Total Fatigue Life of Structural Components

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Abstract: - The present paper considers total fatigue life of structural components. Mathematical models used to simulate the initiation and propagation processes are quite different. In this work the initiation phase is modeled by using strain-life and cyclic stress-strain curves while the propagation phase uses crack growth rate versus strain energy density theory. Fatigue crack growth rate depends not only on the load amplitude, but also on the morphology of crack path. Based on strain energy density theory, a fatigue crack growth model is developed to predict the lifetime of fatigue crack growth for cracked structural components. The validity of the model is established with two cases: a center-crack plate and plate with a hole.

Key Words: - Low-cycle fatigue, crack initiation, crack growth, strain energy density, finite elements

1 Introduction

In order to simulate the initiation and propagation processes different mathematical models are used [1].. The initiation phase is usually modeled using strain-life and cyclic stress-strain curves while the propagation phase uses crack growth rate versus stress intensity curves [2,3]. The strain-life method assumes similitude between the material in a smooth specimen tested under strain control and the material at the notch. For a given load sequence, the fatigue damage in the specimen and the notch root are considered to be similar and so their lives will also be similar. The local stress-strain history must be determined, either by analytical or experimental methods. For the stress analysis finite element modeling is usually required. The present study is aimed at developing a fatigue crack propagation model based on the specific energy incorporating the cyclic deformation properties obtained from a low cycle fatigue test. This concept has been used because the highly strained zone ahead of the crack is very much like a small low cycle fatigue specimen. Also it is more advantageous to form a model mainly based on a low cycle fatigue properties since they are easier to obtain experimentally. The highly strained zone (process zone) very near to the crack is taken for the energy balance instead of taking the whole plastic zone with the premise that mainly it is the zone, where damage accumulates [3]. The FEM can be used for stress analysis and stress intensity calculations.

2 Computation Methods of Total Fatigue Life

The total fatigue life can be divided in two phases: Crack initiation and Crack propagation. In the following chapters will be given procedures that are proposed for total life computation.

2.1 Crack Initiation

Cyclic fatigue can be expressed with use of stress (σ_a- N_i), strain (ε_a- N_i) and energy (W_a-N_i) notations. The Wohler curve is a classical fatigue description in the case of a high number of cycles. It can be written as a straight line in log-log scale:

$$\log N_i = A - m \log \sigma_a, \qquad (1)$$

where constants A, m on that curve could be determined from ASTM standard.

Equation (1) can be also expressed as the experimental notation:

$$\sigma_a = \sigma_f^{/} (2N_i)^b, \quad (2)$$

where b is the fatigue strength exponent (b = - 1/m), and

$$\sigma_f^{/} = 10^{(A + \log 2)/m}, \quad (3)$$

where $\sigma_f^{/}$ is the fatigue strength coefficient. Fatigue description in the strain notation using the total strain amplitude can be divided into elastic and plastic parts

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} \, . \quad (4)$$

Using equation (2) we can write the equation for the relationship between the elastic strain amplitude and a number of cycles up to crack initiation:

$$\varepsilon_{ae} = \frac{\sigma_f^{/}}{E} \left(2N_i\right)^b . \quad (5)$$

Similarly as in equation (5) we can write the relationship between the plastic strain amplitude and a number of cycles:

$$\varepsilon_{ap} = \varepsilon_f^{/} \left(2N_i\right)^c , \quad (6)$$

where $\varepsilon_f^{/}$ is the coefficient of the fatigue plastic strain, and c is the exponent of the fatigue plastic strain. Thus, the total strain amplitude relationship in terms of cycles to crack initiation N_i, is written as:

$$\varepsilon_a = \frac{\sigma_f^{/}}{E} \left(2N_i\right)^b + \varepsilon_f^{/} \left(2N_i\right)^c . \quad (7)$$

The strain – fatigue life relationship as expressed by equation (7) was modified by Morrow in order to incorporate the effects of mean stress for the plain fatigue condition [1]. Changing formula to crack initiation life (7) results in the following:

$$\varepsilon_a = \frac{\sigma_f^{/} - \sigma_m}{E} \left(2N_i\right)^b + \varepsilon_f^{/} \left(2N_i\right)^c , \quad (8)$$

where σ_m is mean stress. If we assume that the cyclic strain curve is of Ramberg–Osgood type, the plastic strain amplitude can be expressed as:

$$\varepsilon_{ap} = \left(\frac{\sigma_a}{k^{/}}\right)^{\frac{1}{n^{/}}} , \quad (9)$$

where $k^{/}$ is the coefficient of the cyclic strain hardening, and $n^{/}$ is the exponent of the cyclic strain hardening. Thus, we obtain the relationship between the total strain amplitude and the stress amplitude

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{k^{/}}\right)^{\frac{1}{n^{/}}} . \quad (10)$$

One typical cyclic stress-strain curve is shown in Fig. 1. Point A is corresponding to twice of the cyclic yield stress and strain ($\sigma_y^{/}$, $\varepsilon_y^{/}$) and point B is the point corresponding to stress and strain ranges with $2N_i$=1, i.e. $\Delta\sigma = 2\sigma_f^{/}$ and $\Delta\varepsilon = ((2\sigma_f^{/}/E) + 2\varepsilon_f^{/})$.

Needed energy absorbed till fracture W_c can be defined if we know the cyclic stress strain

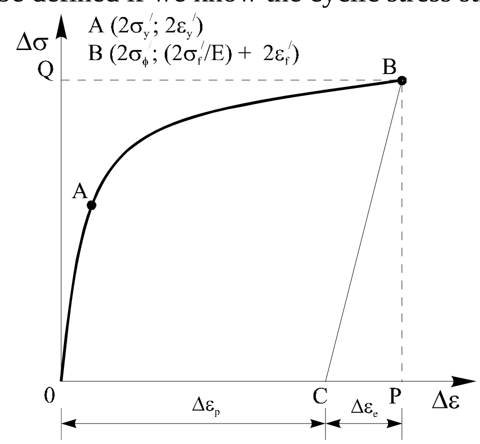

Fig. 1. Cyclic stress – strain curve.

curve (fig.1), as the area below the curve. W_c is the area OABCO which can be defined when we subtract from the area OQBPO the sum of two areas, area OABQO andareaCBPC:

$$W_c = 2\sigma_f^{/} \left(\frac{2\sigma_f^{/}}{E} + 2\varepsilon_f^{/}\right) - \int_0^{2\sigma_f^{/}} \Delta\varepsilon \, d(\Delta\sigma) - \frac{1}{2}\left(2\sigma_f^{/} \frac{2\sigma_f^{/}}{E}\right)$$

after integrating:

$$W_c = 2\sigma_f^{/} \varepsilon_f^{/} - \frac{4n^{/}}{1 + n^{/}} \left(\frac{1}{k^{/}}\right)^{1/n} \left(\sigma_f^{/}\right)^{\frac{1 + n^{/}}{n^{/}}} - \frac{2\sigma_f^{/}}{E} . \quad (11)$$

Last term can be neglected since it is too small when compared to other two, and so:

$$\varepsilon_f^{/} = \left(\frac{\sigma_f^{/}}{k^{/}}\right)^{1/n^{/}} , \quad (12)$$

and relation for energy absorbed till fracture W_c became:

$$W_c = \frac{4}{1 + n^{/}} \sigma_f^{/} \varepsilon_f^{/} . \quad (13)$$

2.2 Crack Propagation

The cyclic stress and plastic strain components of Hutchinson, Rise and Rosengren crack tip singularity fields [4], ahead of crack tip is given by equations (14) and (15)[4] in the case of a small scale yielding conditions :

$$\Delta\sigma_{ij} = \Delta\sigma_y^{/} \left(\frac{\Delta K_I^2}{\alpha^{/}\, \Delta\sigma_y^{/\,2}\, I_{n^/}\, r} \right)^{\frac{n^/}{(1+n^/)}} \overline{\sigma_{ij}}\left(\theta; n^/\right) \quad (14)$$

$$\Delta\varepsilon_{ij} = \frac{\alpha^{/}\Delta\sigma_y^{/}}{E} \left(\frac{\Delta K_I^2}{\alpha^{/}\, \Delta\sigma_y^{/\,2}\, I_{n^/}\, r} \right)^{\frac{1}{(1+n^/)}} \overline{\varepsilon_{ij}}\left(\theta; n^/\right) \quad (15)$$

where ΔK_I, $\Delta\sigma_y^{/}$, are the range of stress intensity factors under mode I loading and cyclic yield stress ($\Delta\sigma_y^{/} \approx 2\sigma_y^{/}$), respectively. For the central-crack plate ΔK_I can be expressed as:

$$\Delta K_I = \beta\,\Delta\sigma\sqrt{\pi\, a}\,,$$

where a is crack length, $\Delta\sigma$ - stress range and $\beta = \sqrt{\frac{w}{\pi a}\, tg\frac{\pi a}{w}}$ .Terms r and θ are the radial and angular positions, respectively, of any point from the crack tip, as shown in Fig. 2. Further the $\overline{\sigma_{ij}}\left(\theta; n^/\right)$ and $\overline{\varepsilon_{ij}}\left(\theta; n^/\right)$ are non-dimensional angular distribution functions. Term $\alpha^{/}$ is given by:

$$\alpha^{/} = \frac{2E}{\left(2k^{/}\right)^{1/n^/}\,\Delta\sigma_y^{/\,\left(n^/-1\right)/n^/}} \quad (16)$$

and $I_{n^/}$ is the non-dimensional parameter of exponent $n^{/}$.

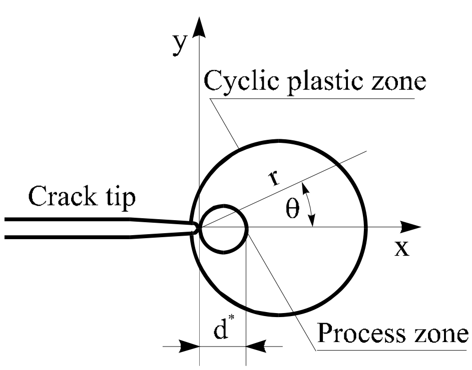

Fig. 2. Zones in front of a crack tip.

From equations (14) and (15) multiplication of equivalent stress and strain along the crack line ($\theta = 0$) is given by:

$$\Delta\sigma_{eq}\,\Delta\varepsilon_{eq} = \frac{\Delta K_I^2\;\overline{\sigma_{eq}}\left(0; n^/\right)\overline{\varepsilon_{eq}}\left(0; n^/\right)}{E\, I_{n^/}\, r} .(17)$$

Since we defined the relation for multiplied equivalent stresses it is possible to define the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [4,5], like:

$$\Delta W_p = \left(\frac{1-n^/}{1+n^/}\right)\Delta\sigma_{eq}\,\Delta\varepsilon_{eq}\,, \quad (18)$$

or

$$\Delta W_p = \left(\frac{1-n^/}{1+n^/}\right)\frac{\Delta K_I^2\;\overline{\sigma_{eq}}\left(0; n^/\right)\overline{\varepsilon_{eq}}\left(0; n^/\right)}{E\, I_{n^/}\, r} (19)$$

Last equation presents distribution of plastic strain energy density per cycle ahead of the crack tip. In Eq. (19) unknown values are the angular distribution functions of equivalent stress and strain and parameter $I_n^{/}$. The region near the crack tip is widely recognized as divided in two important zones. The area near the crack tip is known as the process zone [6] and in that region damage mainly accumulates. Another region is the area between cyclic plastic zone and process zone (Fig.2).

As a very important parameter of the first region is the length of the process zone ahead of crack tip d^* (Fig.2), which can be analyzed as a constant by same authors [7] and as function of ΔK_I by the others [6,8]. As a function of ΔK [8], d^* can be expressed by:

$$d^* = \frac{\Delta K_I^2 - \Delta K_{th}^2}{\pi\, E\, \sigma_y^{/}} \quad (20)$$

where ΔK_{th} is range of threshold stress intensity factor. Since we defined the length d^* it is possible to determine the plastic energy ω_p dissipated per cycle per unit growth. Needed relation for ω_p can be obtained by integrating the relation for the plastic strain energy density (19) and if r is substituted by d^*, or:

$$\omega_p = \int_0^{d^*}\left(\frac{1-n^/}{1+n^/}\right)\frac{\Delta K_I^2\,\psi\left(n^/\right)}{E\, I_{n^/}\, d^*}\, dr \quad (21)$$

after integrating,

$$\omega_p = \left(\frac{1-n'}{1+n'}\right)\frac{\Delta K_I^2\,\psi}{E\,I_{n'}}, \qquad (22)$$

where $\psi = \left(\overline{\sigma_{ij}}\left(0;n'\right)\overline{\varepsilon_{ij}}\left(0;n'\right)\right)$. As external load is increased from zero at the crack tip, it is blunted first and start to open when the stress intensity factor reaches the threshold value K_{th}. Further loading makes the crack tip more blunt and when load reaches the maximum value, the crack moves by some distance. Fatigue crack growth takes place when crack is open, so the driving parameter of crack is $K_{max} - K_{th}$ instead of $K_{max} - K_{min}$ (because, $\Delta K_I = K_{max} - K_{min}$):

$$\omega_p = \left(\frac{1-n'}{1+n'}\right)\frac{\psi\left(K_{max} - K_{th}\right)^2}{E\,I_{n'}}. \qquad (23)$$

Crack would grow by a length δa in a cycle when energy absorbed in the same cycle W_c equals plastic energy dissipated in the process zone, i.e. energy absorbed per unite growth of crack equal to the plastic energy dissipated within the process zone per cycle. Previously stated can be presented by the following formula:

$$W_c\,\delta a = \omega_p. \qquad (24)$$

The expression for δa is:

$$\delta a = \frac{da}{dN_p} = \left(\frac{1-n'}{1+n'}\right)\frac{\psi\left(K_{max} - K_{th}\right)^2}{E\,I_{n'}\left(\frac{4}{1+n'}\sigma'_f\,\varepsilon'_f\right)} \qquad (25)$$

or

$$\delta a = \frac{da}{dN_p} = \frac{\left(1-n'\right)\psi}{4\,E\,I_{n'}\,\sigma'_f\,\varepsilon'_f}\left(K_{max} - K_{th}\right)^2. \qquad (26)$$

The above crack relation is a particular case for R = 0 where $R = K_{min}/K_{max}$. For R = 0, $K_{max} = \Delta K$, $K_{th} = \Delta K_{th}$, where is ΔK_{th} a material constant and it is sensitive to stress ratio R. An important aspect in the fatigue design of structural elements is the ″design of safe cracks″ on the basis of the crack growth threshold. Of much concern, however, is the effect of stress ratio on the fatigue threshold stress intensity range, ΔK_{th}. Regarding the R-effect on ΔK_{th}, many relations, mostly empirical, have been proposed, some of which are [9]:

$\Delta K_{th} = K_{max}$ (1 - R); $\Delta K_{th} = \Delta K_{th0}$ (1 - R)$^{1/2}$; $\Delta K_{th} = \Delta K_{th0}$ (1 - R)$^{\gamma}$; $\Delta K_{th} = \Delta K_{th0}$ (1 – R^2) and $\Delta K_{th} = \Delta K_{th0}$ [(1 - R)/(1 + R)]$^{1/2}$, (27)

where ΔK_{th0} is the threshold stress intensity range at R = 0 and γ is a material constant which varies from zero to unity. For most of materials constant γ can be 0.71 [8]. Despite the large number of proposed relations (27) between ΔK_{th} and R, a general relation does not seem to exist. Such a general relation would be most welcome.

Substituting the value of K_{max} and K_{th} for a general stress ratio R, the fatigue crack growth relations are expressed as given:

$$\delta a = \frac{da}{dN_p} = \frac{\left(1-n'\right)\psi}{4\,E\,I_{n'}\,\sigma'_f\,\varepsilon'_f}\left(\Delta K - \Delta K_{th0}\left(1-R\right)^{\gamma}\right)^2, \qquad (28)$$

$$\delta a = \frac{da}{dN_p} = \frac{\left(1-n'\right)\psi}{4\,E\,I_{n'}\,\sigma'_f\,\varepsilon'_f}\left(\Delta K - \Delta K_{th0}\left(\frac{1-R}{1+R}\right)^{1/2}\right)^2. \qquad (29)$$

It is clear from equations (28) and (29) that with an increase in stress ratio R, ΔK_{th} decreases and ΔK increases, increasing the fatigue crack growth rate but the influence is stronger in stage I (near threshold region of da/dN_p vs ΔK_I plot) where ΔK_I and ΔK_{th} are comparable than in other regions of the plot. From fatigue crack growth relations (26) and (27) or (28) and (29) can be seen that they require only mechanical and fatigue properties E, σ'_f, ε'_f and n', which presents great advantage by application of this procedure.

3 Numerical Results

To illustrate proposed computation procedures in fatigue lives estimations two numerical examples are included: (1) plate with hole under constant amplitude cyclic loads and (2) a center-crack plate under cyclic loads.

Example 1: Initial fatigue life calculation

In this example, crack initiation fatigue life estimation of the plate with central hole was carried out. For stress analysis FEM was used [10]. The structural element was subjected to constant amplitude axially loading. Material characteristics of medium strength steel under cyclic loading are: b = -0.081; c = - 0.67; $\sigma_f^{/}$ = 1165.6 MPa; $\varepsilon_f^{/}$ = 1.142; $n^{/}$ = 0.123; $k^{/}$ = 1062.1 MPa; S_y = 648.3 MPa; S_u = 786.2 MPa; E = 2.069 10^5 MPa; Geometry characteristics are: w = 25.4 mm; 2R = 12.8 mm; t = 7.68 mm; L = 100 mm.

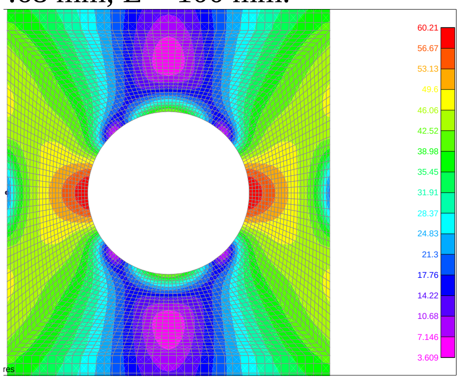

Fig. 3. Stress distribution for the plate with central hole.

Here, we use Morrow criteria (Eq. 8) to determine number of cycles up to crack initiation. Obtained results were compared with available experimental results [11].

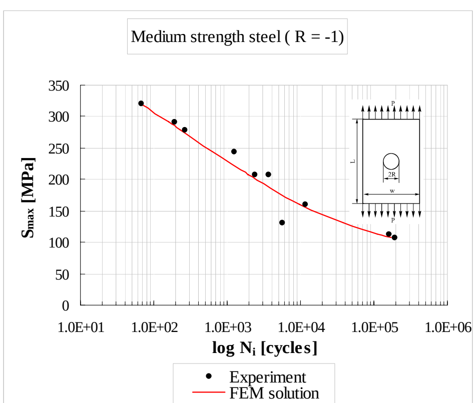

Fig. 4. Fatigue life up to crack initiation of the plate with central hole using FEM (Medium strength steel, R = -1, experiment [11]).

A comparison of experimental data [11] with the predictions shows excellent agreement.

Example2a:Crack growth rate calculation

A fatigue crack growth rate was determined from analytical method using equations (26) and (27) in order to compare with experimental results. A central-crack plate is made of 8630 Steel. Material characteristics of 8630 Steel under cyclic loading are: $\sigma_f^{/}$ = 1936 MPa; $\varepsilon_f^{/}$ = 0.42; $n^{/}$ = 0.195; $k^{/}$ = 2267 MPa; $S_y^{/}$ = 334 MPa; E = 207 10^3 MPa; ΔK_{th0} = 13; $I_n^{/}$ = 3.082; ψ = 0.94794. Geometry characteristics are: w = 500 mm; 2a = 24 mm; L = 100 mm.

Based on known characteristics of material and geometry, calculated values of (da/dN_p) and ΔK_I are presented in Fig.5 (for different relations for ΔK_{th}).

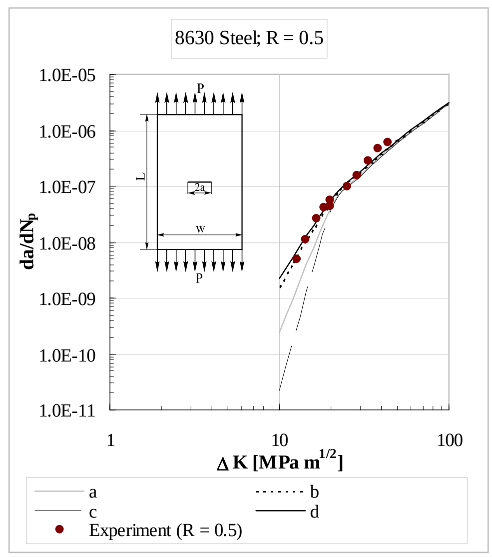

Fig. 5. Comparison of the predicted crack growth rate with experimental data [12].

In Fig.5 all curves have different ΔK_{th}, so:

a) $\Delta K_{th} = \Delta K_{th0}\,(1 - R)^{1/2}$, b) $\Delta K_{th} = \Delta K_{th0}\,(1 - R)^{\gamma}$, c) $\Delta K_{th} = \Delta K_{th0}\,(1 - R^2)$ and d) $\Delta K_{th} = \Delta K_{th0}\,((1 - R)/(1 + R))^{1/2}$.

Predictions from Eq. (29) show the best agreement with experimental data (Fig. 5).

Example2b:Crack growth life calculation

In this example fatigue crack growth prediction was considered. Structural element is a central-crack plate. External loading is axial with constant amplitude. Material characteristics of 2219 T851 Al alloy under cyclic loading are: $\sigma_f^{/}$ = 613 MPa; $\varepsilon_f^{/}$ = 0.35; $n^{/}$ = 0.121; $k^{/}$ = 710 MPa; $S_y^{/}$ = 334 MPa; E = 71 10^3 MPa; ΔK_{th0} = 30; $I_n^{/}$ = 3.067; ψ = 0.95152, K_c = 60 MPa $\sqrt{m}$. Geometry characteristics are:

w = 500 mm; 2a = 24 mm; L = 100 mm.

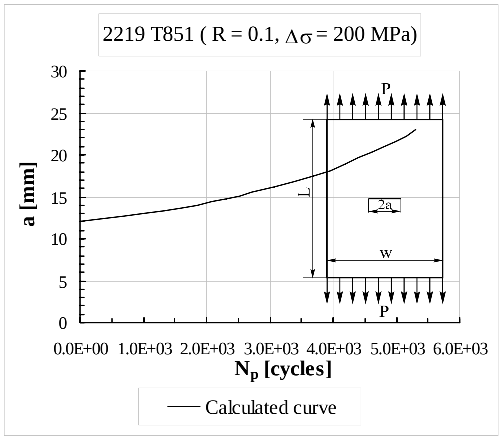

Fig. 6. Crack length versus number of cycles (σ_{max} = 222.22 MPa, σ_{min} = 22.22 MPa).

Figure 6 shows the relationships (integrating eq.29) between crack length a and number of cycles for crack propagation N_p.

4 Conclusion

This research presents efficient computation procedure for total fatigue life of structural components. The mathematical models used to simulate the initiation and propagation processes are quite different. This procedure uses minimal number of material properties. In this work the initiation phase is modeled using strain-life and cyclic stress-strain curves and FEM, while the propagation phase uses crack growth rate versus strain energy density theory. Based on strain energy density theory, a fatigue crack growth model is developed to predict the lifetime of fatigue crack growth for cracked structural components. Computation results are compared with experiments. Good correlation between numerical and experimental results is obtained.

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