}{\lambda_r - \lambda_i} \cdot \frac{(\Psi_{kr} - \Psi_{lr}) \cdot \Psi_{jr}}{a_r} \qquad (28)$$

– if u is a linear stiffness between two d.o.f.s de k and l:

$$\frac{\partial\lambda_i}{\partial k_{kl}} = -\frac{(\Psi_{ki} - \Psi_{li})^2}{a_i}$$

$$\frac{\partial\Psi_{ji}}{\partial k_{kl}} = (\Psi_{ki} - \Psi_{li}) \cdot \sum_{r=1, r\neq i}^{2N} \frac{1}{\lambda_r - \lambda_i} \cdot \frac{(\Psi_{kr} - \Psi_{lr}) \cdot \Psi_{jr}}{a_r} \qquad (29)$$

4.2 Interpretation of Parametric Sensitivity

Analyzing the sensitivity expressions for the system poles and the system mode shapes, respectively, some observations yield, which are further presented [7].

It is very important the remark that, in order to calculate the λ_i system pole sensitivity it is sufficient to know the corresponding modal vector (relation 21). On the other hand, the calculation of modal shapes sensitivity needs knowledge about all the mode shapes (relation 22). Generally, the modal model consists of a limited number of modes, which negatively influence the accuracy of modal shapes sensitivity calculus.

Since the sensitivity analysis gives first order approximations, the results can be considered as accurate only for little change of the modal parameters. Problems about accuracy could also be generated by the existence of very close resonance frequencies.

Another important issue regarding sensitivity analysis accuracy is that the results are strongly influenced by the estimation accuracy of the modal parameters. They are further introduced in the sensitivity relations, for calculus. The quality of parameter estimation is limited, on one hand, by the number of measured modes, the impossibility of measuring the rotational d.o.f.s, the measurement precision, the approximate feature of the estimation methods; all these problems occur when the experimental data are used. On the other hand, when analytical data are used, other issues may occur and limit the quality of parameter estimation: number of computed modes, limited model d.o.f.s, inaccuracies regarding the modeling methods, neglected damping.

5 Conclusion

Human body vibrations due to different categories of causes represent nowadays an important research issue for the scientists. A way for their diminishing and control consists in developing active damping systems to be adapted to the vibration sources. Some alternatives regarding the systems isolation against human body vibration (interval 0-2 Hz - exposed occupations: aeronautic,maritime transport staff; interval 2-20 Hz - drivers of trucks, vehicles, tractors, escavators, bulldozers, concrete platforms, workers around fixed machines that transmit vibrations through the floor; interval 20-200 Hz - exposed occupations: all occupations that use vibratory machines and tools, acting on the hand-arm system: miners, workers form machine building) have been discussed and the use of parametric sensitivity in damping active control has been proposed.

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