The MINLP approach to structural optimization

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Abstract: - The paper presents the Mixed-Integer Non-linear Programming optimization approach (MINLP) to structural optimization. The MINLP is a combined continuous/discrete optimization technique, where a structural topology, discrete materials and standard sizes are optimized simultaneously with the continuous parameters (e.g. costs, mass). The MINLP optimization is performed through three steps: i.e. the generation of a mechanical superstructure, the modelling of an MINLP model formulation and the solution of the defined MINLP problem. The Modified Outer-Approximation/Equality-Relaxation (OA/ER) algorithm and a two-phase MINLP strategy are applied. The optimization is performed by a user-friendly version of the MINLP computer package MIPSYN. Two examples are presented at the end of the paper.

Key-Words: - Topology optimization, Discrete sizes optimization, Mixed-Integer Non-linear programming, MINLP

1 Introduction

The paper presents the Mixed-Integer Non-linear Programming approach (MINLP) to structural optimization. The MINLP is a combined continuous/discrete optimization technique. It handles with continuous and discrete binary 0-1 variables simultaneously. While continuous variables are defined for the continuous optimization of parameters (dimensions, stresses, strains, weights, costs, etc.), discrete variables are used to express discrete decisions, i.e. usually the existence or non-existence of structural elements inside the defined structure. Different materials and standard sizes may also be defined as discrete alternatives. Since the MINLP performs continuous and discrete optimizations simultaneously, the MINLP approach also finds optimal continuous parameters (mass, costs, stresses, etc.), a structural topology with an optimal number and a configuration of structural elements, discrete materials and standard sizes simultaneously.

The MINLP optimization approach is proposed to be performed through three steps. The first one includes the generation of a mechanical superstructure of different topology, material and standard dimension alternatives, the second one involves the development of an MINLP model formulation and the last one consists of a solution for the defined MINLP optimization problem.

The MINLP continuous/discrete optimization problems of structural optimization are in most cases comprehensive, non-convex and highly non-linear. This paper reports the experience in solving this type of problem by using the Outer-Approximation/Equality-Relaxation (OA/ER) algorithm [1], [2]. A two-phase MINLP optimization is proposed to accelerate the convergence of the mentioned algorithm. The optimizations are carried out by an MINLP computer package MIPSYN, the successor of PROSYN [1] and TOP [2-4].

Two examples are presented at the end of the paper. The first one shows the material and standard dimension optimization of a composite floor system and the second one presents the topology and standard sizes optimization of a single-storey industrial building.

2 Mechanical superstructure

The MINLP optimization approach requires the generation of an MINLP mechanical superstructure composed of various topology and design alternatives that are all candidates for a feasible and optimal solution. While topology alternatives represent different selections and interconnections of corresponding structural elements, design alternatives include different materials and standard dimensions.

The superstructure is typically described by means of unit representation: i.e. structural elements and their interconnection nodes. Each potential topology alternative is represented by a special number and a configuration of selected structural elements and their interconnections; each structural element may in addition have different material and standard dimension alternatives.

Therefore, the main goal is to find within the given superstructure a feasible structure that is optimal with respect to topology, material, standard dimensions and all defined continuous parameters.

3 MINLP model formulation

It is assumed that a general non-linear and non-convex continuous/discrete optimization problem can be formulated as an MINLP problem in the form:

$$\min \quad z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + f(\boldsymbol{x})$$

$$\text{s.t.:}$$

$$\begin{aligned} \boldsymbol{h}(\boldsymbol{x}) &= \boldsymbol{0} \\ \boldsymbol{g}(\boldsymbol{x}) &\leq \boldsymbol{0} \qquad (\textbf{MINLP}) \\ \boldsymbol{By} + \boldsymbol{Cx} &\leq \boldsymbol{b} \end{aligned}$$

$$\boldsymbol{x} \in X = \{\boldsymbol{x} \in R^{n} : \boldsymbol{x}^{lo} \leq \boldsymbol{x} \leq \boldsymbol{x}^{up}\}$$

$$\boldsymbol{y} \in Y = \{0,1\}^{m}$$

where $\boldsymbol{x}$ is a vector of continuous variables specified in the compact set X and $\boldsymbol{y}$ is a vector of discrete, binary 0-1 variables. Functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ are non-linear functions involved in the objective function z, equality and inequality constraints, respectively. All functions $f(\boldsymbol{x})$, $\boldsymbol{h}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ must be continuous and differentiable. Finally, $\boldsymbol{By}+\boldsymbol{Cx}\leq\boldsymbol{b}$ represents a subset of mixed linear equality/inequality constraints.

The above general MINLP model formulation has been adapted for structural optimization. It is postulated that it helps us construct an MINLP mathematical optimization model for any structure.

In the context of structural optimization, continuous variables $\boldsymbol{x}$ define structural parameters (dimensions, strains, stresses, costs, mass...) and binary variables $\boldsymbol{y}$ represent the potential existence of structural elements within the defined superstructure. An extra binary variable y is assigned to each structural element. The element is then selected to compose the structure if its subjected binary variable takes value one (y=1), otherwise it is rejected (y=0). Binary variables also define the choice of discrete/standard materials and sizes.

The economical (or mass) objective function z involves fixed costs (mass) in the term $\boldsymbol{c}^{T}\boldsymbol{y}$, while the dimension dependant costs (mass) are included in the function $f(\boldsymbol{x})$. Non-linear equality and inequality constraints $\boldsymbol{h}(\boldsymbol{x})=\boldsymbol{0}$, $\boldsymbol{g}(\boldsymbol{x})\leq\boldsymbol{0}$ and the bounds of the continuous variables represent the rigorous system of the design, loading, resistance, stress, deflection, etc. constraints known from the structural analysis. Logical constraints that must be fulfilled for discrete decisions and structure configurations, which are selected from within the superstructure, are given by $\boldsymbol{By}+\boldsymbol{Cx}\leq\boldsymbol{b}$. These constraints describe relations between binary variables and define the structure's topology, materials and standard dimensions. It should be noted, that the comprehensive MINLP model formulation for mechanical structures may be found elsewhere [3, 5].

4 Solving an MINLP problem

After the MINLP model formulation is developed, the defined MINLP optimization problem is solved by the use of a suitable MINLP algorithm and strategies. A general MINLP class of optimization problem can be solved in principle by the following algorithms and their extensions:

-the *Nonlinear Branch and Bound*, NBB, proposed and used by many authors, e.g. E.M.L. Beale [6], O.K. Gupta and A. Ravindran [7];

-the *Sequential Linear Discrete Programming* method, SLDP, by G.R. Olsen and G.N. Vanderplaats [8] and M. Bremicker et al. [9];

-the *Extended Cutting Plane* method by T. Westerlund and F. Pettersson [10];

-*Generalized Benders Decomposition*, GBD, by J.F. Benders [11], A.M. Geoffrion [12];

-the *Outer-Approximation/ Equality-Relaxation algorithm*, OA/ER, by G.R. Kocis and I.E. Grossmann [13];

-the *Feasibility Technique* by H. Mawengkang and B.A. Murtagh [14]; and

-the *LP/NLP based Branch and Bound* algorithm by I. Quesada and I.E. Grossmann [15].

4.1 Modified OA/ER algorithm

The OA/ER algorithm consists of solving an alternative sequence of Non-linear Programming (NLP) optimization subproblems and Mixed-Integer Linear Programming (MILP) master problems. The former corresponds to continuous optimization of parameters for a mechanical structure with a fixed topology (and fixed discrete/standard materials and dimensions) and yields an upper bound to the objective to be minimized. The latter involves a global approximation to the superstructure of alternatives in which a new topology, discrete/standard materials and dimensions are identified so that its lower bound does not exceed the current best upper bound. The search of a convex problem is terminated when the predicted lower bound exceeds the upper bound, otherwise it is terminated when the NLP solution can be improved no more. The OA/ER algorithm guarantees the global optimality of solutions for convex and quasi-convex optimization problems.

The OA/ER algorithm as well as all other mentioned MINLP algorithms do not generally guarantee that the solution found is the global optimum. This is due to the presence of nonconvex functions in the models that may cut off the global optimum. In order to reduce undesirable effects of nonconvexities the Modified OA/ER algorithm was proposed by Z. Kravanja and I.E. Grossmann [1], see also S. Kravanja et al. [2], by which the following modifications are applied for the master problem: the deactivation of linearizations, the decomposition and the deactivation of the objective function linearization, the use of the penalty function, the use of the upper bound on the objective function to be minimized as well as the global convexity test and the validation of the outer approximations.

4.2 Two-phase MINLP optimization

The optimal solution of complex non-convex and non-linear MINLP problem with a high number of discrete decisions is in general very difficult to be obtained. The optimization is thus proposed to be performed sequentially in two different phases to accelerate the convergence of the OA/ER algorithm. The optimization starts with the topology optimization of a structure, while discrete materials and sizes are relaxed temporary into continuous parameters. When the optimal topology is found, standard materials and sizes are in the second phase re-established and the discrete material and dimension optimization of the structure is then continued until the optimal solution is found.

5 Computer package MIPSYN

The optimization of the structures is proposed to be carried out by a user-friendly version of the MINLP computer package MIPSYN, the successor of PROSYN [1] and TOP [2-4, 16]. MIPSYN is the implementation of many advanced optimization techniques, most important of which are the Modified OA/ER algorithm and MINLP strategies. In terms of complexity, the MIPSYN's synthesis problems can range from a simple NLP optimization problem of a single structure up to the MINLP optimization of a complex superstructure problem. MIPSYN runs automatically or in an interactive mode and thus provides the user with a good control and supervision of the calculations. GAMS/CONOPT2 (Generalized reduced-gradient method) [17] is used to solve NLP subproblems and GAMS/Cplex 7.0 (Branch and Bound) [18] is used to solve MILP master problems.

5.1 Optimization models

For each type of structure, a special optimization model must be developed. Each model is constructed on the basis of the mentioned general MINLP-G model formulation. As an interface for mathematical modelling and data inputs/outputs GAMS (General Algebraic Modelling System), a high level language, is used [19].

6 Numerical examples

MINLP optimization approach is illustrated by two examples. The first one shows the material and standard dimension optimization of a composite floor system and the second one presents the topology and standard sizes optimization of a single-storey industrial building.

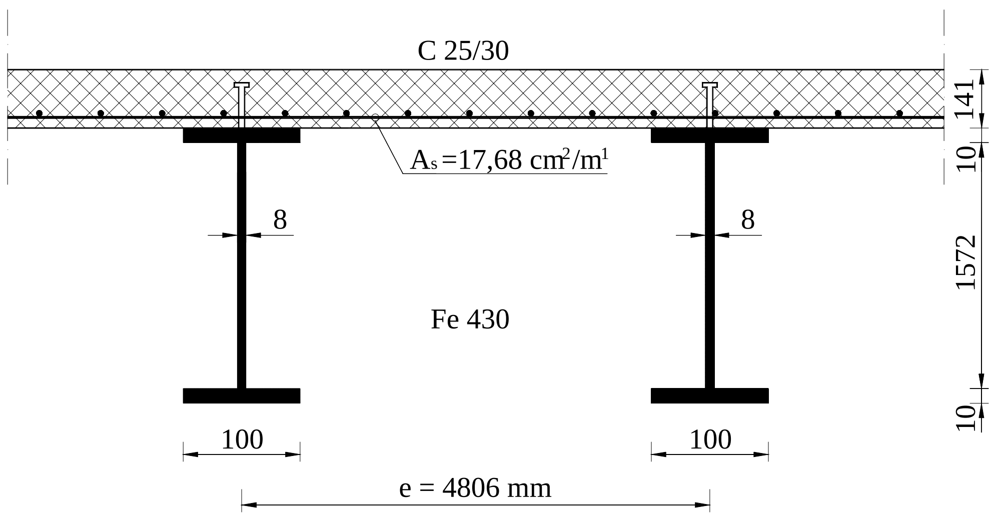

Fig. 1. Cross-section of the optimal composite I beam floor system

6.1 Optimization of a composite floor system

The first example presents the simultaneous material and standard dimension optimization of a composite floor system, which is made from a reinforced concrete slab and 25 m long simply supported structural steel beams of duo-symmetrical welded I sections. The floor system is subjected to the self-weight and the uniformly distributed imposed load of 2.5 kN/m^2.

The objective of the optimization was to find the minimal self-manufacturing costs of the structure, the optimal standard thickness of steel plates for steel webs and flanges, the standard cross-section of the reinforcing steel meshes as well as the optimal concrete strength and structural steel grade.

The MINLP optimization model COMBOPT (COMposite Beams OPTimization) for the composite I beam floor system was developed for the optimization. A high level language GAMS was used for the modelling. The material and labour costs for the composite floor were accounted for in an economical type of objective function, subjected to the constraints for the ultimate and serviceability limit states (dimensioning), defined according to Eurocode 4 [20].

The superstructure of the composite floor comprised 6 different concrete strengths (C25, C30, C35, C40, C45, C50), 3 different structural steel grades (S 235, S 275, S 355), 48 various standard reinforcing steel sections and 9 different standard thickness of sheet-iron plates (from 8 mm to 40 mm) for webs and flanges separately.

The MINLP optimization of the self-manufacturing costs was performed by the computer package MIPSYN (GAMS/CONOPT2 and GAMS/Cplex 7.0). The Modified OA/ER algorithm and the two-phase MINLP optimization were applied. The optimal result of 43.67 EUR per m^2 of the use surface of the composite floor system was obtained in the 3rd MINLP iteration. Beside the optimal self-manufacturing costs, the optimal concrete strength C25/30, steel grade S 275 (Fe 430), intermediate distance between I sections (the topology), depth of the slab and optimal standard thickness of webs and flanges have been obtained, see Fig. 1.

6.2 Optimization of an industrial building

The second example introduces the topology and standard sizes optimization of a single-storey industrial building. The building is 20 meters wide, 40 meters long and 6.5 meters high. The structure is consisted from equal non-sway steel portal frames, which are mutually connected with purlins. Variable imposed loads s=1.60 kN/m^2 (snow) and w=0.137 kN/m^2 (wind) are defined as the uniformly distributed surface load. The material used was steel S 355.

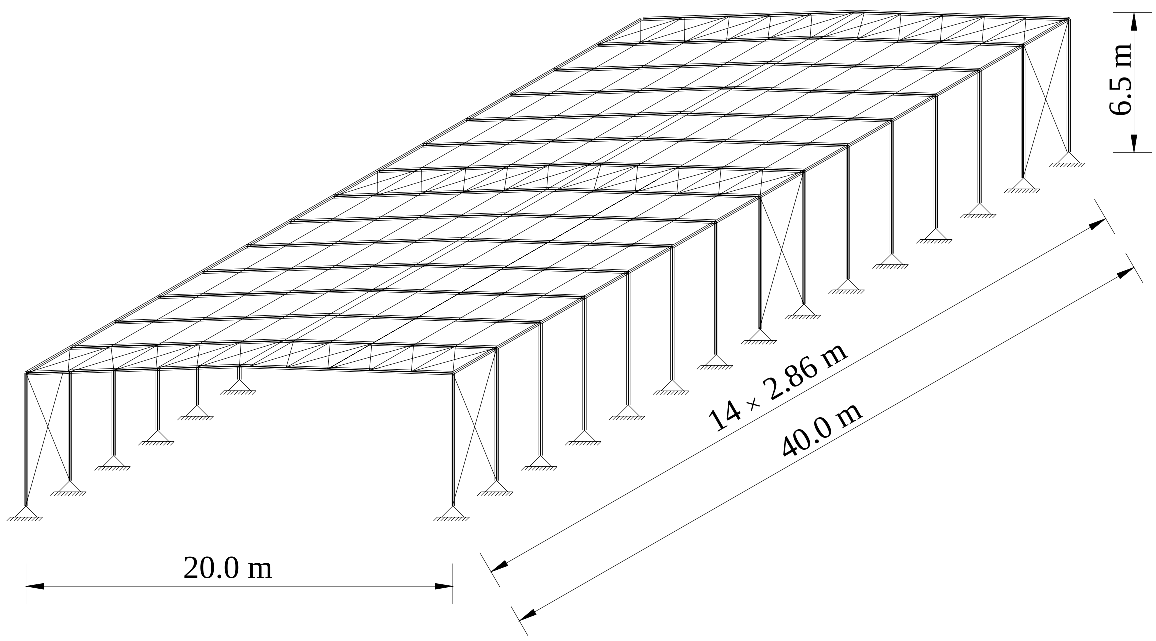

Fig. 2. Optimal topology of the single-storey industrial building

The task of the optimization was to find the minimal structure mass, the optimal topology (the optimal number of portal frames and purlins) and all standard cross-sections.

The MINLP optimization model FRAMEOPT (FRAME OPTimization) was developed. A high level language GAMS was used for the modelling. The objective function of the structure mass was defined. Both, the horizontal concentrated load at the top of the columns (wind) and the vertical uniformly distributed load on the frame beams (snow and wind) were calculated automatically through the optimization considering the calculated intermediate distance between the portal frames. Internal forces were calculated by the elastic first-order theory for the non-sway frame mode. The dimensioning of steel members was performed in accordance with Eurocode 3 [21] for the conditions of both the ultimate and serviceability limit states.

The industrial building superstructure was generated in which all possible structures were embedded by topology variation between 15 to 30 portal frames and 10 to 20 purlins. The superstructure also comprised 24 different standard hot rolled European wide flange I sections, i.e. HEA sections (from HEA 100 to HEA 1000) for each column, beam and purlin separately.

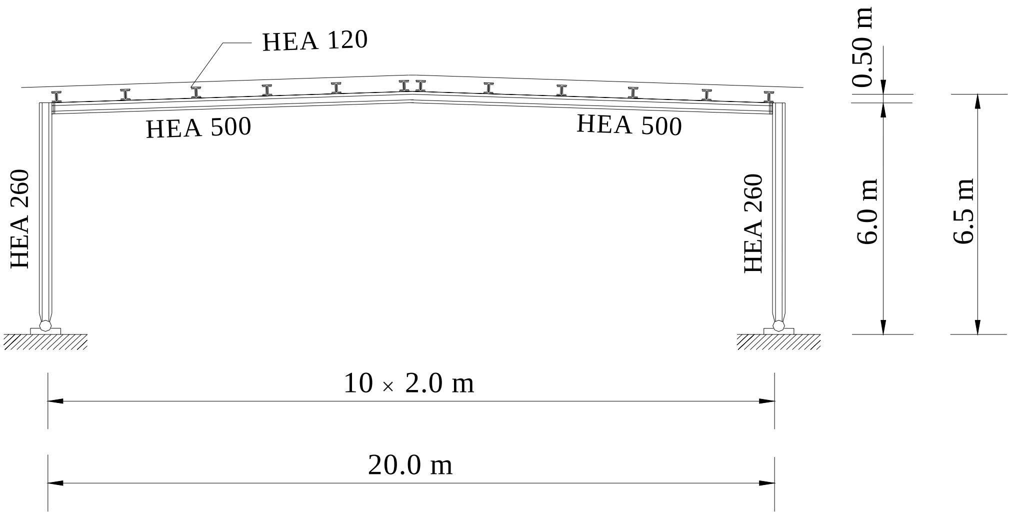

Fig. 3. Optimal steel sections

The MINLP optimization of the structure mass was performed by the computer package MIPSYN (GAMS/CONOPT2 and GAMS/Cplex 7.0). The Modified OA/ER algorithm and the two-phase MINLP optimization were applied. The optimal result of 62.029 tons was obtained in the 3rd main MINLP iteration. The optimal solution includes the obtained optimal topology of 15 portal frames and 12 purlins, see Fig. 2, as well as the optimal standard steel HEA sections of columns, beams and purlins, see Fig. 3.

7 Conclusion

The paper presents the Mixed-Integer Non-linear Programming approach (MINLP) to structural optimization. The Modified OA/ER algorithm and the two-phase MINLP optimization strategy were applied. The optimization is performed by a user-friendly version of the MINLP computer package MIPSYN. Beside the optimal structure costs or mass, the optimal topology with the optimal number of structural elements, the optimal discrete/standard materials and cross-sectional sizes can be obtained simultaneously. Two examples, presented at the end of the paper, clearly show the efficiency of the proposed MINLP approach.

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