Voltage Sensitivity Analysis in MV Distribution Networks

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Abstract: - The paper presents a study on the assessment of node voltage sensitivity in distribution networks with respect to variations of node active and reactive powers. The work provides an analytical tool to quantify node voltages variation due to injections of node powers at any MV distribution network injection point.
Simple analytical expressions have been developed to link node voltages to node active and reactive powers through network electrical parameters, in order to define and calculate appropriate sensitivity coefficients.
A useful graphical representation is also given for the derived expressions, providing immediate access to qualitative and quantitative information on node voltages sensitivity. The proposed examples are referred to typical conductor sections used with underground cable lines and uninsulated overhead lines in MV distribution networks.

Key-Words: - Distribution Systems, Node Voltages, Sensitivity Analysis, Voltage Regulation.

1 Introduction

The aim of the work is to provide a simple analytical tool to quantify node voltage variations due to injections of active and reactive power at one or more nodes of MV distribution networks. In other words, the paper presents a study on the assessment of node voltages sensitivity with respect to variations of node powers.
Voltage sensitivity analysis is the base for the solution of various power system optimisation problems, related, for example, to voltage regulation, loss reduction, network expansion planning, optimal placement of reactive sources and generators, etc. [1], [2], [3].
In particular, the framework of the proposed study is the research of new solutions for innovative management of "active" distribution networks in order to implement real time control of power fluxes in presence of distributed generation (DG). The need for controlling active and reactive power injections to guarantee correct distribution operation in presence of DG calls for analytical tools that are suitable to be used by appropriate automatic control algorithms. This is, for example, the case of on-line control systems used to avoid that voltage and current constraints [4] are violated during normal operation of distribution networks.
Of course, the analytical expressions that will be presented are not intended to substitute existing powerful load flow programs, which are able, among other functions, to perform network sensitivity assessment. The considered expressions are useful to be integrated into *ad hoc* optimisation tools to control node active and reactive powers, e.g. in automatic voltage regulation procedures or in DG installation planning in distribution networks. In such a context, it is required to know network sensitivity in order to assess the effectiveness of possible contribution of distributed generators to voltage regulation procedures by controlling their power output.
In Section 2, the method used to obtain mathematical expressions that link node voltages to node active and reactive powers is described.
Further, linearised equations are described to derive useful closed analytical expressions to calculate node voltages. The difference in the results provided by the two formulations (linear and non linear) is small due to the fact that voltage drops are small as well in distribution networks. Consequently, linearised expressions can be used appropriately in assessing *sensitivity coefficients* for node voltages in this context. These coefficients are the elements of *sensitivity matrices*, called $[S_P]$ and $[S_Q]$, which contain measures of voltage variations due to, respectively, active and reactive node powers.
In Section 3, a graphical representation of the derived voltage sensitivity coefficients is given taking into account practical examples referred to MV networks. Such an approach is useful to get immediate perception of voltage sensitivity variation as node distance from the origin varies. Further, it can easily be highlighted how section and type of conductor (overhead line or underground cable line) influence network sensitivity.

2 Analytical Model

In this section the analytical model used to perform the sensitivity analysis will be described.

Let us consider a three-phase symmetrical, radial distribution network with *n* nodes and *n* branches, where we define as "nodes" the points of load connection, the points of line characteristics change and the junctions, and as "branches" the conductor segments between two nodes.

The nodes can be numbered according to the following rule [5]: the "origin" of the network (typically a HV/MV primary substation) takes the number *0*, while the other nodes are numbered sequentially imposing that a "receiving" node takes a number higher than the "sending" node nearer to it. The terms "receiving" and "sending" are used under the assumption that in a traditional radial network, i.e. without distributed generators, the power flow is directed from a lower to a higher number. The branches are identified by the same number as their receiving node, as shown in Fig. 1.

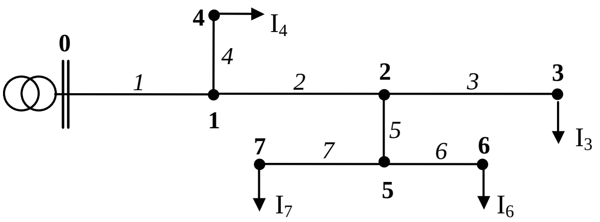

Fig. 1. One-line diagram of a three-phase symmetrical, radial distribution network.

This numbering method allows a simple storage of the network structure in a single square matrix (called incidence matrix, [*A*]) whose dimension is (*nxn*). In particular, the rows corresponds to the n branches and the columns to the nodes.

The elements of [A] describe the network topology and are equal to *1* if the node corresponding to column *j* is fed through the branch corresponding to row *i*, *0* otherwise.

The calculation of the branch flows is easily obtained applying the "mesh method" for network analysis. It can be easily shown that:

$$\left[\overline{J}\right]=\left[A\right]\cdot\left[\overline{I}\right] \tag{1}$$

where:

$\left[\overline{I}\right]$ is the vector of the load currents, dimension (*nx1*);

$\left[\overline{J}\right]$ is the vector of the branch currents, dimension (*nx1*).

The network complex impedance is equal to:

$$\left[\overline{Z}\right]=\left[A\right]^t\cdot\left[\overline{Z_b}\right]\cdot\left[A\right] \tag{2}$$

where $\left[\overline{Z_b}\right]$ is the diagonal matrix, dimension (*nxn*), whose elements are the complex impedances of the corresponding branches.

The main diagonal elements of $\left[\overline{Z}\right]$, $\left(\overline{Z}_{ii}\right)$, are equal to the sum of the branch impedances forming the path from the origin to the node *i*.

The off-diagonal elements, $\left(\overline{Z}_{ij}\right)$, are equal to the sum of the branch impedances forming the path from the origin to the common node of the paths formed by the origin and the nodes *i* and *j*, respectively.

Let us consider node h and its voltage phasor, $\overline{V}_h$. We will assume that $\overline{V}_0$ is known.

Let $\overline{\Delta V_h}$ be the voltage drop across branch h and $\overline{\Delta U_h}$ the total voltage drop from node 0 to node h:

$$\overline{\Delta U_h}=\overline{V}_0-\overline{V}_h \tag{3}$$

$$\left[\overline{\Delta U}\right]=\left[A\right]^t\cdot\left[\overline{\Delta V}\right] \tag{4}$$

where:

$\left[\overline{\Delta U}\right]$ is the column vector $(n\,x\,1)$ whose elements are the voltage drops indicated by $\overline{\Delta U_h}$;

$\left[\overline{\Delta V}\right]$ is the column vector $(n\,x\,1)$ whose elements are the voltage drops indicated by $\overline{\Delta V_h}$.

Vector $\left[\overline{\Delta V}\right]$ can be expressed as follows:

$$\left[\overline{\Delta V}\right]=\left[\overline{Z_b}\right]\cdot\left[\overline{J}\right] \tag{5}$$

As known, cross parameters can usually be neglected in analysis of distribution networks.

Substituting expression (5) in (4), we obtain the following expression for vector $\left[\overline{\Delta U}\right]$:

$$\left[\overline{\Delta U}\right]=\left[A\right]^t\cdot\left[\overline{Z_b}\right]\cdot\left[\overline{J}\right] \tag{6}$$

then, if we consider expression (1), we also have:

$$\left[\overline{\Delta U}\right]=\left[A\right]^t\cdot\left[\overline{Z_b}\right]\cdot\left[A\right]\cdot\left[\overline{I}\right] \tag{7}$$

Considering the definition of $\left[\overline{Z}\right]$ given by (3), expression (7) is equivalent to:

$$\left[\overline{\Delta U}\right]=\left[\overline{Z}\right]\cdot\left[\overline{I}\right] \tag{8}$$

Node voltages vector $\left[\overline{V}\right]$ is the given by the following:

$$\left[\overline{V}\right]=\left[\overline{V}_0\right]-\left[\overline{\Delta U}\right]=\left[\overline{V}_0\right]-\left[\overline{Z}\right]\cdot\left[\overline{I}\right] \tag{9}$$

For the *i-th* node, the complex power $\overline{S}_i$ is defined as:

$$\overline{S}_i=\overline{V}_i\,\overline{I}_i^*=P_i+jQ_i \tag{10}$$

where:

$\overline{V}_i$ is the voltage phasor at node *i*;

$\overline{I}_i$ is the current phasor at node *i*;

$\overline{I}_i^*$ is the complex conjugate of $\overline{I}_i$;

P_i is the net real power in the *i-th* node;

Q_i is the net reactive power in the *i-th* node.

If we express (10) by means of the corresponding matrixes we obtain:

$$\left[\bar{S}\right]=\left[\bar{V}\right]\cdot\left[\bar{I}^*\right]=[P]+j\cdot[Q] \qquad (11)$$

where $[P]$ and $[Q]$ are the column vectors, dimension $(n\,x\,1)$, whose elements are, respectively, the node active and reactive powers.
The complex conjugate of $\left[\bar{S}\right]$ is given by:

$$\left[\bar{S}^*\right]=\left[\bar{V}^*\right]\cdot\left[\bar{I}\right]=[P]-j\cdot[Q] \qquad (12)$$

From this equation, it is possible to obtain an expression for node currents vector $\left[\bar{I}\right]$:

$$\left[\bar{I}\right]=\frac{[P]-j\cdot[Q]}{\left[\bar{V}^*\right]} \qquad (13)$$

Consequently, substituting expression (13) in (9), $\left[\bar{V}\right]$ can be written as:

$$\left[\bar{V}\right]=\left[\bar{V_0}\right]-\left[\bar{Z}\right]\cdot\left(\frac{[P]-j\cdot[Q]}{\left[\bar{V}^*\right]}\right) \qquad (14)$$

Finally, considering that $\left[\bar{Z}\right]=[R]+j\cdot[X]$, where $[R]$ and $[X]$ are (nxn) matrixes, respectively, real and imaginary part of network impedance matrix, we obtain the following expression for $\left[\bar{V}\right]$:

$$\left[\bar{V}\right]=\left[\bar{V_0}\right]-([R]+j\cdot[X])\cdot\left(\frac{[P]-j\cdot[Q]}{\left[\bar{V}^*\right]}\right) \qquad (15)$$

2.1 Linear Expressions and Sensitivity Coefficients

Simplified linear expressions can be derived from (15) under the following hypotheses (commonly accepted in distribution networks analysis):
- the phase difference between node voltages is negligible and, as a consequence, if phasor $\bar{V_0}$ is chosen on the real axis, only the real part of voltage $[V]=real\left[\bar{V}\right]$ is considered;
- constant current models are considered for loads (node powers are referred to system nominal voltage instead of actual node voltage).

Consequently, expression (15), can be written as:

$$[V]=[V_0]-\frac{[R]\cdot[P]+[X]\cdot[Q]}{V_{nom}} \qquad (16)$$

The rms value of voltage at node i, V_i, can expressed as follows:

$$V_i=V_0-\frac{1}{V_{nom}}\cdot\left(\sum_{j=1}^{N}R_{ij}\cdot P_j+\sum_{j=1}^{N}X_{ij}\cdot Q_j\right) \qquad (17)$$

Such expressions have been compared to the ones derived from non linear equations (15). The difference in the results provided by the two formulations (linear and non linear) is small due to the fact that voltage drops are small as well in distribution networks. Consequently, linearised expressions can be used appropriately in assessing sensitivity coefficients for node voltages in this context.
As obvious, the voltage at the *i-th* node not only depends on the *i-th* node powers, but also on the powers injected or absorbed at the other network nodes:

$$V_i=V_i(P_1,P_2,...,P_n,Q_1,Q_2,...,Q_n) \qquad (18)$$

The total differential of function V_i is given by:

$$dV_i=\sum_{j=1}^{n}\frac{\partial V_i}{\partial P_j}\cdot dP_j+\sum_{j=1}^{n}\frac{\partial V_i}{\partial Q_j}\cdot dQ_j \qquad (19)$$

where we find the sensitivity coefficients, $\frac{\partial V_i}{\partial P_j}$ and $\frac{\partial V_i}{\partial Q_j}$, which, from (17), be expressed as:

$$\begin{cases}\dfrac{\partial V_i}{\partial P_j}=-\dfrac{1}{V_{nom}}\cdot R_{ij}\\[2mm] \dfrac{\partial V_i}{\partial Q_j}=-\dfrac{1}{V_{nom}}\cdot X_{ij}\end{cases} \qquad (20)$$

with $i, j=1,2,...,n.$

The above derivatives can be regarded as *voltage sensitivity coefficients* with respect to node powers variation. Their physical meaning is the following:
- for $i=j$ it provides voltage variation at the *i-th* node due to unity variation of the *i-th* power;
- for $i\neq j$ it provides voltage variation at the *i-th* node due to unity variation of the *j-th* power.

Considering the n equations given by expression (19) we have:

$$\begin{bmatrix}dV_1\\ ...\\ dV_n\end{bmatrix}=\begin{bmatrix}\frac{\partial V_1}{\partial P_1} & ... & \frac{\partial V_1}{\partial P_n} & \frac{\partial V_1}{\partial Q_1} & ... & \frac{\partial V_1}{\partial Q_n}\\ ... & ... & ... & ... & ... & ...\\ \frac{\partial V_n}{\partial P_1} & ... & \frac{\partial V_n}{\partial P_n} & \frac{\partial V_n}{\partial Q_n} & ... & \frac{\partial V_n}{\partial Q_n}\end{bmatrix}\cdot\begin{bmatrix}dP_1\\ ...\\ dP_n\\ dQ_1\\ ...\\ dQ_n\end{bmatrix} \qquad (21)$$

We can define a *sensitivity matrix*, $[S]$, $(n\,x\,2n)$, whose elements are the sensitivity coefficients defined by (20):

$$[S]=\begin{bmatrix}\frac{\partial V_1}{\partial P_1} & ... & \frac{\partial V_1}{\partial P_n} & \frac{\partial V_1}{\partial Q_1} & ... & \frac{\partial V_1}{\partial Q_n}\\ ... & ... & ... & ... & ... & ...\\ \frac{\partial V_n}{\partial P_1} & ... & \frac{\partial V_n}{\partial P_n} & \frac{\partial V_n}{\partial Q_n} & ... & \frac{\partial V_n}{\partial Q_n}\end{bmatrix} \qquad (22)$$

Matrix $[S]$ can be written as:

$$[S]=[S_P|S_Q] \quad (23)$$

in which the two sub-matrixes $[S_P]$ and $[S_Q]$, $(n\ x\ n)$, are highlighted:

$$\begin{cases} [S_p]=\begin{bmatrix} \frac{\partial V_1}{\partial P_1} & \dots & \frac{\partial V_1}{\partial P_n} \\ \dots & \dots & \dots \\ \frac{\partial V_n}{\partial P_1} & \dots & \frac{\partial V_n}{\partial P_n} \end{bmatrix} = -\frac{1}{V_{nom}}[R] \\ [S_Q]=\begin{bmatrix} \frac{\partial V_1}{\partial Q_1} & \dots & \frac{\partial V_1}{\partial Q_n} \\ \dots & \dots & \dots \\ \frac{\partial V_n}{\partial Q_1} & \dots & \frac{\partial V_n}{\partial Q_n} \end{bmatrix} = -\frac{1}{V_{nom}}[X] \end{cases} \quad (24)$$

Assessment of the elements of sensitivity matrix $[S]$, indicated by

$$(S_P)_{ij}=\frac{\partial V_i}{\partial P_j}=-\frac{1}{V_{nom}}\cdot R_{i\,j} \quad (25)$$

and

$$(S_Q)_{ij}=\frac{\partial V_i}{\partial Q_j}=-\frac{1}{V_{nom}}\cdot X_{i\,j} \quad (26)$$

allows to quantify voltage variations at each network node due to active and reactive power variations at any other node.

Expressions (25) and (26) are of general validity and can be applied to all radial distribution networks or radially operated distribution networks.

Sensitivity coefficients $(S_p)_{ij}$ and $(S_Q)_{ij}$ can be expressed as functions of line longitudinal parameters, (line resistance and reactance per kilometre) by introducing the following quantities:

$$r_{ij}=\frac{R_{ij}}{L_{ij}} \quad (27)$$

and

$$x_{ij}=\frac{X_{ij}}{L_{ij}} \quad (28)$$

where:

R_{ij} and X_{ij} are, respectively, real and imaginary part of impedance $\overline{Z}_{i\,j}=R_{i\,j}+j\cdot X_{i\,j}$;

$L_{i\,j}$ is:

- for $j=i$ the sum of the branch lengths forming the path from the origin (node *0*) to node *i*;
- for $j\neq i$ the sum of the branch lengths forming the path from the origin to the common node of the paths formed by the origin and nodes *i* and *j*.

Parameters r_{ij} and x_{ij} represents the weighted average, with respect to branches length, of the longitudinal parameters per kilometre of the branches belonging to the common path from the origin to nodes *i* and *j*.

Parameter r_{ij} is a function of:

- branches length;
- branches conductor section;
- branches conductor resistivity.

Parameter x_{ij} is a function of:

- branches length;
- branches conductor section;
- frequency;
- geometric characteristics of electrical lines.

Substituting expressions (27) and (28) in (25) and (26) we obtain:

$$(S_P)_{ij}=-\frac{1}{V_{nom}}\cdot L_{i\,j}\cdot r_{ij} \quad (29)$$

$$(S_Q)_{ij}=-\frac{1}{V_{nom}}\cdot L_{i\,j}\cdot x_{ij} \quad (30)$$

Such expressions show that the sensitivity coefficients depend on network extension (path length) and, respectively, on r_{ij} and $x_{ij.}$

Such expressions show that the most sensitive networks are the ones with extended lines and/or characterised by high value longitudinal parameters (r_{ij} and x_{ij}). Note that the ratio between $(S_P)_{ij}$ and $(S_Q)_{ij}$ is given by $r_{ij}\,/\,x_{ij}$.

Assuming the same section and type of conductor for all the network branches, expressions (27) and (28) can be written as:

$$X_{i\,j}=L_{i\,j}\cdot x \text{ and } R_{i\,j}=L_{i\,j}\cdot r$$

where *r* and *x* are line resistance and reactance per kilometre.

Since expressions (29) and (30) can be written as:

$$(S_P)_{ij}=-\frac{1}{V_{nom}}\cdot L_{i\,j}\cdot r \quad (31)$$

$$(S_Q)_{ij}=-\frac{1}{V_{nom}}\cdot L_{i\,j}\cdot x \quad (32)$$

then

$$\frac{(S_P)_{ij}}{(S_Q)_{ij}}=\frac{r}{x} \quad (33)$$

3 Graphic representation of $(S_P)_{ij}$ and $(S_Q)_{ij}$

Assuming the same section and type of conductor for all the network branches, it is possible to provide an interesting graphical representation of $(S_P)_{ij}$ and $(S_Q)_{ij}$.

Plotting sensitivity coefficients vs. $L_{i\,j}$, we obtain straight lines whose slop is, respectively, $-\frac{1}{V_{nom}}\cdot r$ and $-\frac{1}{V_{nom}}\cdot x$.

Such a graphical representation is useful to get immediate perception of sensitivity variation with node distance from the origin. Further, it can be easily highlighted how section and type of conductor (overhead line or underground cable line) influence the network sensitivity.

In the following, the graphical representation of voltage sensitivity coefficients $(Sp)_{ij}$ and $(S_Q)_{ij}$ for a 20 kV network is given, considering both cases of underground cable lines and uninsulated overhead lines with conductor section of 35mm^2 (Fig. 2) and 95 mm^2 (Fig. 3).

The following parameters have been used:

section=35mm^2: cable line r = 0.675 Ω/km
cable line x = 0.2 Ω/km
overhead line r = 0.519 Ω/km
overhead line x = 0.388 Ω/km

section= 95mm^2:cable line r = 0.249 Ω/km
cable line x = 0.18 Ω/km
overhead line r = 0.193 Ω/km
overhead line x = 0.357 Ω/km

From both figures it can be noted that the greater the node distance from the origin, the higher the voltage sensitivity. As for maximum distances considered for peripheral nodes, 15 km has been taken as a realistic value for actual distribution networks.

In Fig. 2 it is apparent that sensitivity with respect to active power injection is greater than the one with respect to reactive power injection. This is especially true for cable lines, in which r/x = 3.38 for the considered case.

In the nodes that are 15 km far from the origin the sensitivity coefficients reach the highest values:

cable line: S_P = - 0.507 kV/MW;
S_Q = - 0.150 kV/MVAR;

overhead line: S_P = - 0.390 kV/MW
S_Q = - 0.291 kV/MVAR

Further, it can be noted that an uninsulated overhead line is more sensitive than an underground cable line with respect to reactive power injections, *vice versa* with respect to active power injections. For the overhead lines considered in the example we have r/x = 1.34.

In conclusion, considering a conductor section of 35mm^2, passing from cable to overhead lines, S_P reduces by about 20%, while S_Q increases by 94%.

Fig. 3 shows that cable lines are more sensitive to injections of active power than to injections of reactive power (r/x=1.38). On the other hand, overhead lines are more sensitive to injections of reactive power than to injections of active power (r/x=0.54).

Further, cable lines are more sensitive than overhead lines with respect to active power injections, while overhead lines are more sensitive than cable lines with respect to reactive power injections.

Note that a greater conductor section determines lower values of voltage sensitivities.

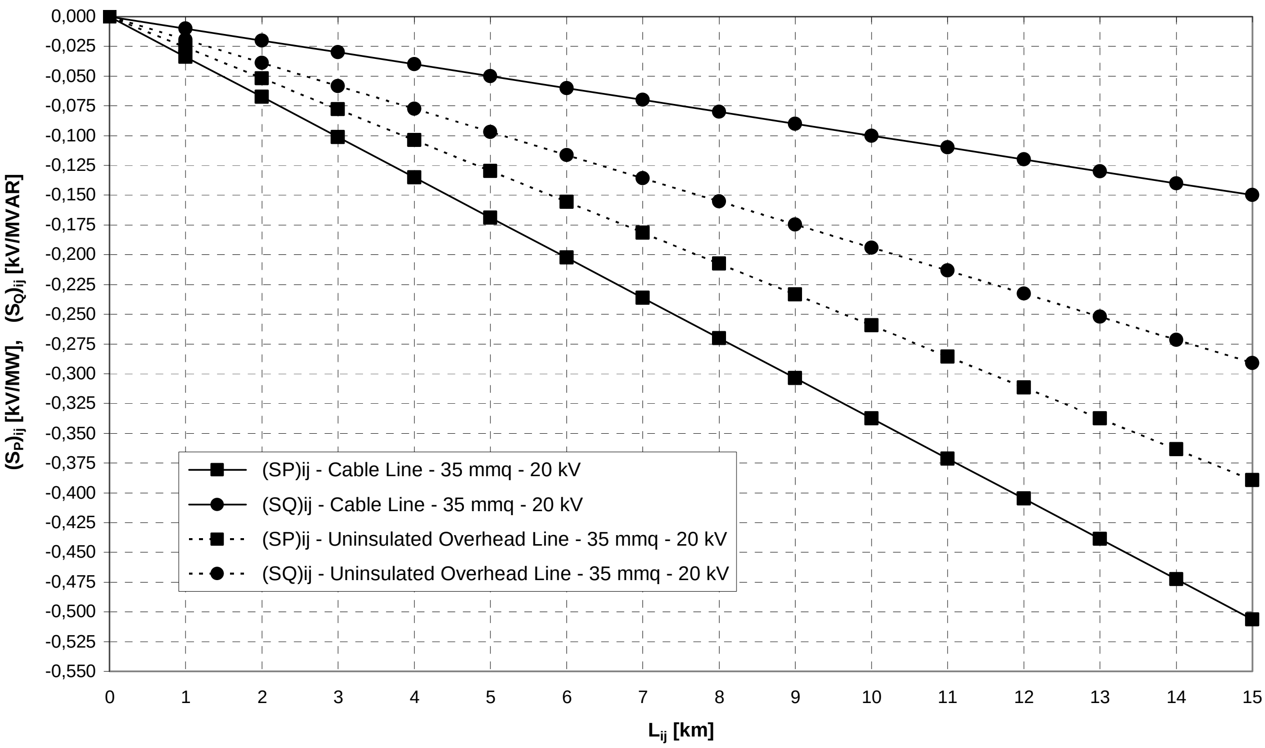

Fig. 2. Graphical representation of $(S_P)_{ij}$ and $(S_Q)_{ij}$ for a 20 kV network with underground cable lines or uninsulated overhead lines, conductor section 35mm^2.

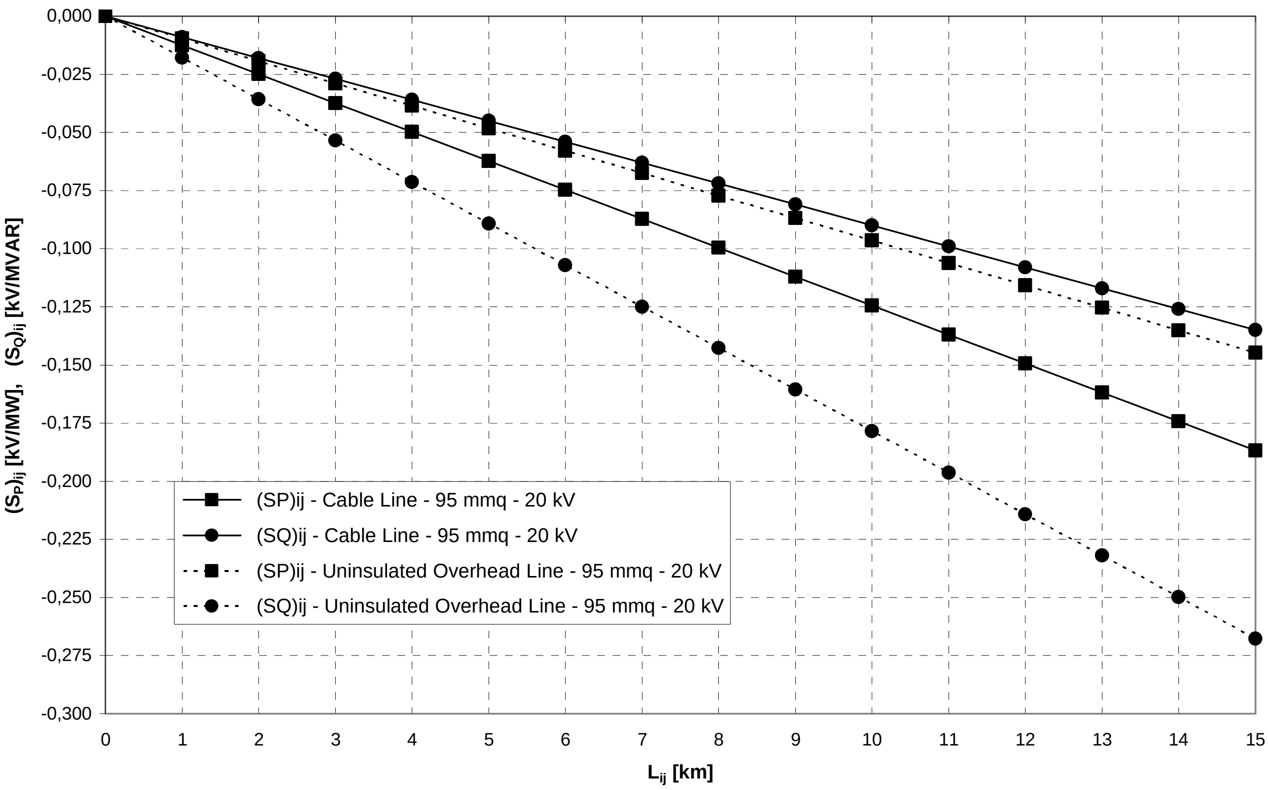

Fig. 3. Graphical representation of $(S_P)_{ij}$ and $(S_Q)_{ij}$ for a 20 kV network with underground cable lines or uninsulated overhead lines, conductor section 95mm^2.

4 Conclusions

The paper presented a simple analytical tool to quantify node voltage variations due to injections of active and reactive powers at one or more nodes of MV distribution networks. Sensitivity coefficients have been derived for node voltage with respect to variations of bus power.

A graphical representation of the coefficients has been discussed through practical examples. Such a representation clearly highlights how node distance from the origin, section and type of conductor (overhead line or underground cable line) influence the network sensitivity.

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