A Method for the Modeling and Analysis of Permanent Magnet Synchronous Machines

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Abstract: - This paper introduces a method for the modeling and analysis of a Permanent Magnet Synchronous Machine (PMSM). The model is based on two dimensional Finite Element (FE) magnetic field analysis. The numerical data of FE solutions are used for obtaining the variations of the self and the mutual inductances and the flux linkages depending on the angular position rotor. In order to have a more general approach, a salient pole like machine is chosen for the analysis. In the paper, the basic steps of the modeling and the solution of the obtained model are given. The model is obtained in terms of natural a,b,c variables and solved by numerical methods. On the contrary of the usual approach, the results of the proposed model are unbalanced and non-sinusoidal.

Key-Words: - Modeling and simulation, permanent magnet machines, generalized machine theory

1 Introduction

It is well known that the generalized machine theory (GMT) is a very useful tool for the modeling and analysis different types of electrical machines [1],[2]. Following the basic principles of electrical circuits theory and virtual work approach, all rotating machines and electromechanical systems can be modeled and analyzed by using the same methodology[3],[4]. Although GMT has been introduced for the conventional types of machines such as symmetrical induction machines and wound rotor synchronous machines, it can be applied to newer machine types such as permanent magnet synchronous machines, synchronous reluctance machines and permanent magnet assisted synchronous machines [5]. An electromechanical system which have a N separate winding is modeled by N mutually coupled electrical circuits and a mechanical terminal equation. Electrical terminal equations are first built in terms of natural phase variables which, in general, includes rotor (armature) position dependent variations of flux linkages, self and mutual inductances. The model is then transformed a new frame of reference which allows a compact and "easy to solve" model. For example, the salient pole synchronus machine, which is originally modeled in a,b,c phase variables, is usually modeled in well known two axis frame of reference fixed on the rotor (d,q,0). Under certain assumptions these two models give the same results for the same operating conditions and d,q,0 formulation allows one to one back transformation to the original phase variables.

The classical theory of salient pole synchronous machines is based on some basic assumptions which are well suited to wound rotor synchronous machines. Some of these assumptions are as follows [2].

-The stator windings have a harmonic- free MMF waveform,
-The rotor position dependent reluctance variations of the magnetic paths of the phase windings are sinusoidal,
-The Mutual and the self inductance terms of the phase windings have no higher order space harmonic (higher than 2nd) terms,
-Speed voltages (due to PM's) are also harmonic-free.

However for some of the newer machine types such as PMSM, SyncREL or PMSyncREL designs, the above mentioned assumptions, more or less, may not be fulfilled. In such cases, to obtain more realistic results the model without any pre-assumtion written in terms of natural a,b,c variables should be used. Using d,q,0 model for such machines may cause differences of the responses between the model and the actual system. Basic purpose of this study can be outlined as to investigate the validity of the basic assumptions of the generalized machine theory and introduce a methodology which can be applied to such machines[3],[4,[5],[6],[7].

2 Buried PM Machine

In order to get a unified approach for the modelling a buried magnet type PM machine will be used. This choice is logical since it is a salient pole - like machine and it can be used either as a PMSM or a SyncREL (when PM's are not present). When the torque due to PM's is weaker than the reluctance torque, this machine becomes a PMASyncREL..

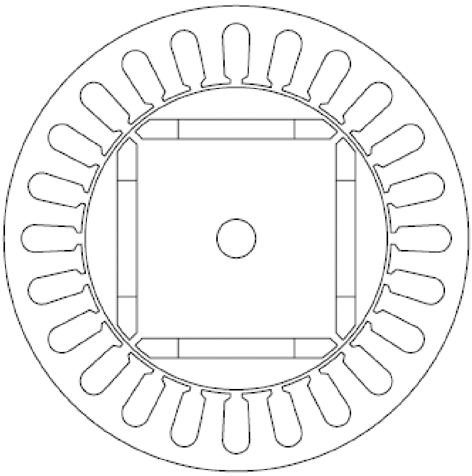


Fig.1 Four pole buried type PM machine model.

In order to develop a general model machine model shown in Fig. 1. is adopted. The embedded PM's in the rotor are chosen in order to have more saliency effects. Slightly distributed (two slot per phase- per pole) stator windings are supposed to be wound. The steps of the modeling and the analysis procedure are given as follows.

2D FE flux distribution is calculated in which step by step increasing rotor positions are considered both for the inductance terms of the phase windings and for the flux linkages of the windings due to PM.

After having the numerical data for each of the rotor positions from the above solutions, the numerical Fourier transformation is then applied in order to have periodic and differentiable functions for the inductance and the flux linkage variations.

The model is obtained in terms of a,b,c natural phase variables by using basic circuit analysis and machine theory methods.

3 Mathematical Model

Following the methodology of GMT, as an electromechanical system, the machine is modeled by magnetically coupled three phase electrical network equations and an additional mechanical terminal

$$[L]\frac{d[I]}{dt} = \left\{[V] - [R][I] - \omega_r \frac{\partial[\lambda]}{\partial\theta_r} - \omega_r \frac{\partial[L]}{\partial\theta_r}[I]\right\} \quad (1)$$

$$T_e = p\left\{\frac{1}{2}[I]^T \frac{\partial[L]}{\partial\theta_r}[I] + [I]^T \frac{\partial[\lambda]}{\partial\theta_r}\right\} \quad (2)$$

$$\frac{d\omega_r}{dt} = (T_e - T_l)/J \quad (3)$$

$$\frac{d\theta_r}{dt} = \omega_r \quad (4)$$

equation. In compact matrix form terminal equations of the machine can be given as follows.

Where,

[L] : 3x3 inductance matrix, which includes the self and the mutual inductances,
[V] : Three phase source voltage vector, which are applied to electrical terminals of three windings,
[R] : 3x3 ohmic resistance matrix which includes the resistance of the three windings on the main diagonal,
[λ] : Flux linkage vector of the windings,
[I] : Current vector of winding currents,
Te : Torque due to PM –current interaction and reluctance difference of the rotor,
ω_r: Rotor speed (in electrical rad/sec),
θ_r : Rotor position (in electrical rad.),
T_l: Load torque.

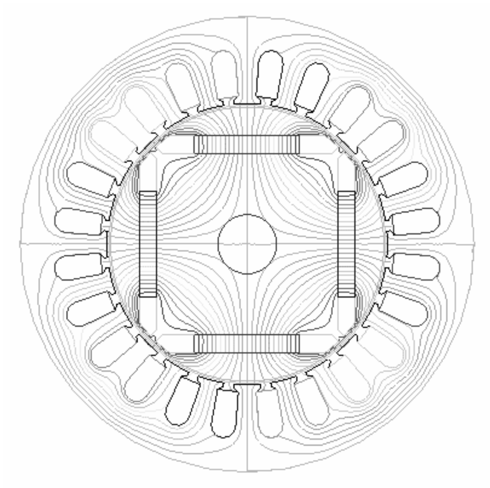
From (1) and (2) it is clear that the first step for put up the mathematical model is the determination of the [L] and [λ] matrices which are rotor position dependent. To obtain the rotor position dependent variation of the winding inductances and the flux linkage terms, two field solutions are required. Smaller rotor position increments allow model to include slotting effects and the cogging torque can be precisely predicted. One of the field solution, in which only single phase winding is excited, is used to compute the self and mutual inductances, the second solution is used to calculate flux linkages due to PM's. In the first solution the PM's are modelled as simple airgaps.
In Fig. 2 the distribution of fluxes due to PM's and the magnitude of the magnetic induction around the air gap

are shown. By using the data of this distribution, flux linkages of the windings can be calculated as,

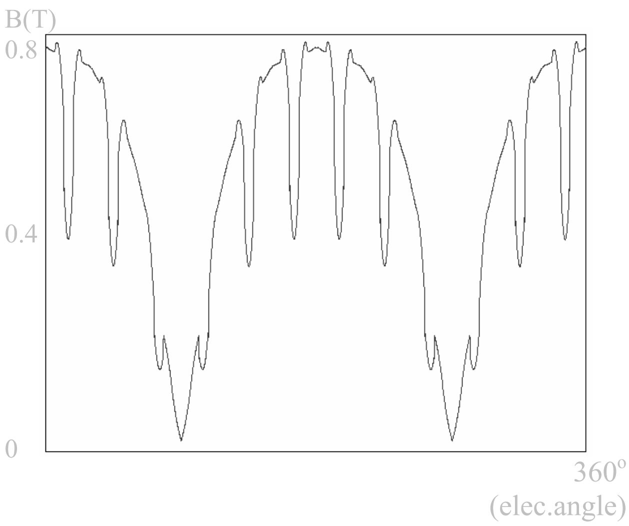
$$\lambda = \int_{\alpha}^{\beta} B(\theta) r l d\theta \, . \qquad (5)$$

Where,
α, β : Relative angular positions of the pairs that constitutes a individual winding,
θ : Integration variable,
r : Radius of air gap,
l : Axial length of the machine.

Since the winding is distributed into adjacent slots, flux linkages for the inner coil and the outer coil are calculated separately and total flux linkage is determined as the sum of individual coils.



a



b

Fig. 2a) Field plot of the machine due to PM excitation.
b) Magnitude of the magnetic induction around the air gap.

For the determination of the self and the mutual inductances of the windings, another field solution is required. In this solution, the self and the mutual fluxes are calculated for the excitation of a phase winding. In Fig.3. three sample flux distributions are shown. In Fig.4. the magnitude of the magnetic induction due to excitation of phase "a" is shown. By using this data, position dependent terms of the self and the mutual inductances can be calculated.

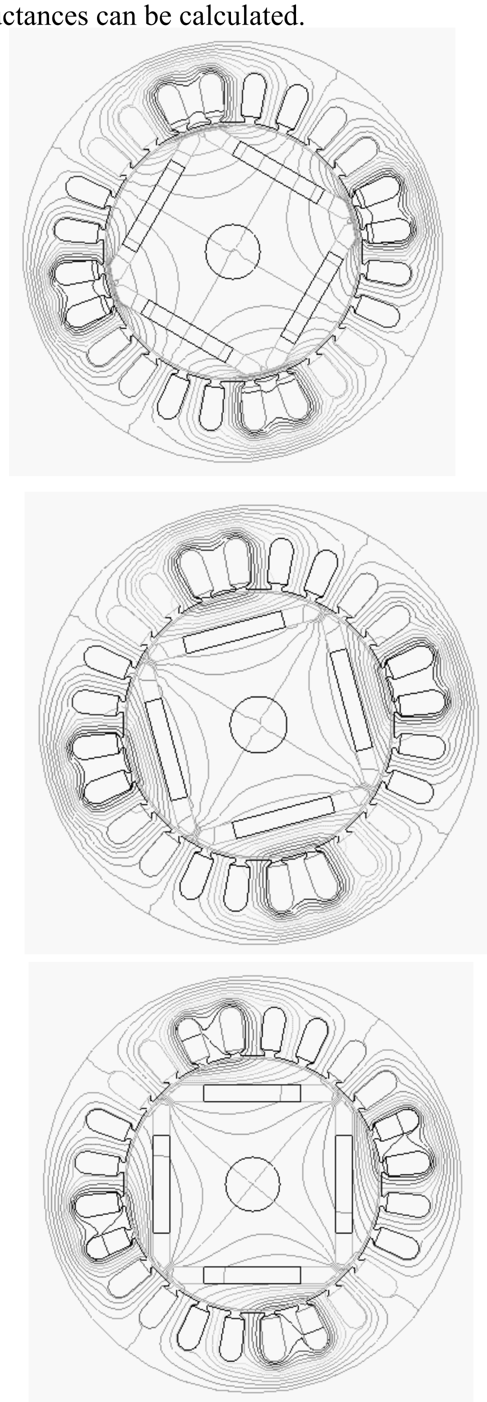


Fig.3. Field plots of the machine due to phase "a" excitation
a) minimum flux position,
b) maximum flux position,
c) An intermediate position.

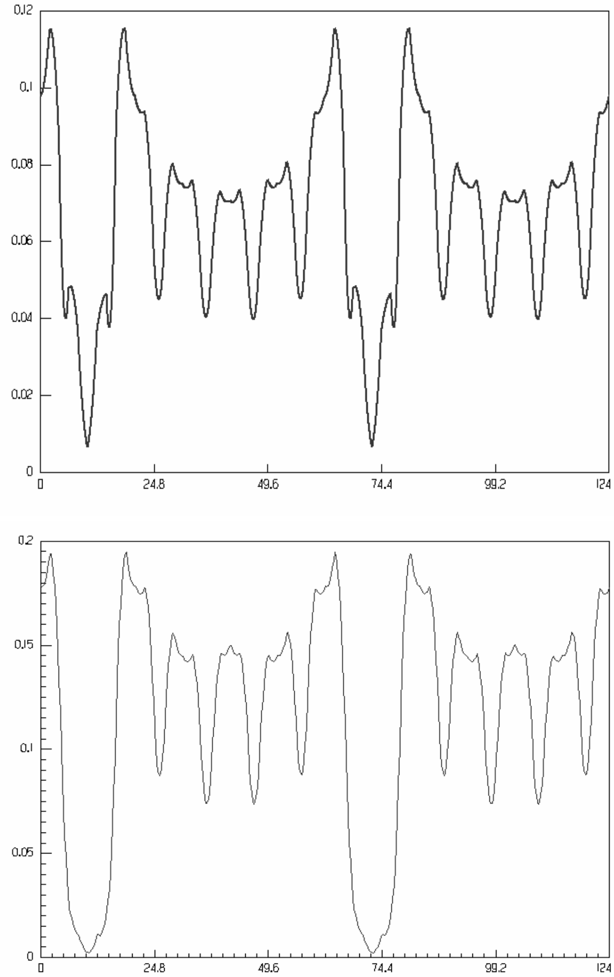


Fig. 4 Air gap induction magnitudes.
a) For minimum flux position,
b) for maximum flux position.

In Fig.5. the results of inductance calculations are shown. Albeit the variation of self inductance waveform is close the sinusoidal form it has higher order harmonic terms. Besides, the mutual inductance waveform is more distorted and includes significant harmonics.

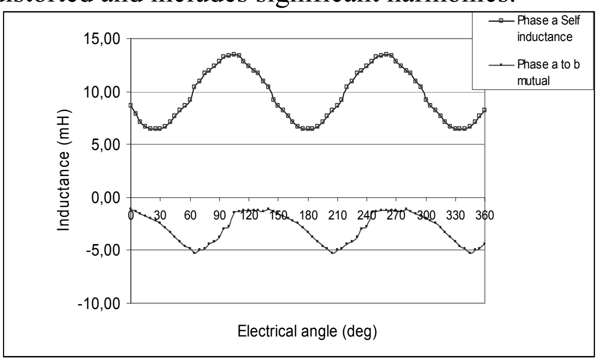


Fig. 5 Calculated variations of the self and the mutual inductances.

By using numerical analysis, the self inductance function for the phase a is determined as,

$Laa=9.42 -3.379\cos(2\theta_r) -0.0144\cos(4\theta_r)-0.1707\cos(6\theta_r)$ mH.

Similarly the mutual inductance function can be approximated as,

$Lab=-2.35-1.19(\cos2\theta_r-2\pi/3)-0.234\cos(4\theta_r-4\pi/3)-0.123\cos(6\,\theta_r-6\pi/3)$ mH.

Finally, the flux linkage for the phase "a" approximated as,

$\lambda a=1.941\cos\theta_r -0.163\cos3\theta_r-0.031\cos5\theta_r 2.77.10^{-3}\cos9\theta_r\ 7.636.10^{-3}\cos11\theta_r+0.292.10^{-3}\cos13\theta_r$ Vs.

These three functions can be modified to obtain complete inductance matrix and the flux linkages for the other phase windings. Note that flux linkage approximation contains harmonic terms up to 13^{th}. Even higher harmonics seem negligible, when their derivative is taken with respect to rotor position, harmonic order comes as a multiplier and contribution of these harmonics become significant. As it seen, the higher order harmonic terms, which must be truncated in the conventional GMT approach, are included in the mathematical model. This inclusion creates an unbalanced three phase system which can only be analyzed correctly by using natural a,b,c phase variables.

3 Solution of Mathematical Model

Inspection of (1) shows that, in order to numerically solve the model, the inverse of the [L] matrix should be computed for the each time step. Afer the inverse matrix is calculated, both sides of (1) are multiplied by $[L]^{-1}$ and a set of differential equations are obtained. Even it is time consuming, this approach gives the chance of analyzing the machine in natural phase variables without any truncation of the higher order harmonics.

The solution of the model is made for the sinusoidal phase voltages in order to see the effects of the higher order harmonic terms which are included in the model. Note that this simulation is made for inspection of the response of the system rather than a performance assessment. The set of balance voltages are considered to be applied into machine terminals. Since rotor has no cage, a low voltage and frequency set is used.
Supply voltages are assumed to be,

Va=20cos(31.4t),
Vb=20cos(31.4t-120^o),
Vc=20cos(31.4t-240^o) Volts.

For the phase resistances and inertia of the motor,

Ra=Rb=Rc=10 Ohm,
J=0.0001 kgm^2
are choosen. No load run up is modeled. A dedicated software which uses 4th order Runge-Kutta algorithm is used with fixed time step.

In Fig. 6. the variation of phase currents, in Fig.7 the variations of speed voltages are shown. As it is clearly seen, balanced set of supply voltages result in unbalanced and non-sinusoidal set of winding currents. Besides, the speed voltages which depend on the derivative of the flux linkages, are also non-sinusoidal and they include slotting effects. The same comments are valid for the torque and the speed. The waveforms of the torque and speed demonstrate that due to interaction of non-sinusoidal currents with the speed voltages generate a torque which has pulsations as well.

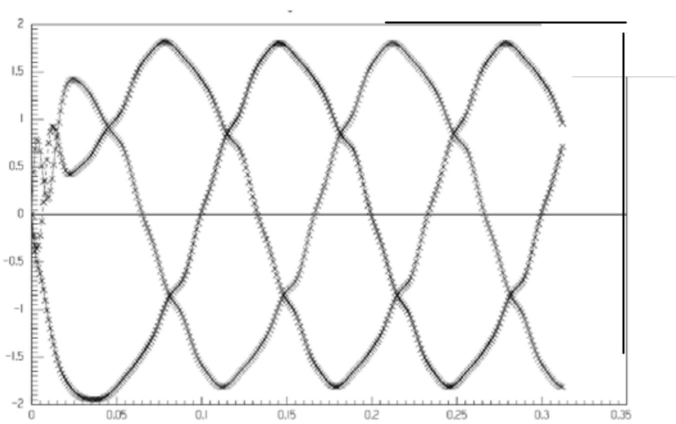


Fig.6 The variation of currents.

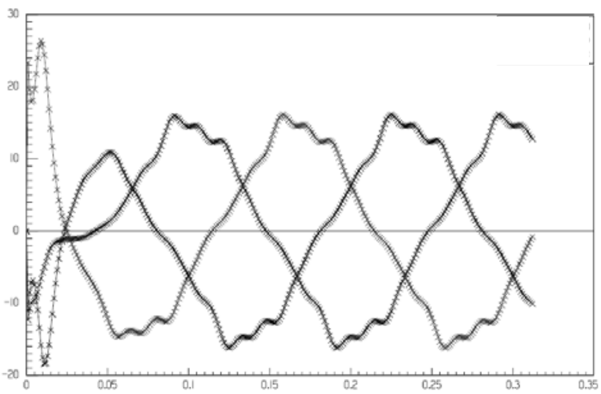


Fig 7 Variation of speed voltages.

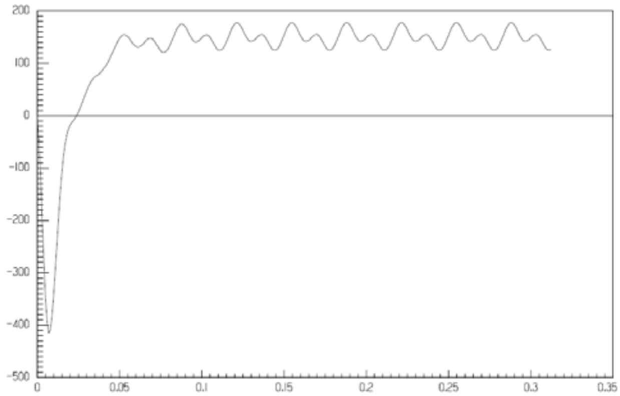


Fig.8 Variation of mechanical speed (rpm).

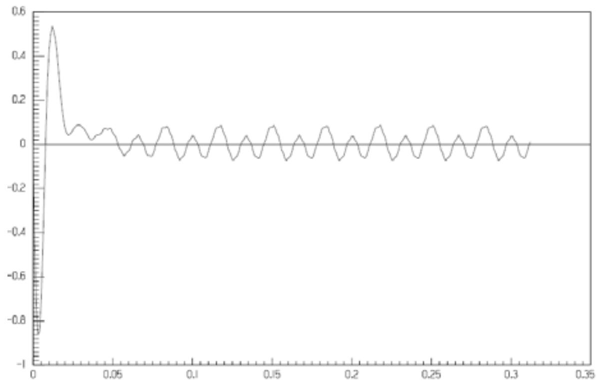


Fig.9 Variation of torque.

4 Conclusion

A method for the mathematical modeling of a PM machine has been introduced. Although a buried type machine has been considered in the paper, the method can be apply any type of PM or SynRel machine.It has been shown that, by using a FE software, basic functions for the flux linkages and inductances can be obtained. The model can be solved numerically and more realistic results can be obtained. The results showed that, in general, basic assumptions of d,q,0 formulation are not exactly valid for this type of machine.

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