# THE LOW LOSS WINDING THEORY

VÍT BRŠLICA Department of Electrical Engineering University of Defense in Brno Kounicova 65, CZ 612 00 Brno CZECH REPUBLIC http://www.unob.cz

*Abstract:* - The usual machine design is based on good electric and magnetic materials exploitation, which is described, by flux density and current density. The limiting factor is only the temperature rise. Every power range has its typical densities values. Small machines design results always in relatively low efficiency and high winding losses. For the battery-operated machines is high efficiency very important. Nowadays many programs are to disposal for computer aided design and optimization, but there is no clear procedure where the winding losses are directly inputted. The method for numerical calculation derived in this paper enables easy dimensions specification of machine with very low and predefined rates of winding losses. The complete design is presented in example.

Key-Words: Electric machine design, winding losses, optimization, low power, high efficiency

# **1** Introduction

This paper was prepared as the result of the bicycle electric drive project. The original idea was to rebuild the vacuum cleaner machine on reduced voltage (and speed). After the basic measurements were completed, the project was scratched due to awful machine's parameters and consequently the presented analysis was prepared.

The battery-operated vehicle has very limited energy to disposal; therefore all power losses must be watched carefully. From the complete chain of battery-to-wheel energy conversion is this paper focused to the motor analysis. For the light traction (bicycle and mini-bike category up to 40 - 60 km/h) the rated power is about 1 kW  $\pm$  3 times more or less it is from 300 W to 3 kW.





The low power motors are traditionally supposed to have low efficiency, typically about 50 - 60% (Fig.1.) and at supply from mains it makes no problem. Neither in thermal problems, because low power motors are usually easy to cool with big enough surface according to their volume, nor in the price of wasted energy.

Also the efficiency dependence on the load ratio is very important, because the full power is in vehicle used only rarely and most of time its motor works with partial load about 10-20% of rated value. The standard dependence efficiency vs. load ratio is rapidly decreasing at low load. Both brings bad results in energy budget of batteryoperated vehicle, where every wasted kilowatt-hour, which is not converted to mechanical energy, is very expensive, because it represents the growth of total weight of vehicle and reduced operational radius.

## 2 Motor main dimensions

The volume of motor depends not on its power P, but on its torque M.

$$\mathbf{P} = \mathbf{M} \boldsymbol{\omega} \tag{1}$$

Where is:

 $\omega$  the angular velocity of rotor.

It is well known the formula with Carter's factor K, which is after adaptation to SI units:

$$P = K D^2 l_i \omega$$
 (2)

where is:

- D outer rotor diameter
- $l_i$  active wire length or length of iron packet  $K = \frac{1}{2} \alpha \pi AB$  (3)
- $\alpha$  pole pitch cover factor (or filed form factor)
- A linear current density (on the rotor surface)

B flux density in the air gap (max. value)

In the linear current density A is hidden standard **current density J**, together with slot depth and the slot-filling factor.

#### 2.1. Optimal Speed

There are two possible solutions of traction drives. Gearless motors for direct drive have always the low velocity and big mass. Special motor construction is necessary.

Gearing solution represents more components of drive, higher costs and maintenance, but it brings high-speed low -mass and low-volume motor at the same power. In last years the rapid progress is evident in gearing technology in many parameters – the single-stage ratio, noise and efficiency (cog belts for example).

The higher torque the lower speed and oppositely, low torque means low volume, but also high frequency in the armature coils and in armature magnetic circuit "AC iron". Another way said low volume of active parts can be reached at high speed only and it is therefore by speed limited.

Low torque means also thin shaft and little bearings with lower mechanical losses.

It must be said here that the air gap diameter (D) is limited by circumference velocity because of aerodynamic losses (friction on rotor surface) and centrifugal force (mechanical strength of rotor).

There is not the angular speed  $\boldsymbol{\omega}$  but the surface speed  $\mathbf{v}$  is important for the mechanical losses (aero-dynamical friction).

Power density [W/kg] grows with speed evidently, but the self-ventilating effect grows as well. However the temperature rise must be checked explicitly. The limit surface speed is about 30 - 35 m/s.

#### 2.2. Excitation losses and PM field

High efficiency motors are preferably solved with zero excitation input, it means PM (permanent magnet) field and armature supply control to maximal power factor, which minimizes not only joule losses in armature winding, but reduces also the flux decreasing.

The speed control cannot use the flux reduction and at high speed it brings wasting increase of iron losses, because the frequency increase cannot be compensated by flux decrease.

The low loss winding of field coils brings a bit worse efficiency at low speed, but better iron losses at high speed.

#### 2.3. Number of Poles and Frequency

The low speed motors are from principle of big diameter, they are supplied by low frequency and the pole number increase is easy to do, without extreme frequency and allied troubles in iron losses. The frequencies up to 400 Hz are acceptable. The highquality materials application enables important iron losses reduction.

To keep low frequency and also because of small diameter the preferable number of poles is four.

For much higher frequencies the ferrite magnetic circuit should be designed with much lower flux density B. But it is not the question of discussed power range, but smaller powers.

#### 2.4. The Kind of Machine

The typical low power machine represents the DC or the universal one (Fig.2) with mechanical commutation and serial connected field coils, used in tools and other home devices as are the drill, vacuum cleaner or wash machine. All these machines are typically operated at high speed, which brings low torque and low dimensions (low volume and mass). The bulk production brings low price, therefore such machines are easy to obtain.

Their iron sheets are from very cheap material, therefore the iron losses are not optimal and field coils current is unnecessarily high.



Fig.2. Cross-section area of universal motor

DC machines for the automotive industry, estimated for use in vehicles are also characterized with relatively poor efficiency.

The other kinds of machines (Asynchronous or Synchronous) differ in main dimensions non-essentially.

The aspect of sliding contact is not important in efficiency account; the voltage drop on brush is comparable to voltage on the semiconductor power (5)

(7)

switch. Therefore the higher value of voltage must be preferred when parameters are postulated.

# **3** Winding Losses Criterion Theory

Joule losses in armature  $\Delta P_{ja}$  can be described by basic formula:

$$\Delta P_{ja} = R I^2 \tag{4}$$

Instead this formula the same value can be calculated using current density J (local value):

 $\Delta P_{ja} = \rho l_z S_{cu} J^2$ Where is:

ρ resistivity,

 $l_z$  one half of the coil turn length,

S<sub>cu</sub> wire cross section area

The machine power can be calculated from speed and Maxwell force of all the armature wires similar way as is described in (2):

 $P = F_{\Sigma} v = F_{\Sigma} \omega r$  (6) Where is:

v circumferential speed,

r outer rotor radius (r = D/2)

 $F_{\rm x}$  amount of circumferential forces

$$F_{\Sigma} = \alpha B l_i J S_{cu}$$

Where is:

 $\alpha$  field form factor (usually  $\alpha = 2/3$ )

B air gap flux density maximal value (under pole shoe)

l<sub>i</sub> active length of wire (iron length)

J current density in armature winding

S<sub>cu</sub> total area of "copper" in the cross section

Substitution (7) in an equation (6) gives the machine power:

$P = \alpha B l_i J S_{cu} v = \alpha B l_i J S_{cu} \omega r$	(8)
Using the substitution	
$l_i = \beta l_z$	(9)

 $\beta$  Active to total length of wire

The coefficient  $\beta$  varies practically within the range:

$$\beta = (1/1, 2 - 1/1, 5) = (5/6 - 2/3)$$
(10)

The rated winding losses formularization can be:

$$\mathbf{p_{ja}} = \frac{\rho \mathbf{l_z} \mathbf{S_{cu}} \mathbf{J}^2}{\alpha \beta \mathbf{B} \mathbf{l_z} \mathbf{J} \mathbf{S_{cu}} \omega \mathbf{r}} =$$
$$= \frac{\rho \mathbf{J}}{\alpha \beta \mathbf{B} \omega \mathbf{r}} = \frac{\rho \mathbf{J}}{\alpha \beta \mathbf{B} \mathbf{v}}$$
(11)

Where:

 $\mathbf{p}_{ja} = \Delta P_{ja} / P \tag{12}$ 

It means, that only electric and magnetic utilization is determining the losses rate and from this formula can be derived the resulting relation for current density:

$$\mathbf{J} = \frac{1}{\rho} \mathbf{p}_{j\mathbf{a}} \alpha \beta \mathbf{B} \mathbf{v}$$
(13)

#### **3.1. Design fundamentals**

Above stated equation can be applied separately in any case of machine design, but it can be with advantage transformed into the set of tables or graphs for comfortable utilization. The presented results are prepared for common magnetic flux density 0,8 T and the coefficients  $\alpha$  and  $\beta$  values are supposed to be equal 2/3 each.

B=0.9 I		B=	0	.8	Т	٦
---------	--	----	---	----	---	---

	pja=	0,01	0,02	0,03	0,04	0,05
[km/h]	v [m/s]			J	[A/mm2]	
10,8	3	0,587	1,173	1,760	2,347	2,933
21,6	6	1,173	2,347	3,520	4,693	5,867
32,4	9	1,760	3,520	5,280	7,040	8,800
43,2	12	2,347	4,693	7,040	9,387	11,733
54	15	2,933	5,867	8,800	11,733	14,667
64,8	18	3,520	7,040	10,560	14,080	17,600
75,6	21	<mark>4,107</mark>	8,213	12,320	16,427	20,533
86,4	24	4,693	9,387	14,080	18,773	23,467
97,2	27	5,280	10,560	15,840	21,120	26,400
108	30	5,867	11,733	17,600	23,467	29,333

Table 1. Current density for various speed and winding losses ratio



Fig.3. Current density for various speed and winding losses ratio

Better review than tabular record gives the graph, which

is presented in Fig.3.

For the lower flux density 0,6T are results only in graphical form in Fig.4. The current density decrease is evident with consequent growth of machine volume.



Fig.4. Current density for various speed and winding losses ratio

### 4. Design Example

To obtain some imagination about influence of derived criteria on the motor proportions and its basic properties, the essential calculi were completed for machines from 250W to 5kW with required parameters:

v = 25 m/s	
B = 0.8 T	
$p_{ja} = 1\%$	

From Table 1 the current density value is:

 $J = 4,93e6 \text{ A/m}^2$ 

Modifying (8), the main dimensions are to be identified

$$\mathbf{P} = \frac{2}{3} \operatorname{\omega r} \mathbf{B} \, \mathbf{l}_{i} \, \mathbf{J} \, \mathbf{S}_{cu} \tag{14}$$

using substitutions (supposing the slot dept h = 0,1 D and slot space equal to teeth space):

$$\begin{split} l_{i} &= 2R \qquad \text{squared rotor !} \\ S_{cu} &= k_{p} \ S_{dr} \qquad \text{copper area from slot area} \\ k_{p} &= 0.5 \qquad \text{slot filling factor} \\ S_{dr} &\sim \frac{1}{2} \ (D^{2} - (0.8D)^{2}) \ (\pi/4) = 18\% \ S_{a} \\ S_{cu} &= 0.1 \ \pi \ r^{2} \end{split} \tag{15}$$

Where D is for rotor diameter, r is rotor semi-diameter,  $S_{dr}$  is the area of all slots and  $k_p$  is slot filling factor. The formula for machine power is:

$$\mathbf{P} = \frac{2}{3} \text{ v B 2 r J 0,1 } \pi r^2$$
 (16)

and from here the rotor radius results:

$$\mathbf{r} = \sqrt[3]{\frac{30\mathbf{P}}{4\pi\mathbf{vBJ}}} \tag{17}$$

Because the calculation input is the surface speed of rotation the angular frequency and speed must be calculated consequently.

The rest of design is not presented, it depends on the voltage, number of slots, number of commutator segments, number of poles and afterward it includes also stator design.

$\mathbf{P} = \mathbf{1000W}$	Р	R	D	ω	n
	[W]	[m]	[mm]	[rad/s]	[1/min]
v = 25 m/s	250	0 0,018	36,5	1372	13098
	500	0,023	45,9	1089	10396
B = 0,8 T	1000	0,029	57,9	864	8251
	1500	0,033	66,2	755	7208
pja = 1%	2000	0,036	72,9	686	6549
	2500	0,039	78,5	637	6080
A/m <sup>2</sup>	3000	0,042	83,5	599	5721
J = 4,93e6	3500	0,044	87,9	569	5435
	4000	0,046	91,9	544	5198
	4500	0,048	95,5	523	4998
	5000	0,049	99,0	505	4825

Table 2. Main dimension and speed for  $p_{ja} = 1\%$ 

Moreover there is in Table 2 the survey of basic rotor parameters for all the series of DC machines for light traction from 250W to 5 kW with very realistic values of rotor diameter and speed.



Fig.5. Current density for various speed and winding losses ratio – ferrite rotor

Similar way can be calculated alternative geometry with longer slim rotor (l>D), or lower magnetic flux.

Although the iron losses are not included into optimizing criterion, some orientation is very interesting. As was mentioned in the chapter 2.4 the material of rotor package is most important for resulting value, all depends on iron volume, frequency and flux density.

The survey in Table 3 allows having basic imagine on the volume and mass of rotor iron and proofs, that for good materials also the iron losses can be under one per cent.

The current densities from winding losses criteria for above mentioned ferrite core rotor, or powder iron cores when low value of diameter D leads to high frequency are derived in Fig.5.

$\mathbf{P} = 1000 \ \mathbf{W}$	Р	500	1000	1500
	D	46	58	66
	ω	1089	864	755
2 <b>p</b> = 4				
Slot depth h	mm	4,6	5,8	6,6
Teeth area	$mm^2$	513,1	814,4	1067,2
Rotor yoke height h <sub>jr</sub>	mm	5,8	7,3	8,3
Rotor yoke area	mm <sup>2</sup>	664,5	1054,9	1382,3
Rotor iron area	mm <sup>2</sup>	1177,6	1869,3	2449,5
V <sub>fe</sub> (iron volume)	ccm	54,1	108,2	162,3
Frequency	Hz	346,7	275,2	240,4
m <sub>fe</sub> (iron mass)	g	421,9	843,8	1265,7
f/50		6,9	5,5	4,8
$p_{1,50} = 1,8$	W/kg	12,5	9,9	8,7
P <sub>fe</sub>	W	5,3	8,4	11,0
P <sub>fe</sub> /P <sub>n</sub>	%	1,1	0,8	0,7

Table 3. Main rotor-iron data for machines from Table 1

### 5. CONCLUSIONS

The possible design of low power machines with efficiency over 90 percent is exactly proved. The way to low ratio of winding losses is in volume reduction together with speed increase.

The higher speed is chosen, the lower winding losses ratio results if reasonable values of current density are kept.

Presented procedure is suitable for preparing the first chart and fast balance-sheet, in the form of table sets or graphs similar to Table 1 and Fig.1, for various flux densities and other parameters. Consequently the optimizing methods based on CAD and similar numerical methods can by applied.

To reach the high efficiency, not only one component of losses should be minimized, but also all together losses are to be optimized. However volume minimizing bring the decrease of all components.

Other losses components are not included into account; neither iron nor mechanical losses are incorporated into suggested optimization calculus. However they are included (or hidden) in the values of speed and flux density. With the choice of good materials for magnetic core and good mechanical components (bearing) the no load losses can be also kept on acceptable level in units of percent.

Presented method can also help in rewinding or relabeling of machines to better parameters. Although all the derivation is presented on DC rotor, it is easy applicable on AC machines as well. In the modern, frequency controlled drives is the rotor speed not important and can be also over 3000 rpm. Thanks inserted converters the arbitrary value of speed can be easily reached.

#### References:

- Bršlica, V.: High efficiency PM motors for low power traction, *IXth International Workshop on Electric Machines*, Prague, Czech Republic, 2001, pp 85 – 90
- [2] Cigánek, L.: Stavba elektrických strojů (Electrical Machines Construction), SNTL Praha 1958
- [3] Bršlica, V.: The Low Power Machine Design for Specified Winding Losses XIVth International Symposium on Electric Machines, Prague, Czech Republic, 2006, pp 5
- [4] Alexejev A.E.: Konstrukce elektrických strojů, Praha 1952
- [5] Сергеев, П. С.: Проектирование электрических машин (Electrical machines design) Энергия Москва 1969
- [6] Kopylov I.P.: Stavba elektrických strojů, (Electrical Machines Construction) SNTL Praha - Mir Moskva, 1988
- [7] Henry-Baudot, J.: Elektrické stroje pro automatizaci (Electrical Machines for Automatics) SNTL Praha 1971
- [8] Reinboth, H.: Vlastnosti a použití magnetických materiálů, (Magnetic materials properties and use) SNTL Praha 1975