Lyapunov Energy Function for an SMES

VALENTIN AZBE, RAFAEL MIHALIC
Faculty of Electrical Engineering
University of Ljubljana
Trzaska 25, 1000 Ljubljana
SLOVENIA
http://www.fe.uni-lj.si/research/lpee.html

Abstract: - A superconducting magnetic energy storage (SMES) can provide dynamic power-flow control, and in this way improve electric-power-system stability. In order to assess its influence on the system's dynamic behavior or to determinate the device's control strategy using direct methods, proper energy functions for this device are needed. In this paper the energy functions for a series connection and for a parallel connection of SMES into the electric-power system have been developed. The energy functions were constructed as additional terms that can be added to any existing structure-preserving energy function. Tests within a single-machine infinite-bus system proved the correctness of the proposed energy functions. The application of new energy functions was demonstrated on the problem of transient-stability assessment.

Key-Words: - FACTS devices, Lyapunov energy functions, Lyapunov direct methods, power-system control, power-system transient stability, superconducting magnetic energy storage

1 Introduction

Superconducting Magnetic Energy Storage (SMES) systems store energy in the magnetic field created by the flow of direct current in a superconducting coil which has been cryogenically cooled to a temperature below its superconducting critical temperature. A typical SMES system includes three parts: superconducting coil, power conditioning system and cryogenically cooled refrigerator. SMES loses the least amount of electricity in the energy storage process compared to other methods of storing energy.

Due to the energy requirements of refrigeration and the high cost of superconducting wire, SMES is currently used for short duration energy storage. Therefore, SMES is devoted to load levelling, damping system oscilations or improving power quality. It could also perform a variety of functions such as automatic generation control, fast spinning reserve, black start, improved transmission efficiency, voltage control and transient-stability improvement. Its application can be similar to an application of FACTS devices with additional possibility of injecting active power into (or out of) the electric-power system (EPS).

In this paper we have attempted to develop the energy functions for an SMES. Our derivations were demonstrated on the phenomenon of transient stability as a typical task where energy functions are traditionally applied in direct methods for transient-stability assessment. With the increased importance of online dynamic security assessment, direct methods might be applied to avoid the time-consuming repetition of solving a system's nonlinear differential equations. Regardless of how SMES acts during and after a fault, its influence should be considered in direct methods that apply proper energy functions. Of course, the application of energy functions is much wider, e.g., for the control strategies of SMES [1].

We tried to construct the energy functions for both of the possible connection of an SMES into the EPS—i.e., for a series connection and for a parallel connection. The energy functions were constructed for the model of the EPS with structure-preserving topology, and therefore they are in the form of an additional term that can be added to any existing structure-preserving energy function. The same principles were applied for construction of energy functions for FACTS devices [2]-[5].

The paper is organized as follows: Section 2 describes an SMES's characteristics and the derivation of its energy functions. Section 3 presents numerical examples of the transient-stability assessment of a single-machine infinite-bus test system that prove the correctness of the proposed energy functions. Section 4 draws conclusions.

2 Characteristics of an SMES

A superconducting coil of an SMES is normally connected to the electric-power system via a power conditioning system, i.e., semiconductor converter. Generally, it can be connected either in series or in parallel to the system. Because the effect on power system dynamics depends essentially on the way of its connection, the injection model and construction procedure for the energy function are presented separately for the series and for the parallel connection of an SMES. In order to be able to derive the energy functions for the SMES, it has to be represented by the injection model that represents its behavior in an EPS with its basic characteristics.

2.1 Series connection of SMES

If SMES is connected in series to the system, it is connected via a voltage-source converter (VSC) and a series transformer, similarly to a static synchronous series compensator (SSSC). Therefore, the injection model for the SMES in series connection is the same as a general SSSC injection model presented in [6].

2.1.1 Injection model for a series connection

Fig. 1 presents an SMES scheme, its phasor diagram and the injection model. Like in an SSSC, the series-injected voltage magnitude of a VSC, U_{T}, does not depend on the current through the line or on the bus voltage. Therefore, U_{T} is limited only by the construction of a VSC and a series transformer and can be treated as an SMES's control parameter. In contrast to an SSSC, the angle of the series-injected voltage, φ_{T}, can be set to any value because an SMES can inject active and/or reactive power. The active and reactive powers injected according to Fig. 1 (c) can be denoted as:

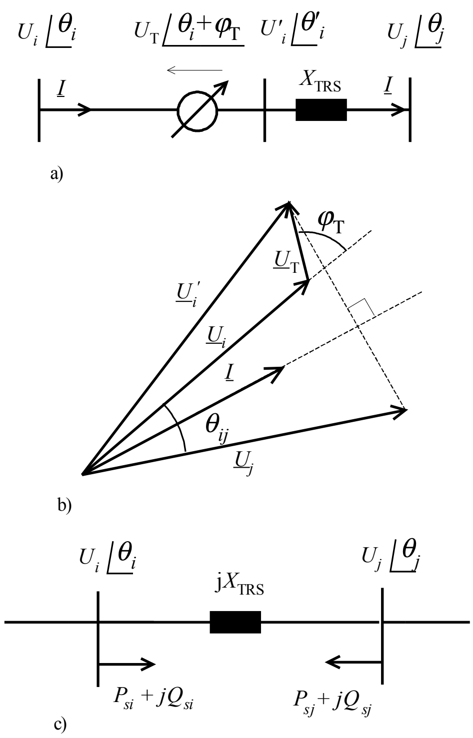

Fig. 1. Series connection of an SMES; a) scheme; b) phasor diagram; c) injection model.

$$P_{si} = \frac{U_i U_{\mathrm{T}}}{X_{\mathrm{TRS}}} \sin\left(\varphi_{\mathrm{T}}\right) \tag{1}$$

$$P_{sj} = -\frac{U_j U_{\mathrm{T}}}{X_{\mathrm{TRS}}} \sin\left(\theta_{ij} + \varphi_{\mathrm{T}}\right) \tag{2}$$

$$Q_{si} = \frac{U_i \times U_{\mathrm{T}}}{X_{\mathrm{TRS}}} \cos\left(\varphi_{\mathrm{T}}\right) \tag{3}$$

$$Q_{sj} = -\frac{U_j \times U_{\mathrm{T}}}{X_{\mathrm{TRS}}} \cos\left(\theta_{ij} + \varphi_{\mathrm{T}}\right) \tag{4}$$

where $\theta_{ij} = \theta_i - \theta_j$, according to Fig. 1. These power injections are the basis for the construction of the energy function.

2.1.2 Construction of an Energy Function

The following step is the construction of an energy function for an SMES that can be added to the structure-preserving energy function (SPEF) presented in [7]. In [7] the SPEF is constructed for the EPS without SMES and we denote it as $V_{\text{without SMES}}$. This energy function can be treated as the sum of the kinetic energy V_{k} and the potential energy V_{p}:

$$V_{\text{without SMES}} = V_{\mathrm{k}} + V_{\mathrm{p}} \tag{5}$$

Using the above energy function formulation the system can be illustrated with a mechanical analogy as a ball rolling on a potential energy surface, as in [8]. This visualization is presented in Fig. 2.

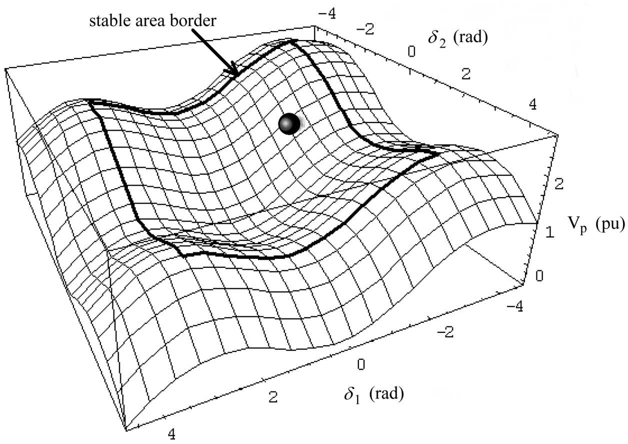

Fig. 2. A ball on a potential-energy surface.

The potential energy V_{p} of a given post-fault system depends on the machine angles. In the case of a three-machine system the potential energy V_{p} can be presented as a surface on a three-dimensional chart, where the horizontal axes represent the angle of two machines according to the third machine (i.e., δ_1 and δ_2 according to Fig. 2). The potential-energy surface has a local minimum at the stable equilibrium point, which corresponds to the machine angles during the post-fault steady-state operation. Around this stable equilibrium point the potential-energy surface forms a bowl-shaped area, which is the area of stable system operation.

The kinetic energy of the system is equated to the kinetic energy of a ball that rolls along the potential-energy surface according to the generator swing trajectory. In steady-state operation the ball stands still at the stable equilibrium point. However, when the fault occurs, the ball is pushed toward the edge of the bowl-shaped area of the potential-energy surface until the fault is cleared. Depending on the total of the kinetic and potential energies of the ball at the time of fault clearing, the ball can either escape from the bowl over the saddle (i.e., an unstable case) or it can continue to oscillate within the bowl (i.e., a stable case). To assess the stability of the system the kinetic energy is compared with the potential energy at the border of the stable area.

To construct an energy function for an SMES that can be added as an additional term to the $V_{\text{without SMES}}$ we follow the construction procedure presented in [7]. According to this procedure, the active-power injections P_{si} and P_{sj} are multiplied by the time derivative of the voltage angle and the reactive-power injections Q_{si} and Q_{sj} are divided by the voltage magnitude and multiplied by the time derivative of the voltage magnitude:

$$P_{si}\dot{\theta}_i = \frac{U_i U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\varphi_\mathrm{T}) \cdot \dot{\theta}_i \tag{6}$$

$$P_{sj}\dot{\theta}_j = -\frac{U_j U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\theta_{ij} + \varphi_\mathrm{T}) \cdot \dot{\theta}_j \tag{7}$$

$$\frac{Q_{si}}{U_i}\dot{U}_i = \frac{\dot{U}_i U_\mathrm{T}}{X_\mathrm{TRS}} \cos(\varphi_\mathrm{T}) \tag{8}$$

$$\frac{Q_{sj}}{U_j}\dot{U}_j = -\frac{\dot{U}_j U_\mathrm{T}}{X_\mathrm{TRS}} \cos(\theta_{ij} + \varphi_\mathrm{T}) \tag{9}$$

Then, (6)–(9) are summed and we obtain the following expression:

$$\begin{aligned} &\frac{U_i U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\varphi_\mathrm{T}) \cdot \dot{\theta}_i - \frac{U_j U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\theta_{ij} + \varphi_\mathrm{T}) \cdot \dot{\theta}_j \\ &+ \frac{\dot{U}_i U_\mathrm{T}}{X_\mathrm{TRS}} \cos(\varphi_\mathrm{T}) - \frac{\dot{U}_j U_\mathrm{T}}{X_\mathrm{TRS}} \cos(\theta_{ij} + \varphi_\mathrm{T}) \end{aligned} \tag{10}$$

Equation (10) is the result of the construction procedure presented in [7]. Now we have to find the first integral of (10), which represents the Lyapunov energy function for an SMES. It is well known that there is no procedure for finding the Lyapunov energy function and that it always has to be found intuitively. Considering the similarity of an SMES to an SSSC and that the already-known energy functions for some FACTS devices are equal to half or to the total sum of the reactive power injected into the system, we can search for the Lyapunov energy function for an SMES in this direction. Let us denote the sum of the SMES's reactive-power injections (3) and (4) as Q_inj and make its derivative:

$$\begin{aligned} \frac{dQ_\mathrm{inj}}{dt} &= \\ &\frac{\dot{U}_i \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \cos(\varphi_\mathrm{T}) + \frac{U_i \cdot \dot{U}_\mathrm{T}}{X_\mathrm{TRS}} \cos(\varphi_\mathrm{T}) - \frac{U_i \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\varphi_\mathrm{T}) \cdot \dot{\varphi}_\mathrm{T} \\ &- \frac{\dot{U}_j \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \cdot \cos(\theta_{ij} + \varphi_\mathrm{T}) - \frac{U_j \cdot \dot{U}_\mathrm{T}}{X_\mathrm{TRS}} \cdot \cos(\theta_{ij} + \varphi_\mathrm{T}) \\ &+ \frac{U_j \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \cdot \sin(\theta_{ij} + \varphi_\mathrm{T}) \cdot (\dot{\theta}_{ij} + \dot{\varphi}_\mathrm{T}) \end{aligned} \tag{11}$$

Comparing (10) and (11) it is clear that some parts are equal. Now we can reproduce the first integral of (10), which represents the energy function for an SMES in series connection:

$$\begin{aligned} V_\text{SMES-series} &= Q_\mathrm{inj} \\ &+ \int \Bigg\{ \left[\frac{U_i \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\varphi_\mathrm{T}) - \frac{U_j \cdot U_\mathrm{T}}{X_\mathrm{TRS}} \sin(\theta_{ij} + \varphi_\mathrm{T}) \right] \cdot (\dot{\theta}_i + \dot{\varphi}_\mathrm{T}) \\ &\left[\frac{U_i}{X_\mathrm{TRS}} \cos(\varphi_\mathrm{T}) - \frac{U_j}{X_\mathrm{TRS}} \cos(\theta_{ij} + \varphi_\mathrm{T}) \right] \cdot (\dot{U}_\mathrm{T}) \Bigg\} dt \end{aligned} \tag{12}$$

The first square bracket in (12) is equal to the active power injected, and we denote it as P_inj. The second square bracket represents the quotient $Q_\mathrm{inj}/U_\mathrm{T}$. Therefore (12) can be rewritten as:

$$V_\text{SMES-series} = Q_\mathrm{inj} + \int \left\{ P_\mathrm{inj} \cdot (\dot{\theta}_i + \dot{\varphi}_\mathrm{T}) + \frac{Q_\mathrm{inj}}{U_\mathrm{T}} \cdot \dot{U}_\mathrm{T} \right\} dt \tag{13}$$

Equation (13) includes the terms that depend on the time derivative of the SMES's control parameters. It can be expected that voltage magnitude U_T should be set to its maximum possible (i.e. constant) value in order to achieve maximum improvement in power system's dynamics (e.g. transient-stability improvement). Consequently U_T can be considered as a constant. The second controlled parameter can be—as SMES's most basic control parameter—active power P_inj. Considering constant P_inj and constant U_T, the integral in (13) can be easily solved and the energy function for an SMES is:

$$V_{\text{SMES-series}} = Q_{\text{inj}} + P_{\text{inj}} \cdot \left(\theta_i + \varphi_{\text{T}}\right) \tag{14}$$

The energy function (14) can be applied not only when P_{inj} and U_{T} are constant, but even when they are sectional-constant, in a way that is presented in [2]. With sectional-constant parameters any control strategy can be approximated.

The SPEF for the system without SMES $V_{\text{without SMES}}$ (5) can now be upgraded to represent the SPEF for the system with an SMES in series connection:

$$V_{\text{with SMES-series}} = V_{\text{without SMES}} + V_{\text{SMES-series}} \tag{15}$$

2.2 Parallel connection of SMES

Like a series-connected SMES, a parallel connected SMES is also coupled with an EPS via a converter. This converter is similar to a STATCOM, except that it can inject to the system not only reactive power but also an active power. The angle between the injected current and the bus-voltage can take any value and is not limited to the angles 90º and –90º, as it is in the case of a STATCOM.

2.2.1 Injection Model of a parallel connection.

Fig. 3 presents the scheme, the phasor diagram and the injection model of an SMES in parallel connection. Power injections are the product of the bus voltage and the injected current:

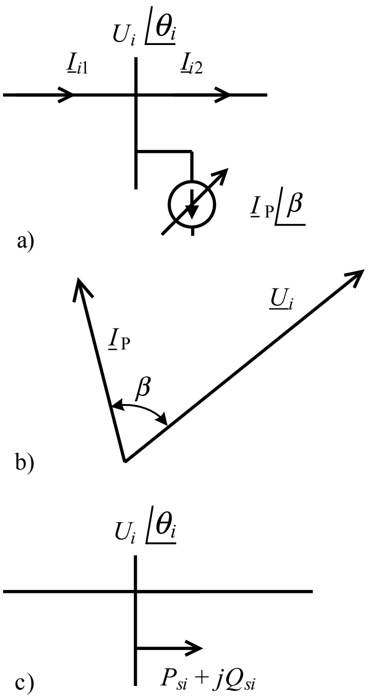

Fig. 3. Parallel connection of an SMES; a) scheme; b) phasor diagram; c) injection model.

$$P_{si} = I_{\text{P}} \cdot U_i \cdot \cos\left(\beta\right) \tag{16}$$

$$Q_{si} = I_{\text{P}} \cdot U_i \cdot \sin\left(\beta\right) \tag{17}$$

These equations are the basis for the construction of an energy function for an SMES in parallel connection.

2.2.2 Construction of an Energy Function

In a similar way to the SMES in series connection we construct its energy function for a parallel connection to be added as an additional term to the SPEF $V_{\text{without SMES}}$ (5). The first steps are the same: the active power injection, P_{si} , is multiplied by the time derivative of the voltage angle, and the reactive-power injection Q_{si} is divided by the voltage magnitude and multiplied by the time derivative of the voltage magnitude. Next, both terms are summed together and we get the following expression:

$$I_{\text{P}} \cdot U_i \cdot \cos\left(\beta\right) \cdot \dot{\theta}_i + I_{\text{P}} \cdot \sin\left(\beta\right) \cdot \dot{U}_i \tag{18}$$

The first integral of (18) can be rewritten as:

$$\int I_{\text{P}} \cdot \cos\left(\beta\right) \cdot U_i \; d\theta_i + \int I_{\text{P}} \cdot \sin\left(\beta\right) \; dU_i \tag{19}$$

The first part of (19) is the integral of the active-power injection over the angle θ_i. Assuming that $\underline{I}_{\text{P}}$ should be set to its maximum possible (i.e. constant) value in order to achieve maximum improvement of power-system dynamic and that the second controlled parameter of an SMES—its most basic parameter—P_{si} is constant, (19) can be rewritten as:

$$P_{si} \cdot \theta_i + \int I_{\text{P}} \cdot \sin\left(\beta\right) \; dU_i \tag{20}$$

Because the angle β in the integral part of (20) is not constant, we transform this integral as:

$$\begin{aligned} &\int I_{\text{P}} \cdot \sin\left(\beta\right) \; dU_i = \\ &= \int \left[I_{\text{P}} \cdot \sin\left(\beta\right) \cdot \dot{U}_i + I_{\text{P}} \cdot U_i \cdot \cos\left(\beta\right) \cdot \dot{\beta} \right. \\ &\qquad \left. - I_{\text{P}} \cdot U_i \cdot \cos\left(\beta\right) \cdot \dot{\beta} \right] dt \end{aligned} \tag{21}$$

The first two terms in square brackets can be denoted as the time derivative of the injected reactive power Q_{si}, while the third part of (21) inherits a term for injected active power that is constant. Therefore, we can rewrite (21) as:

$$\int \left(\frac{d}{dt} \left(I_{\mathrm{P}} \cdot \sin(\beta) \cdot U_i \right) \right) dt - \int P_{si}\, d\beta = \\ = I_{\mathrm{P}} \cdot \sin(\beta) \cdot U_i - P_{si} \cdot \beta = \\ = Q_{si} - P_{si} \cdot \beta \tag{22}$$

Now we can insert (22) into (20) and we get the energy function for an SMES in parallel connection with a constant current magnitude, I_{P}, and constant injected active power, P_{si}:

$$V_{\text{SMES-parallel}} = Q_{si} + P_{si} \cdot (\theta_i - \beta) \tag{23}$$

Again, like in series connection, the energy function (23) is constructed for constant control parameters. Applying the procedure presented in [2], this energy function can be applied even when the control parameters are sectional-constant. Assuming that with sectional-constant parameters any control strategy can be approximated, (23) can be applied for any control strategy.

The SPEF for the system without SMES $V_{\text{without SMES}}$ (5) can now be upgraded to represent the SPEF for the system with an SMES in parallel connection:

$$V_{\text{with SMES-parallel}} = V_{\text{without SMES}} + V_{\text{SMES-parallel}} \tag{24}$$

3 Numerical examples of transient-stability assessment using the Lyapunov direct method

The proof of correctness and a demonstration of the application of the newly constructed energy functions for an EPS comprising SMES—i.e., (15) for SMES in series connection and (24) for SMES in parallel connection—were carried out on an example of transient-stability assessment. In order to obtain the critical energy of the system a potential-energy boundary-surface (PEBS) method [9] was used, in which the critical clearing time (CCT) is the time instant when the total energy of the system along the fault-on trajectory equals the maximum of the potential energy along the same fault-on trajectory.

In order to prove the correctness of the proposed energy functions we compared the results—i.e., the CCTs—obtained by the direct method with the CCTs obtained by the simulation method, i.e., by the time-domain step-by-step simulation. In a single-machine infinite-bus (SMIB) test system the trajectory of the system is uniformly given and therefore the CCTs obtained should be equal, regardless of the method applied [9]. The data for the SMIB test system can be found in [2]. The generator is presented as a classical model with the initial voltage at BUS1 set to 1 p.u. at 30º. The disturbance is a three-phase short-circuit near BUS1, according to Fig. 4 or Fig. 5, and it is assumed to be eventually cleared, i.e., the system's post-fault configuration is identical to the pre-fault one.

3.1 Series connection of SMES

The SMIB test system with an SMES is presented in Fig. 4. The SMES is connected to the system via a series transformer with a short-circuit voltage u_{k}=3.75 % and is rated at 265 MVA. The pre-fault and fault-on values of the SMES's controllable parameters are set to 0. The CCTs were obtained using time-domain simulations, and directly with the use of the energy function (15) that includes newly proposed energy function for an SMES in series connection (14).

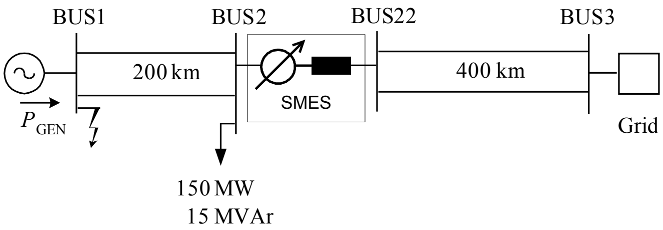

Fig. 4. Series connection of an SMES

The results for various U_{T} limits and various constant injected active power P_{inj} are presented in Table 1. The injected active power P_{inj} considered in this example is the maximum possible power, as it depends on the current through the line in which the SMES is inserted. The results are equal using both methods and in this way they validate the proposed SMES energy function (14).

Table 1. CCTs obtained in a SMIB test system including series-connected SMES.

		Simulation method	Direct method
U_{T} [pu]	P_{inj} [pu]	CCT [ms]	CCT [ms]
0	~	133	133
0.1	0	144	144
0.1	–0.09	149	149
0.2	0	153	153
0.2	–0.22	163	163
0.3	0	161	161
0.3	–0.32	174	174

Comparing the CCTs from Table 1—considering the same U_{T} limits—it can be seen that— according to the negative sign of P_{inj}— in this example the active power should be injected to the SMES in order to improve the transient-stability.

3.2 Parallel connection of SMES
The SMIB test system with an SMES in parallel connection is presented in Fig. 5. The pre-fault and fault-on values of the SMES's controllable parameters are set to 0. Here also, the CCTs were obtained using a time-domain simulation, and directly with the use of the newly proposed energy function (24).

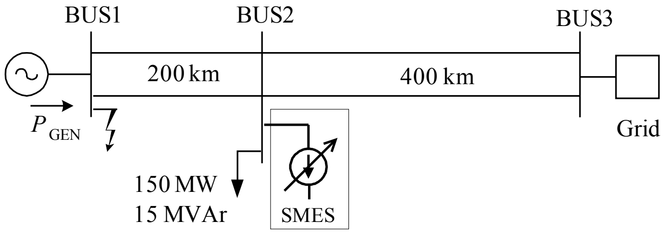

Fig. 5. Parallel connection of an SMES

The energy function (23) was developed for a parallel connection of an SMES to be included in the energy function for the EPS (24). The resulting CCTs are presented in Table 2 for various I_P limits and various active-power injections P_{si}. The voltage magnitude U_i at BUS2 during the first-swing angles' propagation considerably decreases, and consequently the constant active power P_{si} injected within this period is limited to small values. The P_{si} presented in Table 2 are the maximum possible that at the same time give maximum CCTs. The negative sign of P_{si} means that the active power flows from the system to the SMES.

Table 2. CCTs obtained in a SMIB test system including SMES in parallel connection.

		Simulation method	Direct method
I_P [pu]	P_{si} [pu]	CCT [ms]	CCT [ms]
0	0	106	106
0.1	0	113	113
0.1	–0.01	114	114
0.2	0	119	119
0.2	–0.022	120	120
0.3	0	125	125
0.3	–0.033	126	126

4 Conclusion
SMES can improve the power system's stability. In order to assess its influence on the system's dynamic behavior or to determinate the device's control strategy using direct methods, proper energy functions for an SMES is needed. In this paper two energy functions have been developed—i.e. for a series connection and for a parallel connection of an SMES. Both energy functions are constructed as an additional term that can be added to any existing structure-preserving energy function. The correctness of the proposed energy functions was tested with numerical examples on a single-machine infinite-bus test system. The resulting CCTs were compared to the reference CCTs obtained by the simulation method. The equality of the CCTs proves the correctness of the proposed energy functions. Future work will be focused on determining the control strategy of an SMES, based on a maximization of the time derivative of the power system's energy function along the system trajectory.

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